Universitat Pompeu Fabra Barcelona
Department of Economics and Business

Economic Working Paper Series
Working Paper No. 1463

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Updated version: October 2018

(October 2014)

Balance Sheet Channel with Information and Trading Frictions

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First draft: October 2014. This draft: October 2018

Abstract

We consider a model of the balance sheet channel à la Kiyotaki and Moore (1997) but allow agents to trade claims contingent on aggregate states. We show that the interaction of information dispersion about aggregate states with trading frictions in secondary claims markets generates mispricing of aggregate risk, distorts the demand for state-contingent claims and limits aggregate risk-sharing, thereby giving rise to the balance sheet channel. The magnitude of aggregate fluctuations becomes tied to the severity of information-trading frictions and, as they vanish, the balance sheet channel disappears. Thus, the model suggests that the functioning of secondary claims markets has important implications for business cycles. Importantly, the laissez-faire equilibrium is constrained inefficient *because* information-trading frictions generate rent-extraction in secondary claims markets. Optimal policy targets the inefficiency at its source by promoting *both* issuance *and* trade of state-contingent claims in markets.

JEL: E32, E44, G10.

Keywords: balance sheet channel; risk-sharing; information dispersion; trading frictions; mechanism design; financial regulation.

*Affiliation: CREi, UPF, and Barcelona GSE. I acknowledge financial support from the Spanish Ministry of Economy and Competitiveness Grant (ECO2014-54430-P) and the Barcelona GSE Seed Grant. I thank Fernando Broner, Sebastian Di Tella, William Fuchs, Pierre-Olivier Gourinchas, Yuriy Gorodnichenko, Ben Hébert, Christian Hellwig, Pablo Kurlat, Alberto Martin, Demian Pouzo, Victoria Vanasco, Jaume Ventura and seminar/conference participants at Bocconi University, Barcelona GSE, CREi and Universitat Pompeu Fabra, Paris School of Economics, the Federal Reserve Board, the World Bank, EIEF - Rome, UC Berkeley, Universitat de Barcelona, the Society for Economic Dynamics Meetings, the 7th Joint French Macro Workshop, the 4th UTDT Economics Conference and the European Winter Meeting of the Econometric Society for helpful suggestions and comments. Yimei Zou and Joon Sup Park have provided excellent research assistance.

1 Introduction

The global recession of the last decade has once again underscored the important role that financial frictions can play in the amplification and propagation of macroeconomic shocks. We were reminded that the concentration of macroeconomic risks on balance sheets of leveraged agents can cause small shocks to be magnified into full-blown financial crises. For example, the disproportionate exposure of leveraged households to real estate risks is thought to have been responsible for the large and prolonged drop in consumption and employment in the US and Europe, and similar arguments have been made of the contribution of the financial sector.[1] Although the literature on the *balance sheet channel* has long recognized that concentrated risks can give rise to powerful feedback effects, it is not well understood why economic agents may choose to retain so much risk on their balance sheets.[2] Answering this question is especially important for understanding the design of optimal regulatory policy.

With a few exceptions, much of the existing literature on the balance sheet channel shuts down risk-sharing considerations by simply imposing that borrowers and lenders cannot trade claims contingent on aggregate states of the economy. However, the typical agency- or information-based explanations for why the provision of such claims in the market may be limited do not really apply to aggregate states, since atomistic agents cannot influence aggregate outcomes and in many cases there are readily available indicators that can be used for contracting (e.g. GDP, real estate indices, inflation).[3] This is particularly problematic since our canonical models predict that the agents' ability to trade contingent claims, even in the presence of financial constraints, can generate sufficient risk-sharing so as to eliminate the balance sheet channel altogether.[4] Consistent with this, there have been calls for policy-makers to promote risk-sharing through interventions in contracts or in markets, as a way to stabilize the business cycle.[5] Arguably, to even consider whether such interventions are desirable, we need a theory for why risk-sharing may be endogenously limited or absent, even when trade in contingent claims is feasible. The goal of this paper is to provide such a theory.

[1] See Mian and Sufi (2010) and Adrian and Boyarchenko (2013) for the importance of household and financial firm balance sheets respectively in the US Great Recession;Mishkin (1978), Bernanke (1983), and Olney (1999) provide support for the role of balance sheets in the Great Depression.

[2] Balance sheet channel broadly refers to the feedback effects between the health of borrowers' balance sheets (e.g. net worth) and general economic activity (e.g. asset prices, output). See the seminal papers by Bernanke and Gertler (1989), and Kiyotaki and Moore (1997). Some recent contributions include He and Krishnamurthy (2011) and Brunnermeier and Sannikov (2014).

[3] See the seminal papers by Hölmstrom (1979), Townsend (1979) and Myers and Majluf (1984).

[4] This point has been formally made by Krishnamurthy (2003) and, more recently, by Di Tella (2017b).

[5] In a testimony to the US congress, Mian (2013) states that mortgage contracts contingent on real-estate indices would have ameliorated the deleveraging-aggregage demand cycle of the Great Recession. Relatedly, Case et al. (1991) and Shiller (1994) argue that promoting markets for real estate derivatives could have a stabilizing role on business cycles.

Our theory is built on two key ingredients: information dispersion about aggregate states and imperfect competition in secondary claims markets. We show that under general conditions the interaction of these frictions generates mispricing of aggregate risk, distorts the demand for state-contingent claims and limits aggregate risk-sharing, with important and novel implications for business cycles and the design of optimal corrective policy.

Our starting point is a canonical model of the balance sheet channel à la Kiyotaki and Moore (1997), but where we allow agents to trade claims contingent on aggregate states. In the model, financially constrained entrepreneurs (borrowers) undertake productive long-term projects and finance them by issuing claims to investors (lenders). The cashflows of entrepreneurs' projects are exposed to an aggregate shock, which *may* generate fluctuations in entrepreneurs' net worth and trigger liquidations of productive capital, thereby destabilizing the prices of capital and aggregate output through the balance sheet channel. Whether this occurs, however, depends crucially on the extent of risk-sharing between entrepreneurs and investors; namely, the type of claims with which entrepreneurs finance projects.

We depart from the canonical model by introducing frictional secondary markets, in which investors may want to re-trade their claims in order to exploit gains from trade. We introduce a *trading friction* by supposing that secondary markets are not perfectly competitive: an investor who wants to re-trade his claims can only do so with finitely many other investors. We introduce an *information friction* by supposing that investors observe dispersed private signals about the aggregate state of the economy. In what follows, we use the shorthand "information-trading frictions," as it is the interaction of information dispersion with imperfect competition that will matter, and not each friction individually.

In this setting, even though entrepreneurs are risk-neutral, due to a financial friction, they endogenously become risk-averse with respect to aggregate shocks to their net worth.[6] As we explain shortly, a novel feature of our framework is that, even though investors are risk-neutral, due to information-trading frictions, they endogenously become risk-averse with respect to aggregate shocks to their tradable claims. We then ask the following questions. How does the equilibrium resolve the tension between the entrepreneurs' desire to insure the fluctuations in their net worth and the investors' desire to hold claims that are more stable in value? Does the equilibrium resolve this tension efficiently and, if not, what is the underlying source of inefficiency and the optimal corrective policy?

We begin the equilibrium analysis with a useful benchmark economy, in which information-trading frictions are shut down. In this economy, the entrepreneurs' claims are priced (actuarially) fairly in secondary markets, which in turn ensures that full risk-sharing is attained between entrepreneurs and investors. By borrowing more against the good aggregate state –

[6]That financial frictions can generate risk-aversion in firm behavior goes back to Froot et al. (1993).

when their cashflows are high,– and less against the bad aggregate state – when their cashflows are low,– entrepreneurs are able to insulate their net worth from fluctuations, which endogenously stabilizes the equilibrium prices of capital and aggregate output. Though stark, this benchmark is a clear illustration of how the agents' ability to trade contingent claims can mute the impact of aggregate shocks and eliminate the balance sheet channel altogether, as has also been pointed out by Krishnamurthy (2003) and Di Tella (2017b).

We then consider the economy with information-trading frictions. An important step towards equilibrium characterization is the study of trading arrangements by which investors re-trade their claims in secondary markets. We employ a mechanism design approach and allow each investor to design the optimal mechanism by which to re-trade his claims; that is, the investor decides the payments to collect from and the claims to allocate to finitely many other investors, who have dispersed signals about the aggregate state. One of the main results of the paper is that state-contingent claims are systematically mispriced and potentially misallocated in secondary markets, despite the optimal design of trading arrangements. Intuitively, the interaction of information dispersion with trading frictions gives each trader (i.e. buyer) some market power over his private information. This allows the traders to extract rents from trading arrangements, but only if claims traded in these arrangements are contingent on the aggregate state, since only then are the traders' private signals useful for valuing them. By designing optimal trading arrangements, investors minimize but cannot eliminate these rents.[7]

As each investor rationally anticipates that his claims may be mispriced and misallocated in secondary markets, the information-trading frictions alter the aggregate demand for state-contingent claims ex-ante. As a result, the equilibrium prices of claims faced by entrepreneurs become distorted away from fair pricing. Importantly, the direction of this distortion depends on the aggregate supply of claims by the entrepreneurial sector; namely, the price of a claim for the good aggregate state is below actuarially fair when the entrepreneurial sector as a whole issues relatively more claims against that state, and vice versa. Thus, although financial frictions generate benefits for entrepreneurs from insuring net worth fluctuations, information-trading frictions in secondary markets render the provision of such insurance *privately* costly to investors, as reflected in distorted claims prices.

In equilibrium, the magnitude of aggregate fluctuations becomes tied to the severity of information-trading frictions. When these frictions are severe, the claims prices are too distorted, and entrepreneurs optimally choose to finance their projects with non-contingent

[7]This result is in sharp contrast to Cremer and McLean (1988) who famously show that, whenever the traders' types are correlated (as in our setting), traders are unable extract rents from the mechanism. We show, however, that this result does not hold whenever traders can threaten to leave the mechanism *after* learning their mechanism allocations (prices and quantities), which is natural for our application to financial markets; see related literature for details.

claims and expose their net worth to large fluctuations. Here, as in Kiyotaki and Moore (1997) and Brunnermeier and Sannikov (2014), the aggregate shock to entrepreneurs' cash-flows gets amplified and propagated through the balance sheet channel. On the other hand, when information-trading frictions are moderate, the equilibrium features partial risk-sharing. Entrepreneurs optimally finance themselves with contingent claims, but obtain incomplete insurance from investors as the latter still perceive its provision costly. In this case, the effect of the balance sheet channel is dampened; indeed, as information-trading frictions vanish, the equilibrium converges to the benchmark economy where the balance sheet channel is not operative at all. These results suggest that well-functioning secondary claims markets are important for macroeconomic stability; in particular, we should expect more risk-sharing and less amplification/propagation of aggregate shocks whenever agents can insure themselves in deep, well-developed secondary markets, and vice versa. This observation is consistent with the recent calls for policy makers to promote "liquidity" in secondary markets for real-estate derivatives (Case et al., 1991; Shiller, 1994) or indexed government bonds (Caballero, 2003), as a way to stabilize the business cycle. As we discuss next, by taking a stance on the source of secondary market "illiquidity," we are able to give more structure to these arguments.

Our next main result is that the equilibrium is constrained inefficient and warrants policy intervention. To establish this result, we solve the problem of a social planner, who chooses the agents' allocations to maximize social welfare but who is constrained by the same frictions as the agents. This allows us to identify the underlying source of inefficiency in this economy; namely, that information-trading frictions generate rent-extraction in secondary markets. Intuitively, when pricing entrepreneurial claims and designing trading arrangements, each investor tries to minimize the rents that others extract from him in secondary markets. In aggregate, however, these rents simply generate ex-post transfers uand do not lead to a welfare loss. This inefficiency manifests itself in equilibrium through two distortions. First, when designing his trading arrangement, each investor minimizes the mispricing of his claims, and may find it optimal to allocate the claims inefficiently. Second, the potential for mispricing in secondary markets distorts the claims prices faced by entrepreneurs and, thus, leads them to forgo risk-sharing and distort their investment/liquidation decisions.

Optimal policy overcomes the inefficiency with two ex-ante subsidies, financed by levying lump sum taxes on investors and entrepreneurs. First, there is a subsidy that incentivizes investors to design more efficient trading arrangements, in order to correct the misallocation of claims in secondary markets. Second, there is a subsidy that incentivizes entrepreneurs to issue more contingent claims, in order to correct the distortion to risk-sharing due to mispricing of claims. As a result, though there are still fluctuations due to the direct aggregate shock to entrepreneurs' cashflows, optimal policy eliminates the amplification and propagation of the shock through the balance sheet channel. These findings would be difficult to rationalize in canonical models, which are silent on the role of secondary markets in facilitating risk-sharing. Thus, consistent with the theory of second-best, understanding the underlying source of inefficiency that limits risk-sharing is essential for the design of optimal corrective policy.

The information-trading frictions, which are at the heart of this paper, are consistent with different strands of stylized evidence. First, there is growing evidence of substantial information heterogeneity among economic agents about many aggregate variables. For example, Mankiw et al. (2003) find substantial heterogeneity in inflation forecasts among professional forecasters, economists and consumers, and Dovern et al. (2012) document similar findings in a cross-country study of surveys of professional forecasters. More recently, Coibion and Gorodnichenko (2012, 2015) provide extensive evidence that information disagreements are pervasive across a variety of population subgroups and macroeconomic variables. Second, there is ample evidence that trading frictions are pervasive in financial markets. In practice, if an entrepreneur wanted to issue a claim contingent on an aggregate state, she could potentially do so in two ways. She could issue a security, say a contingent bond, which would naturally trade in decentralized, frictional over-the-counter markets (Duffie, 2010; Rocheteau and Weill, 2011). Or, she could issue a derivative contract contingent on the aggregate state. Though some derivatives markets are well-developed, e.g. standardized currency, commodity futures; many others still remain extremely thin, e.g. real-estate, GDP. Shiller (1994) points at history dependence in the formation of market institutions for the slow evolution of these markets.[8] Moreover, the theory developed here implies that these information-trading frictions matter for aggregate growth and volatility. It predicts that aggregate investment and sensitivity of business cycles to shocks depends on the development of secondary markets in which the risks associated to these shocks are traded and the information dispersion in the economy.

Related literature

Krishnamurthy (2003) was the first to formally show that, even in the presence of financial constraints, the entrepreneurs' ability to borrow with contingent claims can generate enough risk-sharing so as to eliminate the balance sheet channel of Kiyotaki and Moore (1997). He proposed limited commitment on the side of the lenders as a way to limit insurance and bring back the balance sheet channel. However, in many models (including this one), lender commitment is non-binding, since debt write-downs (which do not require lenders to make any payments) are sufficient to insure the entrepreneurs against bad aggregate states. More

[8]In referring to the process of derivatives market formation, Shiller (1994) states that "the history of basic economic institutions is one of punctuated equilibrium, where basic economic institutions remain largely unchanged for long periods of time, only to be superseded by new institutions whose advent can only be attributed to innovation." Tufano (1989) provides a narrative for the process of financial innovation.

recently, Di Tella (2017b) considers a model of the balance sheet channel à la Brunnermeier and Sannikov (2014), allows the agents to trade claims contingent on aggregate states, and shows that shocks to idiosyncratic volatility, which tighten the entrepreneurs' financial constraints in bad aggregate states can induce entrepreneurs to become overly exposed to aggregate risk. Bocola et al. (2018) argue that insurance between entrepreneurs and lenders may naturally be limited for crisis states since in those states not only is the entrepreneurs' net worth low but also the lenders' labor income.[9] In contrast to these papers, we focus on the role of secondary market (i.e. information-trading) frictions, and the distortions they introduce to aggregate risk-sharing, investment and economic activity. As we discuss next, understanding the underlying source of these distortions has important normative implications.

Due to its normative focus, this paper relates to a growing literature that studies pecuniary externalities in models with financial constraints (Caballero and Krishnamurthy, 2003; Lorenzoni, 2008; Korinek, 2011; Di Tella, 2017a). Broadly, this literature has focused on externalities arising either because the asset prices enter into financial constraints or because asset price movements have distributional consequences (Dávila and Korinek, 2017). The pecuniary externality we identify is different, as it arises because information-trading frictions generate rent-extraction in secondary claims markets; indeed, the equilibrium of our benchmark economy without information-trading frictions is constrained efficient. As a result, the policy implications we obtain are novel too. For instance, we show that the optimal policy consists of interventions both in contracts and in markets. In contrast, policies such as capital requirements or leverage limits, often found to be optimal in models with financial constraints, are sub-optimal in our setting, as they do not address the underlying source of inefficiency.[10]

In a related paper, Albagli et al. (2017) study the role of equity market frictions in shaping aggregate investment.[11] To do so, they embed the noisy REE model developed by Albagli et al. (2011) into a general equilibrium setting and show that the lack of information aggregation, resulting from noise-trading and limits to arbitrage, leads to rent-seeking behavior by incumbent shareholders, which distorts aggregate investment relative to first-(and second-)best. Our approaches are complementary. Like them, we emphasize how the combination of information dispersion and trading frictions distorts economic activity through rent-extraction, though in our setting rent-extraction operates through a channel different to theirs. Different from them, we focus on distortions that operate through aggregate risk-sharing. Moreover, we employ a mechanism design approach, which helps us ensure that our results come from the primi-

[9]Rampini and Viswanathan (2010, 2016) also study the interaction between financial constraints and risk-sharing, though they do not focus on the balance sheet channel.

[10]Recently, Kurlat (2018) shows that policy implications of the canonical models with financial constraints are reversed if the markets for capital suffer from an adverse selection problem à la Akerlof (1970).

[11]Also, see Bond et al. (2012) for an overview of the literature on real effects of financial markets.

tive information-trading frictions rather than particular restrictions on the trading protocols, which is crucial for understanding the normative properties of our economy.

This paper is related to the classical literature on mechanism design under asymmetric information (Myerson and Satterthwaite, 1983; Hagerty and Rogerson, 1987). It is well-known that in environments with correlated types (as in our case, due to dispersed signals), there exist trading mechanisms that are both efficient and able to extract the mechanism participants' (i.e. the traders') full surplus (Cremer and McLean, 1988; McAfee and Reny, 1992). We show, however, that this result does not hold whenever traders can threaten to leave the trading mechanism *after* learning their mechanism allocations.[12] This generates a set of "ex-post" participation constraints, which we believe are natural for our application to asset markets. As a result, the claims traded in the optimal mechanism can now be both mispriced and allocated inefficiently among agents, a feature that is critical for our main results.

This paper is also related to the large literature on security design in the presence of asymmetric information (Gorton and Pennacchi, 1990; Nachman and Noe, 1994; DeMarzo and Duffie, 1999; Biais and Mariotti, 2005; Dang et al., 2012). We contribute to this literature on two fronts. First, we show that information dispersion can also result in mispricing and misallocation of aggregate risk, albeit the extent to which this occurs depends on the severity of trading frictions in secondary markets, a feature that is absent in standard models with adverse selection.[13] Importantly, to show this, we employ a mechanism design approach, as in a setting with dispersed information (i.e. correlated types) the choice of trading protocols can play an important role. Second, we study the general equilibrium effects of information dispersion through its effect on mispricing and misallocation of aggregate risk, which allows us to identify novel inefficiencies and policy implications. Closer in spirit to our paper is the recent work by Hartman-Glaser and Hébert (2017), who consider a security design game between many banks and households, where banks can offer securities to households indexed to real-estate prices, so as to share aggregate risk. They show that, if banks are better informed about the quality of the index, there can exist an equilibrium in which all banks optimally offer unindexed securities, due to households' fear that indexed securities are 'lemons.' Besides different mechanisms, our focus is not on whether there is indexation or not, but rather on how information frictions affect the pricing and allocation of aggregate risk, what this implies for aggregate investment and, importantly, for the design of optimal policy.

[12] Relatedly, Compte and Jehiel (2009) study a bargaining problem where each participant, who is privately informed about her outside option, can quit the mechanism and take the outside option at any point in time.

[13] Biais and Mariotti (2005) study the interaction between information asymmetries and market power, in a setting that features a privately informed security seller and an uninformed but monopolistic buyer. Relatedly, Axelson (2007) considers a security design problem with imperfect competition and privately informed buyers, where securities are traded in a second-price sealed-bid auction.

More broadly, the idea that the "liquidity" of financial claims with which borrowers finance themselves can affect aggregate economic activity is related to Hollifield and Zetlin-Jones (2017), who show that banks may optimally distort maturity- and risk-transformation in order to make their liabilities more effective as a medium of exchange in frictional goods markets à la Lagos and Wright (2005). The importance of well-functioning secondary claims markets has also been stressed by Broner et al. (2008, 2010), who show that they can help economies mitigate problems due to weak enforcement institutions and reduce sovereign risk.

Finally, this paper belongs to a long tradition in economics of studying the aggregate implications of information dispersion (Lucas, 1972; Lorenzoni, 2009; Angeletos and La'O, 2013; Gaballo, 2017). Relative to this literature, this paper emphasizes the interaction between information dispersion with imperfect competition in financial markets, which also relates our work to the literature on asset pricing with heterogeneous information and limits to arbitrage (Grossman and Stiglitz, 1980; Albagli et al., 2011). In our model, it is the finiteness of traders within each trading arrangement, rather than the presence of noise traders, which prevents information aggregation and distorts asset prices, and which in turn facilitates the mechanism design approach we employ and our normative analysis.

The paper is organized as follows. In Section 2, we present the model. In Sections 3 and 4, we characterize the equilibrium and its implications for risk-sharing, investment and output. In Section 5, we study the efficiency properties of equilibrium and implications for policy. We conclude in Section 6. All proofs are relegated to the Appendix.

2 The Model

There are three periods, indexed by $t \in \{0, 1, 2\}$, and two sets of agents, entrepreneurs and investors, each of unit mass. There are two goods, perishable consumption and durable capital.

Preferences and Endowments. Entrepreneurs are risk-neutral with lifetime utility $U^E = \mathbb{E}\{c_0^E + c_1^E + c_2^E\}$, where c_t^E is an entrepreneur's consumption in period t. Investors are also risk-neutral, but they may be subject to preference shocks at $t = 1$. Their lifetime utility is $U^I = \mathbb{E}\left\{c_0^I + \beta\left(c_1^I + c_2^I\right)\right\}$, where c_t^I is an investor's consumption in period t and where $\beta \in \{0, 1\}$ denotes his preference type, distributed independently among investors with $\lambda \equiv \mathbb{P}(\beta = 0) \in \left[0, \frac{1}{2}\right]$.[14] Thus, λ is also the fraction of investors who are shocked at $t = 1$.

Entrepreneurs have access to long-term productive projects, but have no endowments to fund the projects on their own. They can finance them by issuing claims to investors, each of whom has a large endowment e of the consumption good at $t = 0$.

[14] The sole purpose of these preference shocks is to generate gains from trade in secondary markets.

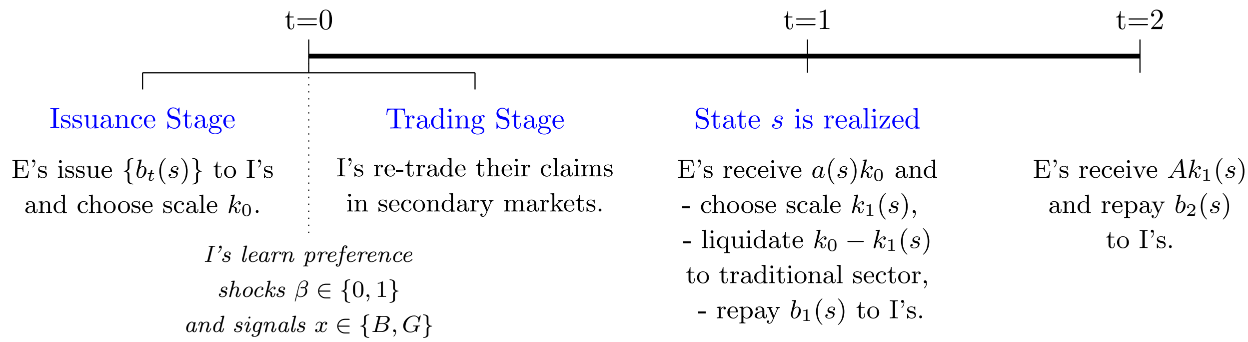

Figure 1: **Timeline**. E's denotes the entrepreneurs, whereas I's denotes the investors. All consumption takes place at the end of each period.

Technology. At $t = 0$, each entrepreneur creates k_0 units of capital by spending $\chi(k_0)$ units of the consumption good, where $\chi(\cdot)$ is increasing and convex with $\chi(0) = 0$. At $t = 1$, she receives ak_0 consumption goods and chooses continuation scale k_1 by liquidating or buying capital in a competitive capital goods market at price p. Finally, at $t = 2$, the entrepreneur receives Ak_1 consumption goods. The intermediate "cashflow" a depends on an aggregate state $s \in \{l, h\}$, where $a(l) < a(h)$ and $\pi(h) \equiv \mathbb{P}(s = h) \in (0, 1)$, whereas the final cashflow A is for simplicity deterministic.

The units of capital liquidated by the entrepreneurial sector can be absorbed by a "traditional" sector, composed of a mass of competitive firms, owned by the investors. These firms are less productive than the entrepreneurs: by employing $\widehat{k}_1$ units of capital at $t = 1$, each firm produces $gA\widehat{k}_1$ units of consumption goods at $t = 2$, where $g \in (0, 1)$.[15]

Financial Markets. Period $t = 0$ is divided into two stages. First, there is an *issuance stage*, where entrepreneurs issue claims to investors in a competitive market that prices each unit of consumption at date t and state s at $q_t(s)$. After the issuance stage, the investors learn their preference shocks and receive dispersed signals about the aggregate state. At this point, there is a *trading stage*, where investors can re-trade claims in a frictional secondary market.[16]

Agents consume at the end of each period, and the economy's timeline is depicted in Figure 1. Our economy is affected by two types of frictions. First, there is a financial friction, which is standard in the literature on the balance sheet channel, as it creates a role for entrepreneurial net worth to affect investment. Second, there are information-trading frictions, which are at the heart of this paper. We describe these next.

[15]This technological assumption is standard in the literature on the balance sheet channel (Kiyotaki and Moore, 1997; Lorenzoni, 2008).

[16]That the issuance market is competitive is convenient, but not essential. What is crucial is that the claims issued by entrepreneurs are re-traded in a frictional market at some point before they mature.

Financial friction. We assume that an entrepreneur can always walk away with her $t = 2$ output. Let $b_t(s)$ denote the claims issued by the entrepreneur for date t and state s. Then, any payments she can credibly promise to make must satisfy the following limited pledgeability constraint: $b_2(s) \leq 0$ for $s \in \{l, h\}$.

Information-trading frictions. We assume that, after the issuance stage, each investor privately learns her preference type $\beta \in \{0, 1\}$ and a signal $x \in \{B, G\}$ about the aggregate state s (information friction). Conditional on the aggregate state, the signals are distributed independently among investors, and $\mathbb{P}(s = h|x = G) > \mathbb{P}(s = h|x = B)$, i.e. investors with Good (Bad) signals are optimistic (pessimistic) about the state. An investor who wants to re-trade his claims in secondary markets must post them in a *trading arrangement*, which is a mechanism that allocates these claims to and collects payments from a finite number $n \geq 1$ of randomly selected other investors, whom we refer to as *traders* (trading friction). Importantly, the format of trading arrangements is not arbitrary and is chosen optimally by each investor.

To facilitate the exposition, in the main analysis, we impose directly that only the impatient investors (fraction λ) post their claims in trading arrangements, whereas the remaining investors (fraction $1-\lambda$) are traders; for convenience, we set $n\lambda = 1-\lambda$ in the main text, so that there is an exact match between trading arrangements and traders. The microfoundation for such sorting of investors is provided in Appendix C.[17] All trades are executed simultaneously.

2.1 Preliminaries

We will assume throughout that the investors' endowment of consumption goods is sufficiently large so that they effectively have "deep pockets."

Assumption 1 *The endowment satisfies:* $e > 2\chi(k)$ *for* $k > 0$ *s.t.* $\chi(k) = (a(h) + A)k$.

This will ensure that in equilibrium the patient investors' consumption is positive at all dates. It will in turn imply that the equilibrium risk-free interest rate equals to one, i.e. claims prices satisfy:

$$q_1(s) = q_2(s) = q(s), \ \sum_s q(s) = 1. \tag{1}$$

Since in equilibrium the price of capital will be bounded by the discounted return to capital in the traditional and the entrepreneurial sectors, another implication of Assumption 1 is that:

$$gA \leq p(s) \leq A \ \forall s. \tag{2}$$

[17] We suppose common knowledge of gains from trade within each trading arrangement, which by arguments akin to Milgrom and Stokey (1982) ensures that there is no trade between investors of same preference type.

To simplify the analysis, we impose properties (1) and (2) in the agents' problems, and we verify them in Appendix B.

We will also make the following technological assumptions, which jointly ensure that the entrepreneurs will never find it optimal to create a unit of capital only to liquidate it prematurely with probability one (w.p.1), or to borrow with non-contingent claims and at the same time avoid liquidations w.p.1.

Assumption 2 *The economy's technology satisfies:*

(i) $\chi(k) < \sum_s \pi(s)a(s)k$ *for* k *s.t.* $\chi'(k) = \sum_s \pi(s)(a(s) + gA)$.

(ii) $a(l)k < \chi(k)$ *for* k *s.t.* $\chi'(k) = \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}(a(h) + A) + \left(1 - \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}\right)(a(l) + gA)$.

(iii) $\sum_s \pi(s)a(s) < a(l) + gA < a(h)$.

These assumptions do not drive our main results, but they simply allow us to focus the equilibrium analysis on the economically most interesting cases.

Finally, it is useful to define a notion of first-best scale of investment, k^{FB}, which would arise in an economy without information-trading frictions and with fully pledgeable cashflows:

$$\chi'(k^{FB}) = \sum_s \pi(s)(a(s) + A). \tag{3}$$

As we will see, k^{FB} will provide an upper bound on the investment scale in our economy.

2.2 Entrepreneurs' Problem

All entrepreneurs are identical. The representative entrepreneur takes as given the prices of claims $\{q(s)\}$ and capital $\{p(s)\}$, and chooses the investment scale k_0, the continuation scales $\{k_1(s)\}$, and the claims $\{b_t(s)\}$ to maximize her lifetime welfare:

$$\sum_s q(s)\left(b_1(s) + b_2(s)\right) - \chi(k_0) + \sum_s \pi(s)\left[(a(s) + p(s))k_0 - p(s)k_1(s) - b_1(s) + Ak_1(s) - b_2(s)\right] \tag{P1}$$

subject to the following set of constraints:

$$\chi(k_0) \leq \sum_s q(s)\left(b_1(s) + b_2(s)\right) \tag{4}$$

$$p(s)k_1(s) + b_1(s) \leq (a(s) + p(s))k_0 \ \forall s, \tag{5}$$

$$b_2(s) \leq 0 \ \forall s, \tag{6}$$

$$0 \leq k_1(s) \ \forall s. \tag{7}$$

The first two constraints simply impose consumption non-negativity. Constraint (4) states that at $t = 0$ the entrepreneur's expenditures on investment cannot exceed the funds raised by issuing claims to investors. Constraint (5) states that at $t = 1$ the entrepreneur's expenditure on claims repayments plus net purchases of capital cannot exceed the output of her project. Constraint (6) is the limited pledgeability friction and constraint (7) requires that the continuation scale be non-negative. The entrepreneur's consumption non-negativity constraint $t = 2$ is implied by limited pledgeability and non-negativity of continuation scale.

2.3 Traditional Sector Firms' Problem

The problem of the traditional sector firms is simple. At $t = 1$ and state s, each firm in this sector takes the price of capital $p(s)$ as given and chooses to purchase $\widehat{k}_1(s) \geq 0$ units of capital to maximize its discounted profits:

$$\Pi(s) = gA\widehat{k}_1(s) - p(s)\widehat{k}_1(s). \tag{8}$$

Since in equilibrium the price of capital will be (weakly) greater than gA, these firms will only demand capital when $p(s) = gA$. To finance capital purchases, they raise funds from the investors, who in equilibrium will have sufficient resources to fund these activities (see Appendix B). Finally, since these firms earn zero discounted profits in each state, the investors' claims on this sector too have value of zero.

2.4 Investors' Problem

At the issuance stage, all investors are identical. The representative investor takes as given the claims prices $\{q(s)\}$, and he chooses to purchase claims $\{d_t(s)\}$ and the trading arrangement in which to re-trade them, denoted by μ.[18] The set of feasible trading arrangements from which the investor chooses is denoted by $\mathcal{M}$ and will be described shortly.

At the trading stage, the investors learn their preferences and signals about the aggregate state. An impatient investor posts the claims $\{d_t\,(s)\}$ in the trading arrangement μ that he has designed, which gives him a net expected payoff $V\left(\{d_t\,(s)\}\,,\mu\right)$. A patient investor does not post his claims, but he participates as a trader in the trading arrangements of other investors, which gives him a net expected payoff of W.

[18] The assumption that the trading arrangements are chosen ex-ante avoids the unnecessary complications of analysing the mechanism design problem of a privately informed designer.

Thereferore, the investor's problem is to choose $\{d_t(s)\}$ and μ to maximize lifetime welfare:

$$e - \sum_s q(s)\left(d_1(s) + d_2(s)\right) + \lambda V\left(\{d_t(s)\}; \mu\right) + (1-\lambda)\left(W + \sum_s \pi(s)\left(d_1(s) + d_2(s)\right)\right), \quad \text{(P2)}$$

where recall that λ is the ex-ante probability that an investor becomes impatient. This problem is subject to two sets of constraints. First, the claims purchases must be consistent with the investor's consumption being non-negative, i.e. $e \geq \sum_s q(s)(d_1(s) + d_2(s))$ and $d_t(s) \geq 0 \; \forall t, s$. Second, the trading arrangement μ must be in the feasible set $\mathcal{M}$, which we describe next.

Feasible Trading Arrangements

When designing a trading arrangement, the investor anticipates that, if impatient, he will have to sell his claims to privately informed traders (the patient investors). He designs an optimal mechanism, which solicits reports from the traders, and then allocates claims to and collects payments from them based on these reports. We focus on direct revelation mechanisms implementable as Bayesian Nash equilibria, in which each trader is willing to report his signal truthfully to the mechanism, given that other traders also report truthfully.

Because the risk-free rate between $t = 1$ and $t = 2$ equals one, for the design problem it suffices to keep track of the total claims held by the investor for state s, which we will denote by $v(s) \equiv d_1(s) + d_2(s)$. Let $i \in \{1, ..., n\}$ index the traders matched with a given trading mechanism, and let θ^i denote the report of trader i, who recall has a private signal $x^i \in \{B, G\}$. The reports and the signals of traders other than i are for short denoted by θ^{-i} and x^{-i} respectively. Upon collecting the traders' reports $(\theta^1, ..., \theta^n)$, the mechanism proposes allocation $\mathcal{A}^i(\theta^i, \theta^{-i}) \equiv \left(\omega^i(\theta^i, \theta^{-i}), \{v^i_s(\theta^i, \theta^{-i})\}\right)$ to trader i, where $\omega^i(\theta^i, \theta^{-i})$ is the payment that trader i makes to the mechanism and $v^i_s(\theta^i, \theta^{-i})$ are the units of claims for state s that the mechanism transfers to the trader; when there is no trade with trader i, we say that the allocation is empty.[19] The trader can accept or reject this allocation, and his payoff from accepting it is:

$$U^i\left(x^i, \mathcal{A}^i\left(\theta^i, \theta^{-i}\right)\right) = \sum_s v^i_s\left(\theta^i, \theta^{-i}\right) \cdot \mathbb{P}\left(s | x^i, \mathcal{A}^i\left(\theta^i, \theta^{-i}\right)\right) - \omega^i\left(\theta^i, \theta^{-i}\right), \quad (9)$$

where $Prob\left(s | x^i, \mathcal{A}^i(\theta^i, \theta^{-i})\right)$ is trader i's belief that the state is s, given his signal and his allocation, where the latter may contain information about other traders' signals and, thus, about the aggregate state.

[19]The implicit assumption that the mechanism does not condition the allocations on the signal of the selling investor is not essential for our main results (see Appendix A).

Naturally, the trading mechanism needs to satisfy a set of participation, incentive compatibility and feasibility constraints.

Participation (PC). We impose a form of ex-post participation constraint by supposing that trader i has the right to choose not to participate in the trading mechanism *after* learning his allocation, i.e. he participates if and only if the payoff in equation (9) is non-negative. Because the mechanism can always choose the empty allocation, it is without loss to focus on mechanisms which ensure that each trader participates in equilibrium:

$$U^i\left(x^i, \mathcal{A}^i\left(x^i, x^{-i}\right)\right) \geq 0 \ \forall i, x^i, x^{-i}. \tag{10}$$

Incentive compatibility (IC). In order for a trader to report her signal truthfully, given that the other traders do so as well, it must be that:

$$x^i \in \arg\max_{\theta^i} \mathbb{E}\left\{\max\left\{0, U^i\left(x^i, \mathcal{A}^i\left(\theta^i, x^{-i}\right)\right)\right\} | x^i\right\} \ \forall i, x^i. \tag{11}$$

Note that the IC's take into account that each trader can potentially deviate to both misreport his signal and then choose not to trade.

Feasibility (FC). The claims that the mechanism allocates to the traders cannot exceed the claims available to the investor:

$$\sum_i v_s^i(\theta^i, \theta^{-i}) \leq v\left(s\right) \ \forall s, \theta^i, \theta^{-i}. \tag{12}$$

Assumption 1 will ensure that in equilibrium the mechanism allocations are consistent with the investors' consumption non-negativity (see Appendix B). If out of equilibrium an investor cannot afford to participate in the mechanism, we assume that he gets the empty allocation.

A trading arrangement is thus defined by $\mu = \{\mathcal{A}^i\left(x^i, x^{-i}\right)\}_{i,x^i,x^{-i}}$, and the set of feasible trading arrangements $\mathcal{M}$ consists of all such μ's satisfying (PC), (IC) and (FC). The expected payoff to the investor who trades his claims in the trading arrangement μ is:

$$V\left(\{v(s)\}, \mu\right) = \mathbb{E}\left\{\sum_{i=1}^{n} \omega^i(x^i, x^{-i})\right\}, \tag{13}$$

whereas the ex-ante expected payoff to a trader from participating in this arrangement is:

$$W = \mathbb{E}\left\{U^i(x^i, \mathcal{A}^i(x^i, x^{-i}))\right\}. \tag{14}$$

Importantly, note that when an investor designs the trading arrangement for his claims, he takes as given the arrangements designed by others and, thus, her payoff W from participating in them. Thus, the optimal trading arrangement maximizes $V(\{v(s)\},\mu)$. In equilibrium, of course, all trading arrangements coincide.

2.5 Equilibrium Notion

We are ready to define an equilibrium of our economy.

Definition 1 *An equilibrium consists of prices $\{q(s), p(s)\}$, a trading arrangement $\mu \in \mathcal{M}$, and allocations $\{k_0, k_1(s), \widehat{k}_1(s), b_t(s), d_t(s)\}$ such that:*

1. *The allocations and the trading arrangement solve the agents' problems P1 and P2,*
2. *The markets for claims and capital clear: $b_t(s) = d_t(s)$ $\forall t, s$ and $k_1(s) + \widehat{k}_1(s) = k_0$ $\forall s$.*

3 Equilibrium

We now characterize the equilibrium of our economy. We first study the entrepreneurs' optimal investment, financing, and continuation decisions, for given asset prices. Then, we study the investors' optimal choice of trading arrangements. Finally, we combine these results to determine the equilibrium pricing of claims and capital.

3.1 Optimal investment, financing and continuation

In this section, we characterize the solution to the entrepreneurs' problem. We solve this problem backwards.

At $t = 2$ and state s, the entrepreneur simply consumes her final output net of repayment, $Ak_1(s) - b_2(s)$, which recall is non-negative since $k_1(s) \geq 0$ and $b_2(s) \leq 0$.

At $t = 1$ and state s, the entrepreneur has resources or net worth $(a(s) + p(s))k_0 - b_1(s)$, and she must decide how many units of capital to purchase and how much to consume.

Lemma 1 *The entrepreneur's optimal continuation scale at $t = 1$ and state s is given by:*

$$k_1(s) \begin{cases} = \frac{(a(s)+p(s))k_0 - b_1(s)}{p(s)} & \text{if } p(s) < A, \\ \in \left[0, \frac{(a(s)+p(s))k_0 - b_1(s)}{p(s)}\right] & \text{if } p(s) = A. \end{cases} \tag{15}$$

The entrepreneur's return to a unit of capital at this date is A. So, she prefers to purchase capital rather than consume if $p(s) < A$, and she is indifferent otherwise (i.e. if $p(s) = A$).

At $t = 0$, the entrepreneur chooses the investment scale k_0 and the claims $\{b_t(s)\}$. Since the entrepreneur is (weakly) constrained at $t = 1$, she will neither consume at $t = 0$ nor save for consumption at $t = 2$.

Lemma 2 *The entrepreneur optimally sets* $\chi(k_0) = \sum_s q(s)b_1(s)$ *and* $b_2(s) = 0\ \forall s$.

To see this, suppose to the contrary that the entrepreneur saves, $b_2(s) < 0$. Then she can do (weakly) better by increasing $b_2(s)$ and reducing $b_1(s)$, since she could thus reduce capital liquidations at $t = 1$ (Lemma 1), while keeping k_0 unchanged. By analogous reasoning, if the entrepreneur consumes at $t = 0$, she can do better by reducing consumption and $b_1(s)$, since she could again reduce capital liquidations at $t = 1$, while keeping k_0 unchanged.

Using Lemmas 1 and 2, the entrepreneur's problem simplifies to the maximization of her expected net worth weighted by her marginal value of funds in each state, $\frac{A}{p(s)}$:

$$\max_{k_0, \{b_1(s)\}} \sum_s \pi(s) \frac{A}{p(s)} \left((a(s) + p(s))k_0 - b_1(s)\right) \tag{16}$$

subject to $\chi(k_0) = \sum_s q(s)b_1(s)$ and $b_1(s) \leq (a(s) + p(s))k_0\ \forall s$.

Consider the entrepreneur's choice of financing, for a given scale k_0. When deciding how to raise funds, the entrepreneur compares across the two states the ratio of her (probability weighted) marginal value of funds in state s, $\pi(s)\frac{A}{p(s)}$, to the price of a claim against that state, $q(s)$. If $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} < \frac{\pi(l)}{q(l)}\frac{A}{p(l)}$, borrowing against the high state is more attractive; as a result, the entrepreneur exhausts her borrowing capacity in that state and borrows the remainder against the low state (and vice versa). If $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} = \frac{\pi(l)}{q(l)}\frac{A}{p(l)}$, she is indifferent to borrowing against high vs low state. This is formalized in the following lemma.

Lemma 3 *The entrepreneur's optimal choice of claims issuance is given by:*

$$b_1(s') \begin{cases} = (a(s') + p(s'))k_0 & \text{if } \frac{\pi(s')}{q(s')}\frac{A}{p(s')} < \frac{\pi(s'')}{q(s'')}\frac{A}{p(s'')}, \\ \in \left[q(s')^{-1}\left(\chi(k_0) - q(s'')b_1(s'')\right), \frac{(a(s')+p(s'))k_0 - b_1(s')}{p(s')} \right] & \text{if } \frac{\pi(s')}{q(s')}\frac{A}{p(s')} = \frac{\pi(s'')}{q(s'')}\frac{A}{p(s'')}, \\ = q(s')^{-1}\left(\chi(k_0) - q(s'')b_1(s'')\right) & \text{if } \frac{\pi(s')}{q(s')}\frac{A}{p(s')} > \frac{\pi(s'')}{q(s'')}\frac{A}{p(s'')} \end{cases} \tag{17}$$

for $s', s'' \in \{l, h\}$ *and* $s' \neq s''$.

Equipped with the entrepreneur's optimal financing decision, we can now determine her optimal investment scale by maximizing (16) with respect to k_0.

Lemma 4 *The entrepreneur's optimal investment scale at* $t = 0$ *is given by:*

$$\chi'(k_0) = \sum_s q(s)(a(s) + p(s)). \tag{18}$$

Thus, the entrepreneur invests until the marginal cost of creating an additional unit of capital equals the market value of cashflows produced by that unit plus its resale.

3.2 Optimal trading arrangements

We are now to study the investor's choice of trading arrangement, which maximizes the value of claims given in (13) subject to (PC), (IC), and (FC) given in (10)-(12).

Let $v(s)$ be the claims held by an investor for state s, and let μ^* denote the optimal trading arrangement by which these claims are traded. The following proposition states the key properties of optimal trading arrangements.

Proposition 1 *The optimal trading arrangement for claims $\{v(s)\}$ has the following features.*

1. *The value of the claims is given by:*

$$V(\{v(s)\}, \mu^*) = \sum_s \pi(s)v(s) - \zeta|v(h) - v(l)|, \tag{19}$$

where $\zeta = \begin{cases} \zeta^+ & \text{if } v(h) \geq v(l) \\ \zeta^- & \text{if } v(h) < v(l) \end{cases}$, *and* $\zeta^+ \in (0, \pi(h))\,, \zeta^- \in (0, 1 - \pi(h))$ *are scalars.*

2. *The ex-ante net expected payoff of a trader participating in it is:*

$$0 \leq W \leq n^{-1}\zeta|v(h) - v(l)|, \tag{20}$$

where the last inequality is an equality if and only if the trading arrangement allocates the claims efficiently.

3. ζ^+, ζ^- *are monotonically decreasing to zero as n grows to ∞, and are equal to zero in the absence of information asymmetries.*

The first result states that contingent claims are systematically mispriced in trading arrangements, despite their optimal design. As a result, the value of contingent claims is less than actuarially fair. The intuition for this result is as follows. If to the contrary the claims were priced fairly, it would have to be that the mechanism allocates the claims to traders w.p.1 and that the traders break even. But then, we show that there is a profitable deviation for a trader to misreport his true signal. In particular, the traders who are more optimistic about the value of the claims have an incentive to mimic those who are more pessimistic; because the latter break even, the former must earn positive profits or rents. The distortion $\zeta|v(h) - v(l)|$ captures the extent to which the traders are able to engage in rent-extraction.

It depends on the direction of the claims because the traders with highest (lowest) signals are the more optimistic ones about the claims when $v(h) > v(l)$ ($v(h) < v(l)$) and, as a result, the distribution of traders' valuations for the claims need not be the same in the two cases.

The second result shows that the mechanism may allocate the claims inefficiently, i.e. it may not allocate all the claims to the traders, which is socially costly since the impatient investors do not value future consumption. This is reflected in the feature that the traders' aggregate payoff nW can be lower than the loss of the mechanism, $\zeta|v(h) - v(l)|$. The reason is that it can be optimal to exclude pessimistic traders in order to reduce the rents earned by the optimistic ones; we illustrate this in Lemma A.3 in the Appendix. As we show in Section 5, the possibility of such misallocation has important policy implications.

The third result is that the mispricing/misallocation of contingent claims is due to the interaction of information dispersion with trading frictions. The combination of the two effectively gives each trader a form of market power and enables him to earn rents, but only when the traded claims are state-contingent. Instead, if information were symmetric, the mechanism could trivially sell the claims to traders at the commonly known expected value. On the other hand, as the number of traders participating in the mechanism grows and information aggregates, a single trader's signal becomes less relevant for valuing the claims, which reduces his ability to extract rents.

Remark 1 *The results in Proposition 1 do not depend on the binary signal structure that we assumed (see proof). However, the binary structure facilitates general equilibrium analysis, as we can obtain an explicit characterization of the mechanism and readily verify that in equilibrium the investors' consumption non-negativity constraints are satisfied at all times.*

3.3 Equilibrium prices of claims and capital

Now that we have determined the value of claims in secondary markets, we can study the determination of equilibrium claims prices at the issuance stage. The following corollary shows how the mispricing of contingent claims in secondary markets distorts the claims prices faced by the entrepreneurs.

Corollary 1 *In equilibrium, the prices of state-contingent claims satisfy:*

$$q(l) = 1 - q(h), \;\; q(h) \begin{cases} = \pi(h) - \lambda\zeta^{+} & \text{if } b_1(h) > b_1(l), \\ \in [\pi(h) - \lambda\zeta^{+}, \pi(h) + \lambda\zeta^{-}] & \text{if } b_1(h) = b_1(l), \\ = \pi(h) + \lambda\zeta^{-} & \text{if } b_1(h) < b_1(l). \end{cases} \tag{21}$$

When the aggregate supply of claims by entrepreneurs satisfies $b_1(h) > b_1(l)$, the market discounts an additional unit of a claim for the high state by $\lambda\zeta^+$, since with probability λ each investor will need to re-trade such a claim and face a discount ζ^+ (Proposition 1). Analogously, when $b_1(h) < b_1(l)$, the market discounts each additional unit of a claim for the low state by $\lambda\zeta^-$. Finally, the investors are happy to hold a non-contingent portfolio when $q(h) \in [\pi(h) - \lambda\zeta^+, \pi(h) + \lambda\zeta^-]$, since the resale value of an additional unit of a claim for the high (low) state is weakly lower than $q(h)$ $(1 - q(h))$.

To complete the determination of asset prices, we use the entrepreneurs' optimality conditions together with the traditional sector firms' demand for capital to pin down the market clearing prices of capital.

Corollary 2 *The equilibrium price of capital in state s satisfies:*

$$p(s) \begin{cases} = A & \text{if } b_1(s) < a(s)k_0, \\ \in [gA, A] & \text{if } b_1(s) = a(s)k_0, \\ = gA & \text{if } b_1(s) > a(s)k_0. \end{cases} \tag{22}$$

When entrepreneurial sector's promised repayments exceed its cashflows, entrepreneurs have to liquidate capital, i.e. $k_1(s) < k_0$. As a result, the price of capital becomes depressed, as capital must be absorbed by the traditional sector, which is less productive than entrepreneurs. When the repayments are lower than cashflows, then the price must be A, since if it were lower there would be excess demand for capital from the entrepreneurs; thus, also $k_1(s) = k_0$. Finally, when repayments are just equal to the cashflows, the price must be in the interval $[gA, A]$ so that in equilibrium capital is not traded, i.e. $k_1(s) = k_0$.

It is useful to briefly summarize what we have learned thus far. First, even though entrepreneurs are risk-neutral, the combination of limited pledgeability with the fact the shock to intermediate cashflows is aggregate makes the entrepreneurs effectively risk-averse with respect to net worth fluctuations. Second, even though investors are risk-neutral, the presence of information-trading frictions in secondary markets make them effectively risk-averse with respect to fluctuations in the value of their tradeable claims, but in a manner that generates a kink in the claims pricing schedule. While the former generates gains from risk-sharing, the latter renders risk-sharing (privately) costly. In what follows, we investigate how this tension gets resolved in equilibrium, and whether there is role for policy.

4 Implications for risk-sharing, investment and output

We now use the results in Sections 3.1-3.3 to study the implications of information-trading frictions for equilibrium risk-sharing, investment, and output.

We begin by considering a benchmark economy, where the information-trading frictions are shut down, which starkly illustrates how risk-sharing between entrepreneurs and investors can eliminate the balance sheet channel and stabilize economic activity. We will then see how the presence of information-trading frictions alters this picture.

4.1 Benchmark: Economy without Information-Trading Frictions

Let us suppose that the parameters are such that $\lambda\zeta = 0$, so that the claims prices are undistorted and given by $q(s) = \pi(s)\ \forall s$. By Corollary 1 and Proposition 1, this would arise if either there were no re-trading needs ($\lambda = 0$) or information were symmetric ($\zeta = 0$).

The first implication is that full risk-sharing must be attained in equilibrium. That is, the entrepreneurs' marginal value of funds, $\frac{A}{p(s)}$, and thus the prices of capital must be equalized across states. If this were not the case, the entrepreneurs would exhaust their borrowing capacity against the state with the higher price of capital (Lemma 3), resulting in more liquidations and a lower price of capital in that state (Lemma 1 and Corollary 2), a contradiction.

Second, the entrepreneurs will only borrow against their intermediate cashflows and avoid capital liquidations, i.e. $b_1(s) \leq a(s)k_0$ and $k_1(s) = k_0\ \forall s$. If they were to borrow more, they would have to liquidate capital (Lemma 1), which would depress its price to gA (Corollary 2), making investment less desirable than k_0 (Lemma 4 and Assumption 2(i)).

Third, the economy's final output is fully insulated from the shock to intermediate cashflows, i.e. $y_2(s) = Ak_0\ \forall s$. This is despite the fact that this shock directly affects the economy's output at $t = 1$, i.e. $y_1(s) = a(s)k_0$ in state s. The notion that risk-sharing can mute the impact of shocks on borrowers' net worth and thereby eliminate the balance sheet channel has been explored in a related framework by Krishnamurthy (2003) and Di Tella (2017b).

Finally, although the balance sheet channel is eliminated, investment and output may be depressed due to limited pledgeability. This occurs if and only if the funding required to invest at the first-best scale exceeds the entrepreneurs' intermediate cashflows, i.e. $\chi(k^{FB}) > \sum_s \pi(s)a(s)k^{FB}$ as defined in equation (3).[20] This last feature is not surprising and is common to models with financial frictions.

[20]If $\chi(k^{FB}) > \sum_s \pi(s)a(s)k^{FB}$, then the entrepreneurs invest all of their intermediate cashflows, $\chi(k_0) = \sum_s \pi(s)a(s)k_0$, and the price of capital is $p = \chi'(k_0) - \sum_s \pi(s)a(s) \in (gA, A)$ by Assumption 2(i). But, if $\chi(k^{FB}) \leq \sum_s \pi(s)a(s)k^{FB}$, then the entrepreneurs invest at the first-best scale and the price of capital is A.

4.2 Economy with Information-Trading Frictions

We now return to the economy with information-trading frictions, i.e. $\lambda\zeta > 0$. It will be convenient to split the parameter space into two regions since, depending on the severity of the information-trading frictions, the economy will either be in a "no risk-sharing" or "partial risk-sharing" regime. To this end, let us define

$$\bar{q} \equiv \min\left\{\frac{\pi(h)g}{\pi(h)g + 1 - \pi(h)}, \frac{\chi'(\bar{k}) - (a(l) + gA)}{(a(h) + A) - (a(l) + gA)}\right\}, \tag{23}$$

where $\bar{k} > 0$ is such that $\chi(\bar{k}) = (a(l) + gA)\bar{k}$. The term $\pi(h) - \bar{q} > 0$ captures the equilibrium "insurance premium" that the entrepreneurs are willing to pay to insure against net worth fluctuations, and it is naturally smaller when either liquidations are not too costly (higher g) or investment is not too attractive (e.g. higher $\chi'(\bar{k})$).

The following proposition shows that, when the information-trading frictions are severe so that $\lambda\zeta^+ > \pi(h) - \bar{q}$, then the equilibrium features no risk-sharing whatsoever.

Proposition 2 (No Risk-Sharing) *Suppose $\lambda\zeta^+ > \pi(h) - \bar{q}$. Then, in equilibrium, the claims are non-contingent, $b_1(h) = b_1(l) = b_1$, the capital is liquidated if and only if the state is low, and the asset prices satisfy $q(h) = \bar{q}$ and $p(l) = gA < A = p(h)$.*

When $q(h) = \bar{q}$, the equilibrium cost of insurance is high enough so that the entrepreneurs are willing to borrow with non-contingent claims, even though this generates large fluctuations in their net worth and, thus, their marginal utility of funds at $t = 1$. In turn, since $\lambda\zeta^+ > \pi(h) - q(h)$, the investors are unwilling to provide insurance, since their private cost to providing it, $\lambda\zeta^+$, is greater than the premium they would receive, $\pi(h) - q(h)$.

Because there is no risk-sharing at all, the shock to intermediate cashflows now does propagate to the economy's final output through capital liquidations in the low state, i.e. $y_2(l) = Ak_0 - (1-g)A(k_0 - k_1(l)) < Ak_0 = y_2(h)$. Moreover, the shock is now amplified through capital price fluctuations, since $k_0 - k_1(l)|_{p(l)=gA} = \frac{b_1 - a(l)k_0}{gA} > \frac{b_1 - a(l)k_0}{A} = k_0 - k_1(l)|_{p(l)=A}$. This idea that fluctuations in borrowers' net worth can amplify and propagate aggregate shocks goes back to the classic works of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), and has more recently been explored by Brunnermeier and Sannikov (2014), among others. In the literature, however, the limits to risk-sharing are often imposed ad-hoc rather than derived from first principles. In Section 5, we will show that understanding the source of the underlying distortion has important policy implications.

Next, we show that, when information-trading frictions are moderate, the equilibrium features partial risk-sharing, which in turn bounds the magnitude of economic fluctuations.

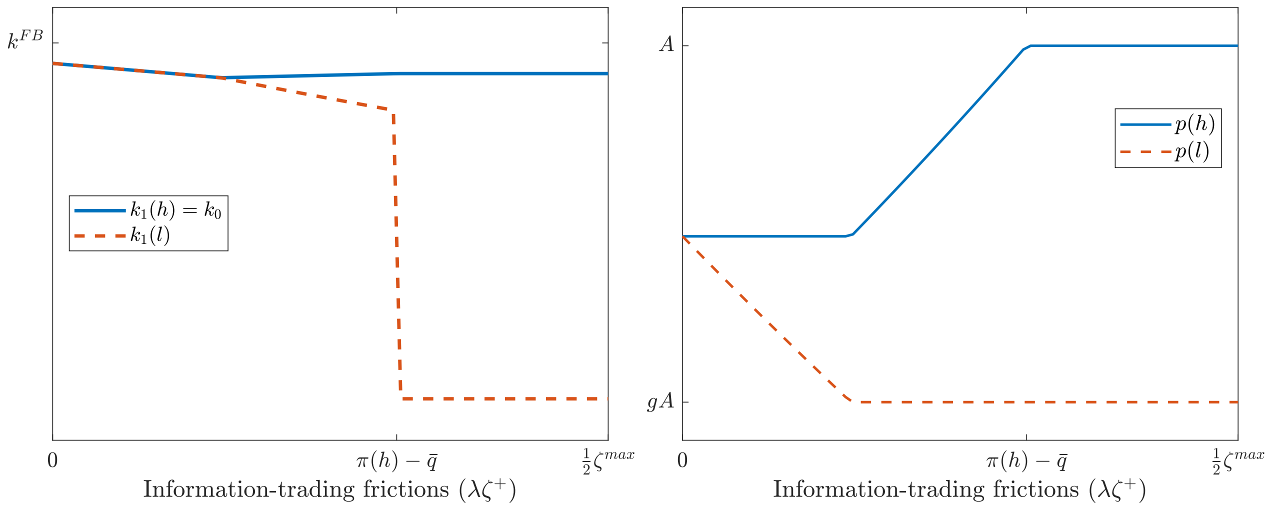

Figure 2: Illustrates the equilibrium effect of the information-trading frictions. The left panel depicts the equilibrium investment scale k_0, and the continuation scale, $k_1(s)$, whereas the right panel depicts the price of capital, $p(s)$. The frictions are minimized at zero and maximized at $\lambda = \frac{1}{2}$ and $n = 1$, in which case $\zeta^+ = \zeta^{\max} \equiv \pi(h) - \max\{\mathbb{P}(s = h|x = B), \mathbb{P}(x = G)\mathbb{P}(s = h|x = G)\}$. For this comparative static exercise, it is useful to relax the exact matching assumption (see Appendix C).

Proposition 3 (Partial Risk-Sharing) *Suppose $\lambda\zeta^+ < \pi(h) - \bar{q}$. Then, in equilibrium, the claims are contingent, $b_1(h) > b_1(l)$, the capital is liquidated only if the state is low, and the asset prices satisfy $q(h) = \pi(h) - \lambda\zeta^+$ and $\gamma p(h) \le p(l) < p(h)$ with $\gamma \equiv \frac{\pi(h)-\lambda\zeta^+}{1-\pi(h)+\lambda\zeta^+}\frac{1-\pi(h)}{\pi(h)}$, where $p(l) = \gamma p(h)$ if $b_1(l) < (a(l) + gA)k_0$.*

Since the entrepreneurs' willingness to pay for insurance (if they were uninsured) is now greater than the investors' cost of providing it, they will borrow with contingent claims, and the equilibrium must feature some risk-sharing. However, because insurance provision is privately costly ($\lambda\zeta^+ > 0$), full risk-sharing still cannot be attained, i.e. there is partial risk-sharing.

Because of partial risk-sharing, the fluctuations in the entrepreneurs' net worth and, thus, in their marginal value of funds at $t = 1$ are diminished relative to the economy with no risk-sharing. The reason is that entrepreneurs must weakly prefer to borrow against the high state, since by Lemma 3 and market clearing we have that $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} \ge \frac{\pi(l)}{q(l)}\frac{A}{p(l)} \iff \gamma p(h) \le p(l)$ with equality if $b_1(l) < (a(l) + gA)k_0$. As a result, the magnitude of equilibrium fluctuations now becomes tied to the severity of information-trading frictions. This can be seen by inspecting γ, which is decreasing in $\lambda\zeta^+$. Indeed, as $\lambda\zeta^+$ goes to zero, γ goes to one and the equilibrium converges to the benchmark economy with full risk-sharing, in which the amplification and propagation of the shock through the balance sheet channel is eliminated.

Figure 2 illustrates the equilibrium effect of information-trading frictions, as described in Propositions 2 and 3. The left panel depicts their effect on the aggregate investment and continuation scales k_0 and $\{k_1(s)\}$, whereas the right panel illustrates the behavior of capital prices. When $\lambda\zeta^+ > \pi(h) - \bar{q}$, there is no risk-sharing whatsoever, which as we can see results in the largest possible fluctuations in the continuation scale (thus, liquidations and aggregate output) as well as capital prices. Otherwise, there is partial risk-sharing and the fluctuations are muted. Indeed, we can see that when the information-trading frictions are small enough, though capital prices fluctuate, the entrepreneurs do not liquidate capital at all; here, the frictions simply depress investment scale ex-ante.

These results suggest that in order to understand the macroeconomic implications of a given shock, it is important to understand the markets where agents can trade claims contingent on that shock. In particular, we should expect more risk-sharing and less amplification/propagation of aggregate shocks, whenever agents can insure themselves in well-developed, deep secondary markets, and vice versa. This observation is consistent with the recent calls for policy makers (or international institutions) to promote liquidity in secondary markets for real-estate derivatives (Case et al., 1991; Shiller, 1994) or indexed government bonds (Caballero, 2003), as a way to stabilize the business cycle. By taking a stance on the source of illiquidity in secondary markets (i.e. information-trading frictions), we will be able to give more structure to these normative arguments (see Section 5).

Figure 2 also shows that the ex-ante investment scale, k_0, which, though non-monotonic due to capital liquidations, always remains depressed below first-best, k^{FB}. To see why this is the case, note from Lemma 4 that:

$$\chi'(k_0) = \sum_s q(s)(a(s) + p(s)) < \sum_s \pi(s)(a(s) + A) = \chi'(k^{FB}), \tag{24}$$

where the inequality holds because $q(h) < \pi(h)$ and $p(l) < p(h) \leq A$ by Propositions 2 and 3. The reason why asset prices are always distorted in this economy is that the entrepreneurs' cashflows in the low state, $a(l)k_0$, are insufficient to finance the first-best scale of investment (Assumption 2(ii)). Interestingly, however, the investment scale in this economy can actually be above or below that of the benchmark economy with full risk-sharing, and this is precisely because a fraction of capital gets liquidated prematurely.

Finally, before we move to study the efficiency properties of equilibrium, we make an important observation with regards to the welfare effect of information-trading frictions.[21]

Proposition 4 (Rent-Extraction) *In equilibrium, the investors' lifetime welfare satisfies:*

$$e \leq U^I \leq e + \lambda\zeta^+(b_1(h) - b_1(l)), \tag{25}$$

[21] The entrepreneurs' welfare is clearly decreasing in the information-trading frictions, since they simply face a larger cost of insurance against net worth shocks.

where the last inequality is an equality if and only if the optimal trading arrangements allocate the claims efficiently.

The investors rationally anticipate that the contingent claims they hold will be mispriced and trade below their fair value in secondary markets (Proposition 1). As a result, when forming their portfolios ex-ante, the investors demand compensation from the entrepreneurs for holding these claims (Corollary 1). In aggregate, however, the mispricing in secondary markets generates ex-post transfers among ex-ante identical investors and, therefore, does necessarily result in a welfare loss. In the extreme, when the claims are allocated efficiently in trading arrangements, there is no welfare loss to the investors at all! As we will see, this observation will play an important role in the welfare analysis of the next section.

The result in Proposition 4 also suggests an interesting political economy problem. Consider, for instance, a market reform that somehow manages to eliminate the information-trading frictions (e.g. establishes a competitive centralized marketplace for the claims) to ensure that the claims are traded fairly in secondary markets. Such a reform will inadvertently have distributional consequences and be opposed by the investors who are able to extract rents due to these frictions. To be supported by all parties, the policy maker would need to find and implement the appropriate compensating transfers, which often proves to be a daunting task in practice.[22] Though such considerations are clearly relevant and important to take into account when seeking practical implementation of policies, we abstract away from them in what follows and study a constrained planning problem, in which the social planner is able to compensate the investors for rents they extract in equilibrium, through transfers.

5 Constrained Efficient Allocations

In this section, we study the efficiency properties of equilibrium. To do so, we first solve the problem of a social planner, who chooses the allocations to maximize the lifetime welfare of the entrepreneurs, subject to the information and incentive constraints faced by the agents, and subject to delivering the investors the same lifetime welfare U as in the laissez-faire equilibrium (Lemmas 5 and 6); thus, we search for Pareto-improvement upon the equilibrium. Second, we derive our main inefficiency result by comparing the planner's solution to the laissez-faire equilibrium (Proposition 5). Finally, we show how to implement the constrained efficient allocations through policy (Proposition 6).

The planner chooses investment scale k_0, continuation scale $\{k_1^{SP}(s)\}$, transfers to the investors at the issuance stage, T_0, and at $t \in \{1, 2\}$, $\{T_t(s)\}$, and allocates $\{k_0 - k_1(s)\}$ to the

[22]See Caselli and Gennaioli (2008) for a study of the political feasibility of financial market reforms.

traditional sector, and the remaining goods each period to the entrepreneurs. In addition, the planner designs the optimal trading arrangements where investors can re-trade their claims to transfers $\{T_t(s)\}$. We suppose that the planner faces the same set of pledgeability and information-trading frictions as the agents. When necessary, in order to avoid confusion, we will use the superscript SP to denote the planner's allocations.

We begin by characterizing the optimal trading arrangement chosen by the planner.

Lemma 5 *The optimal trading arrangement chosen by the social planner allocates the claims to $\{T_t(s)\}$ efficiently. As a result, the investors' lifetime welfare is:*

$$U^I = T_0 + \sum_s \pi(s)(T_1(s) + T_2(s)). \tag{26}$$

The intuition for this result is as follows. Using Proposition 1, if the investors were to re-trade their claims $\{T_t(s)\}$ in the trading arrangements of the laissez-faire equilibrium, then their lifetime welfare would be $U^I \leq T_0 + \sum_s \pi(s)(T_1(s) + T_2(s))$, with equality if the trading arrangement allocates the claims efficiently among investors. Therefore, any trading arrangement that allocates the claims efficiently among investors must be optimal for the planner, as she could then reduce the transfer T_0 and increase the entrepreneurs' welfare.

Using Lemma 5, we can embed (26) as a constraint in the planner's problem, which becomes:

$$\max_{k_0, T_0, \{k_1(s), T_t(s)\}} e - \chi(k_0) - T_0 + \sum_s \pi(s) \left[(a(s)k_0 - T_1(s) + gA\left(k_0 - k_1(s)\right) + Ak_1(s) - T_2(s) \right] \tag{P3}$$

subject to

$$0 \leq T_0 + \chi(k_0) \leq e, \tag{27}$$

$$0 \leq T_1(s) \leq a(s)k_0, \tag{28}$$

$$0 \leq T_2(s) \leq gA(k_0 - k_1(s)), \tag{29}$$

$$0 \leq k_1(s) \leq k_0, \tag{30}$$

$$U \leq U^I = T_0 + \sum_s \pi(s) \left(T_1(s) + T_2(s)\right). \tag{31}$$

Constraints (27)-(29) are the economy's resource constraints combined with the limited pledgeability friction, constraint (30) is the feasibility constraint on capital allocation at $t = 1$, and constraint (31) states that the investors' lifetime welfare must at least be U, and that it is optimal for the planner to allocate the claims to the transfers efficiently.

Before characterizing the solution to problem (P3), it is useful to make the following definitions. First, let $\widetilde{k}_U > 0$ be the largest investment scale the planner can achieve with no liquidations and by delivering investors welfare U, i.e. $\chi(\widetilde{k}_U) = \sum_s \pi(s)a(s)\widetilde{k}_U + e - U$. Second, we define the planner's effective value of capital at $t = 1$ as:

$$p_U^{SP} = \begin{cases} A & \text{if } \sum_s \pi(s)a(s) + A \leq \chi'(\widetilde{k}_U) \\ \chi'(\widetilde{k}_U) - \sum_s \pi(s)a(s) & \text{if } \sum_s \pi(s)a(s) + A < \chi'(\widetilde{k}_U) \leq \sum_s \pi(s)a(s) + gA \\ gA & \text{if } \chi'(\widetilde{k}_U) < \sum_s \pi(s)a(s) + gA \end{cases}. \quad (32)$$

The following lemma completes the characterization of the solution to the planner's problem.

Lemma 6 *The social planner's investment scale is given by:*

$$\chi'(k_0^{SP}) = \sum_s \pi(s)a(s) + p_U^{SP}, \quad (33)$$

and she liquidates capital if and only if $p_U^{SP} = gA$, in which case her continuation scale is:

$$k_1^{SP}(s) = k_0^{SP} - \frac{U - \left(e - \chi(k_0^{SP}) + \sum_s \pi(s)a(s)k_0^{SP}\right)}{gA} \; \forall s. \quad (34)$$

By allocating the claims efficiently in trading arrangements, the planner effectively faces undistorted "claims prices" when transferring consumption goods between the entrepreneurs and the investors. This is because any transfers among investors within trading arrangements do not affect their ex-ante welfare. As a result, the planner attains full risk-sharing by equalizing the effective value of capital across states, just as in the benchmark economy.

Despite attaining full risk-sharing, however, the planner may still choose to liquidate capital prematurely. The reason is that, when the intermediate cashflows are insufficient to deliver the investors a welfare of U, the planner faces a tradeoff between reducing the ex-ante investment (increasing T_0) vs increasing ex-post capital liquidations (increase $\{T_2(s)\}$). The latter strategy becomes optimal when $\chi'(\widetilde{k}_U) < \sum_s \pi(s)a(s) + gA$, which is more likely to occur for higher values of U.[23]

We now utilize the characterization of the planner's problem, in conjunction with the results in Sections 3 and 4, to state the main result of this section.

Proposition 5 (Inefficiency) *The equilibrium of the economy with information-trading frictions is generally constrained inefficient.*[24] *In sharp contrast, the equilibrium of the benchmark economy is constrained efficient.*

[23] Observe that Assumption 2(i) ensures that when $U = e$ the planner does not liquidate capital, since $\chi'(\widetilde{k}_e) \geq \sum_s \pi(s)a(s) + gA$.

[24] The only scenario in which equilibrium could be constrained efficient is when both the allocation of claims is efficient, and entrepreneurs' claims satisfy $b_1(s) = a(s)k_0 \; \forall s$, despite their mispricing (see proof).

In our benchmark economy (i.e. $\lambda\zeta = 0$), the investors do not earn rents and their lifetime welfare is $U = e$. By inspection of the planner's problem for $U = e$, we immediately see that the allocations of the benchmark economy coincide with the planner's and are constrained efficient. This is because the sole source of inefficiency in our economy is that the information-trading frictions permit rent-extraction in secondary markets, and this manifests itself in two distortions. First, when choosing the optimal trading arrangements, investors minimize the mispricing associated with rent-extraction and, as a result, may design arrangements that allocate the claims inefficiently (Proposition 1). Second, anticipating that their claims will be mispriced in secondary markets, investors demand to be compensated for holding contingent claims, which distorts the claims prices (Corollary 1). This mispricing in turn limits risk-sharing, distorts investment at $t = 0$ and continuation at $t = 1$, as shown in Section 4.2.

The planner overcomes the inefficiency because she internalizes that the rents associated with the re-trading of contingent claims are simply ex-post transfers among ex-ante identical investors and, thus, do not affect their welfare (Lemma 5). At the margin, therefore, the planner need not compensate investors for the "mispricing" of claims in the trading arrangements that she designs, and therefore does not distort investment and continuation scales (Lemma 6). Instead, in the laissez-faire equilibrium, when purchasing claims and designing trading arrangements, each investor fails to internalize the positive externality on other investors that he exerts through rents; formally, each investor takes the rent W that he earns from other investors as given when solving problem (P2). This is a form of pecuniary externality that is very different from those identified in the literature on models with financial constraints (Caballero and Krishnamurthy, 2003; Lorenzoni, 2008; Korinek, 2011; Bianchi, 2011; Dávila and Korinek, 2017; Di Tella, 2017a). As a consequence, our policy implications novel too.

Proposition 6 (Optimal Policy) *The constrained efficient allocations can be implemented through ex-ante (i.e. at issuance stage) (i) subsidy that incentivizes entrepreneurs to issue contingent claims, (ii) subsidy that incentivizes investors to design efficient trading arrangements, and (iii) lump sum taxes to finance the intervention and ensure that investors are as well off as in the laissez-faire equilibrium.*

First, the planner wants trading arrangements that allocate the claims efficiently among investors (Lemma 5). She can achieve this by giving each investor a subsidy $\tau_0^{Eff} \cdot \max\{0, d_1(h) + d_2(h) - d_1(l) - d_1(l)\}$, when the investor has purchased claims $\{d_t(s)\}$, but *only if* the investor has designed a trading arrangement that is efficient.[25] The subsidy is set to exactly offset the net private loss to the investor from designing a privately sub-optimal trading arrangement:

[25] It is straightforward to show that Proposition 1 also holds under the additional restriction that the allocation of claims be efficient.

$\tau_0^{Eff} = \lambda(\widehat{\zeta}^+ - \zeta^+) \geq 0$, where $\widehat{\zeta}^+$ is the distortion in the optimal trading arrangement that is restricted to be efficient. Note that this intervention leaves each investor's willingness to pay for entrepreneurial claims and, thus, the equilibrium claims prices as in Corollary 1.

Second, the planner wants to achieve full risk-sharing between entrepreneurs and investors (Lemma 6), and to do so she must correct the distortion in the claims prices (see Corollary 1). The planner can achieve this by giving each entrepreneur a subsidy $\lambda\zeta^+ \cdot \max\{0, b_1(h) + b_2(h) - b_1(l) - b_2(l)\}$, when the entrepreneur has issued claims $\{b_t(s)\}$. Note that this intervention ensures that the entrepreneurs effectively face undistorted claims prices.

Finally, the planner must finance the intervention and at the same time deliver the investors welfare U as in the laissez faire equilibrium. Since in equilibrium $b_1(s) = d_1(s)$ and $b_2(s) = d_2(s) = 0 \; \forall s$, the planner can achieve this by levying lump sum taxes $\widehat{T}_0^I = \lambda\widehat{\zeta}^+ \cdot \max\{0, b_1(h) - b_1(l)\} + e - U$ and $\widehat{T}_0^E = U - e$ on the investors and the entrepreneurs respectively. Thus, the planner effectively extracts from the investors the rent $\lambda\widehat{\zeta}^+ \cdot \max\{0, b_1(h) - b_1(l)\}$ that they earn in the post-intervention equilibrium, but gives them back the rent $U - e$ they earned in the laissez-faire equilibrium. It is then straightforward to show that the investment scale and capital liquidations decisions of the entrepreneurs coincide with those of the planner; namely, optimal policy attains full risk-sharing between entrepreneurs and investors, and it eliminates the amplification and propagation of aggregate shocks through the balance sheet channel.

To conclude, we note that these results are broadly in line with some recent proposals for policy makers to enhance aggregate risk-sharing through interventions in contracts and in markets (Case et al., 1991; Shiller, 1994; Caballero, 2003; Mian, 2013). Looking through the lens of our model, optimal policy should target both the mispricing of claims and their potential misallocation in markets, as well as implement the appropriate compensating transfers in order to ensure that the intervention is Pareto improving. Furthermore, since our theory ties the inefficiency to the severity of information-trading frictions, it implies that optimal policy is potentially both market-specific and time-varying. Therefore, consistent with the theory of second-best, understanding the underlying source of inefficiency that limits risk-sharing is crucial for thinking about the design of the optimal corrective policy.

6 Conclusions

We considered a model of the balance sheet channel à la Kiyotaki and Moore (1997), but where we allowed entrepreneurs and investors to trade claims contingent on aggregate states. Using a mechanism design approach, we showed that the interaction of information dispersion about aggregate states with trading frictions in secondary claims markets generates mispricing of aggregate risk, distorts the demand for state-contingent claims and limits aggregate risk-sharing, thereby giving rise to the balance sheet channel. In equilibrium, the magnitude of aggregate fluctuations becomes tied to the severity of information-trading frictions, suggesting that the functioning of secondary claims markets has important implications for business cycles. Importantly, we showed that the laissez-faire equilibrium is constrained inefficient, due to a novel pecuniary externality arising from rent-extraction in secondary claims markets. We characterized the optimal policy and showed that it targets the inefficiency at its source by promoting (e.g. through subsidies) *both* issuance *and* trade of contingent claims in markets.

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Appendix

A Proofs for Sections 3-5

Proofs of Lemmas 1 - 4. See text. ■

Proof of Proposition 1. In what follows, we establish Proposition 1 for a generalized investor signal structure. Let $X = \{x_1, ..., x_M\}$ denote the set of traders' signals where $M \geq 2$ and where $\mathbb{P}(s = h|x^i = x)$ is increasing in x, i.e. traders with higher signals are more optimistic about the aggregate state. We will also allow the mechanism to have access to some external imperfectly informative signal y about the aggregate state (e.g. public information, seller's signal), which takes values in some set $Y = \{y_1, ..., y_N\} \subset \mathbb{R}$, has the property that $\mathbb{P}(s = h|y) \in (0, 1)$ is increasing in y, and is conditionally independent of the traders' signals. The case of no external signal is captured by assuming that Y is a singleton.[26] The mechanism design problem is in the text, except that the allocations of trader i can also be conditioned on $y \in Y$, i.e. $\mathcal{A}^i(x^i, x^{-i}, y) = \{\omega^i(x^i, x^{-i}, y), v_s^i(x^i, x^{-i}, y)\}$.

<u>Part 1.</u> Any mechanism μ for trading claims $\{v(s)\}$ that satisfies the participation and the feasibility constraints (PC) and (FC), given by (10) and (12), must also satisfy $V(\{v(s)\}, \mu) \leq \mathbb{E}\{v(s)\}$. Thus, $\mathbb{E}\{v(s)\}$ is the upper bound on the value of the mechanism. If a mechanism μ reaches this upper bound, i.e. $V(\{v(s)\}, \mu) = \mathbb{E}\{v(s)\}$, then we say that mechanism μ achieves *full surplus extraction*. Such a mechanism must clearly be optimal.

When the investor's portfolio of claims is non-contingent, i.e. $v(h) = v(l)$, the optimal mechanism clearly achieves full surplus extraction. For example, consider the mechanism with the following allocations: $v_s^i(x^i, x^{-i}, y) = \omega^i(x^i, x^{-i}, y) = n^{-1}v(l)$ for all s, i, x^i, x^{-i}, and y. Thus, non-contingent claims must be traded fairly and allocated efficiently w.p.1. We are therefore left to study the case where $v(h) \neq v(l)$. We will do so in two steps.

First, we show that full surplus extraction is impossible in any mechanism that satisfies the participation, incentive compatibility and feasiblity constraints (10)-(12).

Lemma A.1 *Let μ^* be an optimal mechanism for trading claims $\{v(s)\}$, and assume that $v(h) \neq v(l)$. Then, $V(\{v(s)\}, \mu^*) < \mathbb{E}\{v(s)\}$.*

Proof. Suppose to the contrary that $V(\{v(s)\}, \mu^*) = \mathbb{E}\{v(s)\}$, and consider the associated allocations $\mathcal{A}^i(x^i, x^{-i}, y) = \{\omega^i(x^i, x^{-i}, y), v_s^i(x^i, x^{-i}, y)\}$. Also, suppose that $v(h) > v(l)$; the argument for $v(h) < v(l)$ is analogous.

[26] The arguments that follow can also be extended to continuous signals.

First, full surplus extraction implies that $U^i\left(x^i, \mathcal{A}^i\left(x^i, x^{-i}, y\right)\right) = 0$ for all i, x^i, x^{-i}, y and $\sum_{i \in I} v_s^i\left(x^i, x^{-i}, y\right) = v\left(s\right)$ for all x^i, x^{-i}, y, s, i.e. in the optimal mechanism the traders' rents must be zero and the claims must be allocated efficiently w.p.1. To see this, from (10) and (12) we have:

$$\begin{aligned}
\mathbb{E}\left\{\sum_{i \in I} \omega^i\left(x^i, x^{-i}, y\right)\right\} &\leq \mathbb{E}\left\{\sum_{i \in I}\sum_s v_s^i\left(x^i, x^{-i}, y\right) \cdot \mathbb{P}\left(s | x^i, \mathcal{A}^i\left(x^i, x^{-i}, y\right)\right)\right\} \\
&= \mathbb{E}\left\{\mathbb{E}\left\{\sum_{i \in I}\sum_s v_s^i\left(x^i, x^{-i}, y\right) \cdot \mathbb{P}\left(s | x^i, \mathcal{A}^i\left(x^i, x^{-i}, y\right)\right) | x^i, x^{-i}, y\right\}\right\} \\
&= \mathbb{E}\left\{\mathbb{E}\left\{\sum_{i \in I}\sum_s v_s^i\left(x^i, x^{-i}, y\right) \cdot \mathbb{P}\left(s | x^i, x^{-i}, y\right) | x^i, x^{-i}, y\right\}\right\} \\
&\leq \mathbb{E}\left\{\mathbb{E}\left\{\sum_s v\left(s\right) \cdot \mathbb{P}\left(s | x^i, x^{-i}, y\right) | x^i, x^{-i}, y\right\}\right\} \\
&= \mathbb{E}\left\{v\left(s\right)\right\}, \qquad (35)
\end{aligned}$$

where the first inequality is strict if $U^i\left(x^i, \mathcal{A}^i\left(x^i, x^{-i}, y\right)\right) > 0$ for some i, x^i, x^{-i}, y, and the second inequality is strict if $\sum_{i \in I} v_s^i\left(x^i, x^{-i}, y\right) < v\left(s\right)$ for some x^i, x^{-i}, y, s.

Second, we find a profitable deviation for a trader to misreport his signal. Since by the argument above $U^i\left(x^i, \mathcal{A}^i\left(x^i, x^{-i}, y\right)\right) = 0$ for all i, x^i, x^{-i}, y, incentive compatibility requires that $U^i\left(x^i, \mathcal{A}^i\left(\theta^i, x^{-i}, y\right)\right) \leq 0$ for all $i, x^i, \theta^i, x^{-i}, y$. To reach a contradiction, we will now show that $U^i\left(x^i, \mathcal{A}^i\left(\theta^i, x^{-i}, y\right)\right) > 0$ for some $i, x^i, \theta^i, x^{-i}, y$. Note that:

$$\begin{aligned}
U^i\left(x^i, \mathcal{A}^i\left(\theta^i, x^{-i}, y\right)\right) &= \sum_s v_s^i\left(\theta^i, x^{-i}, y\right) \cdot \mathbb{P}\left(s | x^i, \mathcal{A}^i\left(\theta^i, x^{-i}, y\right)\right) - \omega^i\left(\theta^i, x^{-i}, y\right) \\
&= \sum_s v_s^i\left(\theta^i, x^{-i}, y\right) \cdot \left(\mathbb{P}\left(s | x^i, \mathcal{A}^i\left(\theta^i, x^{-i}, y\right)\right) - \mathbb{P}\left(s | \theta^i, \mathcal{A}^i\left(\theta^i, x^{-i}, y\right)\right)\right)
\end{aligned}$$

for all $i, x^i, \theta^i, x^{-i}, y$, since $U^i\left(\theta^i, \mathcal{A}^i\left(\theta^i, x^{-i}, y\right)\right) = 0$. Observe that (i) $\mathbb{P}\left(h | x^i, \mathcal{A}^i\left(\theta^i, x^{-i}, y\right)\right) \gtreqless \mathbb{P}\left(h | \theta^i, \mathcal{A}^i\left(\theta^i, x^{-i}, y\right)\right)$ whenever $x^i \gtreqless \theta^i$ since the allocation cannot perfectly reveal the state, and (ii) for all x^i, x^{-i}, y, we have that $v_h^i\left(x^i, x^{-i}, y\right) > v_l^i\left(x^i, x^{-i}, y\right)$ for some i because, as we have argued above, $\sum_i v_h^i\left(x^i, x^{-i}, y\right) = v\left(h\right) > v\left(l\right) = \sum_i v_l^i\left(x^i, x^{-i}, y\right)$ for all x^i, x^{-i}, y. From (ii), there exist i and $\widehat{x} < x_M$ such that $v_h^i\left(\widehat{x}, x^{-i}, y\right) > v_l^i\left(\widehat{x}, x^{-i}, y\right)$ for some x^{-i}, y; in other words, the mechanism must allocate more claims in the high than in the low state to some trader with signal other than highest (e.g. in the events that no trader has received signal x_M). Combining with (i), we have that $U^i\left(x_M, \mathcal{A}^i\left(\widehat{x}, x^{-i}, y\right)\right) > 0$, and it is sub-optimal for trader i who has received signal x_M to report his signal truthfully. ■

Next, we show that the losses of the optimal mechanism must be proportional to the contingency of the investor's portfolio of claims.

Lemma A.2 *Let μ^* be an optimal mechanism for trading claims $\{v(s)\}$. Then,*

$$V(\{v(s)\}, \mu^*) = \sum_s \pi(s)v(s) - \zeta|v(h) - v(l)|,$$

where $\zeta = \begin{cases} \zeta^+ & \text{if } v(h) \geq v(l) \\ \zeta^- & \text{if } v(h) < v(l) \end{cases}$, *and* $\zeta^+ \in (0, \pi(h)), \zeta^- \in (0, 1 - \pi(h))$ *are scalars.*

Proof. Suppose that $v(h) > v(l)$; the argument for $v(h) < v(l)$ is analogous. Denote by $\mathcal{A}^i(x^i, x^{-i}, y) = \{v_s^i(x^i, x^{-i}, y), \omega^i(x^i, x^{-i}, y)\}$ the allocations of the optimal mechanism.

Define $\tilde{v}_s^i(x^i, x^{-i}, y) \equiv \frac{v_s^i(x^i, x^{-i}, y) - n^{-1}v(l)}{v(h) - v(l)}$ and $\tilde{\omega}^i(x^i, x^{-i}, y) = \frac{\omega^i(x^i, x^{-i}, y) - n^{-1}v(l)}{v(h) - v(l)}$ for all i, x^i, x^{-i}, y, and consider the modified allocations $\tilde{\mathcal{A}}^i(x^i, x^{-i}, y) = \{\tilde{v}_s^i(x^i, x^{-i}, y), \tilde{\omega}^i(x^i, x^{-i}, y)\}$. Then the design problem can be re-written as follows:

$$V(\{v(s)\}, \mu^*) = v(l) + (v(h) - v(l)) \cdot \max_{\{\tilde{v}_s^i, \tilde{\omega}^i\}} \mathbb{E}\left\{\sum_{i \in I} \tilde{\omega}^i\left(x^i, x^{-i}, y\right)\right\}$$

subject to:

$$\sum_s \tilde{v}_s^i\left(x^i, x^{-i}, y\right) \cdot \mathbb{P}\left(s | x^i, \tilde{\mathcal{A}}^i\left(x^i, x^{-i}, y\right)\right) \geq \tilde{\omega}^i\left(x^i, x^{-i}, y\right) \text{ for all } i, x^i, x^{-i}, y,$$

$$x^i \in \text{argmax}_{\theta^i \in X} \mathbb{E}\left\{\max\left\{0,\ U(x^i, \tilde{\mathcal{A}}^i(\theta^i, x^{-i}, y)\right\} | x^i\right\} \text{ for all } i, x^i,$$

and

$$\sum_{i \in I} \tilde{v}_s^i\left(x^i, x^{-i}, y\right) \leq \tilde{v}(s) \text{ for all } x^i, x^{-i}, y, s,$$

where $\tilde{v}(l) = 0$ and $\tilde{v}(h) = 1$. It follows that:

$$V(\{v(s)\}, \mu^*) = v(l) + (v(h) - v(l)) \cdot V(\{\tilde{v}(s)\}, \tilde{\mu}^*)$$

where $\tilde{\mu}^*$ is the optimal mechanism for trading the claims $\{\tilde{v}(s)\}$. Since this mechanism also satisfies the constraints (10), (11) and (12), full surplus extraction is also not possible in this mechanism (see Lemma A.1), and thus $V(\{\tilde{v}(s)\}, \tilde{\mu}^*) < \pi(h)$. Also, $V(\{\tilde{v}(s), \tilde{\mu}^*\}) > 0$ because the mechanism designer always has the option to trade the claims at the expected valuation of the trader with the lowest signal, which is strictly positive. We define $\zeta^+ \equiv \pi(h) - V(\{\tilde{v}(s)\}, \tilde{\mu}^*) \in (0, \pi(h))$. Analogous arguments imply that $\zeta^- \in (0, 1 - \pi(h))$. ■

Part 2. The traders' participation constraint (10) implies that their ex-ante expected payoff W must be non-negative. It is also clear that the traders' expected payoff cannot exceed the expected losses of the mechanism. That the mechanism may trade the claims with probability less than one and, thus, allocate the claims inefficiently is shown in Lemma A.3 below.

Part 3. That the scalars ζ^+ and ζ^- monotonically decline to zero as n grows to ∞ follows from two simple observations. First, the mechanism designer always has the option to disregard a trader's signal and set his allocations to zero; thus, the value of the mechanism must be non-decreasing in the number of traders. Second, it is straightforward to construct a mechanism (albeit sub-optimal) in which the traders' rents go to zero as n grows to ∞; hence, this must also hold in the optimal mechanism. To this end, consider a mechanism which asks each trader to pay the expected value of the claims allocated to him, conditional on (i) the trader having received the 'worst' possible signal and (ii) the other traders' signals, i.e. $\omega^i(x^i, x^{-i}, y) = n^{-1} \min_x \sum_s \mathbb{P}(s|x^i = x, x^{-i}, y)v(s)$. Clearly, this mechanism satisfies the (PC), (IC), and (FC) constraints, and the unconditional expected value of the claims in the mechanism increases monotonically to $\sum_s \pi(s)v(s)$ with n.

Next, suppose that information is symmetric, i.e. conditional on the state, the investors' signals are perfectly correlated. Then the following mechanism achieves full surplus extraction: $v^i_s(x^i, x^{-i}, y) = \mathbb{1}_{\{x^i = x^j \forall j \neq i\}} \cdot n^{-1}v(s)$ and $\omega^i(x^i, x^{-i}, y) = \mathbb{E}\{v_s(x^i, x^{-i}, y)|x^i, x^{-i}, y\}$, i.e. the claims are allocated to the trader if and only if his report is exactly the same as the others', in which case he is asked to pay his expected valuation of the claims. If $n = 1$, to achieve full surplus extraction we need to allow the mechanism to condition the allocations also on the selling investor's signal, which we can embed in y. The optimal mechanism simply allocates the claims to the trader if and only if the seller's and the trader's reports coincide, in which case the trader receives all of the claims and pays to the mechanism the expected valuation of the claims conditional on his signal.

We now return to the signal structure assumed in the paper to obtain additional properties of the optimal mechanism.

Lemma A.3 *Suppose that investors' signals are binary, i.e.* $X = \{B, G\}$, *then the allocations of the optimal mechanism satisfy* $v^i_h, v^i_l, \omega^i \geq 0\ \forall i$. *Furthermore, the mechanism allocates the claims efficiently if and only if* $\Delta \geq 0$, *where:*

$$\Delta \equiv \begin{cases} \mathbb{P}(h|x^i = B\ \forall i) - \mathbb{P}(h|x^1 = G,\ x^i = B\ \forall i \neq 1)\mathbb{P}(x^1 = G|x^i = B\ \forall i \neq 1) & \text{if } v(h) > v(l) \\ \mathbb{P}(h|x^i = G\ \forall i) - \mathbb{P}(h|x^1 = B,\ x^i = G\ \forall i \neq 1)\mathbb{P}(x^1 = B|x^i = G\ \forall i \neq 1) & \text{if } v(h) < v(l). \end{cases} \tag{36}$$

Proof. Suppose that $v(h) > v(l)$; the proof for $v(h) < v(l)$ is analogous. We solve the relaxed problem in which the (IC) constraint for the trader who has received signal B is slack, and then we verify that this is the case at the optimum.

In the relaxed problem, the (PC) constraint of trader with signal B must clearly be binding,

$$\omega^i(B, x^{-i}) = \sum_s v_s^i(B, x^{-i})\mathbb{P}(s|B, \mathcal{A}^i(B, x^{-i})) \tag{37}$$

for all i, x^{-i}. It is also clear that the (IC) constraint of trader with signal G must be binding,

$$\mathbb{E}\{\omega^i(G, x^{-i})|G\} = \mathbb{E}\left\{\sum_s v_s^i(G, x^{-i})\mathbb{P}(s|G, \mathcal{A}^i(G, x^{-i})) - \max\{0, U^i(G, \mathcal{A}^i(B, x^{-i}))\}|G\right\} \tag{38}$$

for all i. From (37), the payoff to trader with signal G from reporting that he has signal B is:

$$U^i(G, \mathcal{A}^i(B, x^{-i})) = \left(v_h^i(B, x^{-i}) - v_l^i(B, x^{-i})\right)\left(\mathbb{P}(h|G, \mathcal{A}^i(B, x^{-i})) - \mathbb{P}(h|B, \mathcal{A}^i(B, x^{-i}))\right), \tag{39}$$

where $\mathbb{P}(h|G, \mathcal{A}^i(B, x^{-i})) > \mathbb{P}(h|B, \mathcal{A}^i(B, x^{-i}))$. The relaxed problem thus reduces to:

$$\max_{\{v_s^i\}} \mathbb{E}\left\{\sum_s \sum_i v_s^i(x^i, x^{-i})\right\} - \sum_i \mathbb{P}(x^i = G)\mathbb{E}\{\max\{0, U^i(G, \mathcal{A}^i(B, x^{-i}))\}|G\} \tag{40}$$

subject to the feasibility constraint $\sum_i v_s^i(x^i, x^i) \leq v(s)$ for all s, x^i, x^{-i}. Thus, the value of the mechanism increases in the claims it sells, but decreases in the rents it leaves to traders with signal G. By inspection, we see that it is without loss to set $v_h^i(x^i, x^{-i}) \geq v_l^i(x^i, x^{-i}) \geq 0$ and, thus, $U^i(G, \mathcal{A}^i(B, x^{-i})) \geq 0$ for all i, x^i, x^{-i}. Furthermore, since selling claims to traders with G signals does not generate rents, it is optimal to set: $v_s^i(G, x^{-i}) = m(x^1, ..., x^n)^{-1}v(s)$, where $m(x^1, ..., x^n)$ is the number of traders in the mechanism who have received signal G. This implies that traders with B signals receive the empty allocation when there is at least one trader in the mechanism with signal G.

We are therefore left to determine $v_s^i(x^i, x^{-i})$, whenever all traders in the mechanism have signal B. For this case, since selling equal units of claims in both states does not entail rents, it is optimal to set $v_h^i(x^i, x^{-i}) \geq v_l^i(x^i, x^{-i}) = n^{-1}v(l)$. Maximization of (40) with respect to $v_h^i(x^i, x^{-i}) \in [n^{-1}v(l), n^{-1}v(h)]$ yields:

$$v_h^i(x^i, x^{-i}) = \begin{cases} n^{-1}v(h) & \text{if } \Delta \geq 0 \\ n^{-1}v(l) & \text{otherwise.} \end{cases} \tag{41}$$

Thus, the mechanism allocates these claims to the traders iff the expected payoff from doing so exceeds the rents this allocation generates for traders with G signals. Note that, given the computed allocations, when a trader's allocation is non-empty, he perfectly infers the other traders' signals from the allocation.

Finally, we are left to specify the payments for the traders with G signals and verify that indeed the (IC) constraint of traders with B signals is slack. To this end, consider:

$$\omega^i(G, x^{-i}) = \sum_s v_s^i(G, x^{-i})\mathbb{P}(s|G, \mathcal{A}^i(G, x^{-i})) \\ - \left(v_h^i(B, x^{-i}) - v_l^i(B, x^{-i})\right)\left(\mathbb{P}(h|G, \mathcal{A}^i(B, x^{-i})) - \mathbb{P}(h|B, \mathcal{A}^i(B, x^{-i}))\right). \quad (42)$$

These payments satisfy the G signal traders' (PC) constraints (since $v_h^i(B, x^{-i}) \geq v_l^i(B, x^{-i})$) and by construction also their (IC) constraints. We now verify that $U^i(B, \mathcal{A}^i(G, x^{-i})) \leq 0$ for all i, x^{-i}. Consider trader 1 and note (i) $U^1(B, \mathcal{A}^1(G, x^{-1})) = \sum_s v(s)\mathbb{P}(s|B, x^{-1}) - \omega^1(G, x^{-1}) = 0$ when $\Delta \geq 0$ and all other traders have signal B, since then $\omega^1(G, x^{-1}) = \sum_s v(s)\mathbb{P}(s|B, x^{-i})$; (ii) $U^1(B, \mathcal{A}^1(G, x^{-1})) = \sum_s v(s)\mathbb{P}(s|B, x^{-i}) - \omega^1(G, x^{-1}) < 0$ when $\Delta < 0$ and all other traders have received signal B, since then $\omega^1(G, x^{-1}) = \sum_s v(s)\mathbb{P}(s|G, x^{-i})$; (iii) $U^1(B, \mathcal{A}^1(G, x^{-1})) = m(x^1, ..., x^n)^{-1}\sum_s v(s)\mathbb{P}(s|B, x^{-i}) - \omega^1(G, x^{-1}) < 0$ when some other traders have received signal G, since then $\omega^1(G, x^{-1}) = m(x^1, ..., x^n)^{-1}\sum_s v(s)\mathbb{P}(s|G, x^{-i})$. The result follows since we can repeat this argument for any trader $i \in \{1, ..., n\}$. ■

Thus, Lemma A.3 shows that the claims allocations and the payments for all traders are non-negative in the optimal mechanism. Though we conjecture that this result holds for general investor signal structure, we do not have a proof of that. In Appendix B, we will use these properties of the optimal mechanism to verify the assertions made in Section 2.1 that, at the proposed claims prices (see property (1)), the investors' consumptions remain non-negative at all times.

Finally, the lemma also provides the conditions under which the optimal trading arrangements allocate the claims inefficiently in secondary markets. In particular, the optimal mechanism trades with probability less than one whenever the loss generated from excluding the pessimistic traders from trade is smaller than the resulting reduction in the rents earned by the more optimistic traders. For instance, when $n = 1$ and $v(h) > v(l)$, this occurs if and only if $\mathbb{P}(h|x^i = B) < \mathbb{P}(x^i = G)\mathbb{P}(h|x^i = G)$, i.e. when signals are very informative about the state and/or there is a high probability of a trader having received a *G*ood signal. ■

Proof of Corollary 1. In equilibrium, the claims issued by the entrepreneurs must be held by the investors. Thus, for all s, $d_1(s) = b_1(s)$ and $d_2(s) = 0$, which implies that $v(s) = b_1(s)$.

The investor's net expected payoff from purchasing claims $\{v(s)\}$ at the issuance stage is:

$$\Gamma(\{v(s)\}) = -\sum_s q(s)v(s) + \lambda V(\{v(s)\}, \mu^*) + (1-\lambda)\sum_s \pi(s)v(s). \tag{43}$$

From Proposition 1: (i) when $q(h) = \pi(h) - \lambda\zeta^+$, then $\Gamma(\{v(s)\}) = 0$ if $v(h) \geq v(l)$ and $\Gamma(\{v(s)\}) < 0$ otherwise; (ii) when $q(h) = \pi(h) + \lambda\zeta^-$, then $\Gamma(\{v(s)\}) = 0$ if $v(h) \leq v(l)$ and $\Gamma(\{v(s)\}) < 0$ otherwise; (iii) when $q(h) \in (\pi(h) - \lambda\zeta^+, \pi(h) + \lambda\zeta^-)$, then $\Gamma(\{v(s)\}) = 0$ if $v(h) = v(l)$ and $\Gamma(\{v(s)\}) < 0$ otherwise. It thus follows that the claims prices given in Corollary 1 are part of equilibrium.

Next, note that these are the only prices consistent with equilibrium. From Proposition 1: (i) if $v(h) > v(l)$, then $\Gamma(\{v(s)\})$ is decreasing in $q(h)$ and equal to zero when $q(h) = \pi(h) - \lambda\zeta^+$. Therefore, if $q(h)$ were higher, the investors would not want to hold the claims $\{v(s)\}$, but if $q(h)$ were lower, there would be excess demand. By an analogous argument, if $v(h) < v(l)$, then it must be that $q(h) = \pi(h) + \lambda\zeta^-$. Finally, if $v(h) = v(l)$, then $\Gamma(\{v(s)\}) = 0$ for any $q(h)$. However, if $q(h) \notin [\pi(h) - \lambda\zeta^+, \pi(h) + \lambda\zeta^-]$, then the investor would deviate and form a contingent portfolio: if $q(h) < \pi(h) - \lambda\zeta^+$, then $\Gamma(\{v(s)\}) > 0$ for $v(h) > v(l)$; if $q(h) > \pi(h) + \lambda\zeta^-$, then $\Gamma(\{v(s)\}) > 0$ for $v(h) < v(l)$. ■

Proof of Corollary 2. See text. ■

The next two lemmas will be used in the proofs of Propositions 2 and 3.

Lemma A.4 *In equilibrium, $b_1(h) \geq b_1(l)$ and $q(h) < \pi(h)$.*

Proof. Suppose to the contrary that $b_1(h) < b_1(l)$. Then, we must have $p(h) \geq p(l)$ and $q(h) = \pi(h) + \lambda\zeta^-$ (Corollaries 1 and 2). But, Lemma 3 implies that $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)}\frac{A}{p(l)}$ or, equivalently, that:

$$p(h) \leq \frac{\pi(h)}{q(h)} \cdot \frac{1-q(h)}{1-\pi(h)} \cdot p(l) < p(l), \tag{44}$$

a contradiction.

Suppose to the contrary that $q(h) \geq \pi(h)$. By the argument above and Corollary 1, it must be that $b_1(h) = b_1(l) \leq (a(l) + gA)k_0$. By Assumption 2(iii), it must be that $b_1(h) < a(h)k_0$ and, thus, $p(h) = A$ (Corollary 2). Since also $b_1(h) < (a(h) + p(h))k_0$, we must have $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)}\frac{A}{p(l)}$ (Lemma 3). But, because $q(h) \geq \pi(h)$ and $p(h) = A$, this is only possible if $q(h) = \pi(h)$ and $p(l) = A$, which in turn implies that $b_1(l) \leq a(l)k_0$ (Corollary 2). By Lemma 4, the entrepreneurs' investment scale satisfies, $\chi'(k_0) = \sum_s \pi(s)a(s) + A$, which implies that $\chi(k_0) > a(l)k_0 \geq \sum_s q(s)b_1(s)$ by Assumption 2(ii), a contradiction. ■

Lemma A.5 *In equilibrium, $p(h) > p(l)$ and, thus, $b_1(l) \geq a(l)k_0$ and $b_1(h) \leq a(h)k_0$.*

Proof. Suppose to the contrary that $p(l) \geq p(h)$. By Lemma A.4, $q(h) < \pi(h)$ and, thus, it must be $\frac{\pi(h)}{q(h)} \frac{A}{p(h)} > \frac{1-\pi(h)}{1-q(h)} \frac{A}{p(l)}$. But then we must have $b_1(h) \geq b_1(l) = (a(l) + gA)k_0$ (Lemmas 3 and A.4) and, thus, $p(l) = gA$ (Corollary 2). Since $p(h) \geq gA$, we must thus also have $p(h) = gA$. By Lemma 4, the entrepreneurs' investment satisfies $\chi'(k_0) = \sum_s q(s)a(s) + gA$. Since $q(h) < \pi(h)$, we have $\chi(k_0) < \sum_s \pi(s)a(s)k_0 < (a(l) + gA)k_0$ by Assumptions 2(i)&(iii). But we also have $\chi(k_0) = \sum_s q(s)b_1(s) \geq (a(l) + gA)k_0$, a contradiction. Finally, by Corollary 2, if $b_1(l) < a(l)k_0$, then $p(l) = A$; and if $b_1(h) > a(h)k_0$, then $p(h) = gA$. These contradict the previous result that $p(h) > p(l)$. ■

Proof of Proposition 2. Assume that $\bar{q} > \pi(h) - \lambda\zeta^+$.

We first show that $b_1(h) = b_1(l)$. Suppose to the contrary that $b_1(h) > b_1(l)$ (recall $b_1(h) \geq b_1(l)$ by Lemma A.4), then $q(h) = \pi(h) - \lambda\zeta^+$ (Corollary 1). Since $b_1(h) < (a(h) + p(h))k_0$ (Lemma A.5), the entrepreneurs' optimal financing decision implies that $\frac{\pi(h)}{q(h)} \frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)} \frac{A}{p(l)}$ or, equivalently,

$$p(l) \geq \frac{\pi(h) - \lambda\zeta^+}{\pi(h)} \frac{1 - \pi(h)}{1 - \pi(h) + \lambda\zeta^+} p(h), \tag{45}$$

where the inequality is an equality whenever the borrowing constraint is slack in state l, i.e. if $b_1(l) < (a(l) + gA)k_0$ (Lemma 3). If $b_1(l) < (a(l) + gA)k_0$, then (45) holds with equality and, using the definition of $\bar{q}$ in (23) and the assumption that $\bar{q} > \pi(h) - \lambda\zeta^+$, we have that $p(l) < gp(h) \leq gA$, a contradiction. Instead, if $b_1(l) = (a(l) + gA)k_0$, then entrepreneurs liquidate capital in state l and, thus, $p(l) = gA$ (Corollary 2). By Lemma 4, the entrepreneurs' investment scale satisfies:

$$\chi'(k_0) = q(h)(a(h) + p(h)) + (1 - q(h))(a(l) + gA). \tag{46}$$

Since also $\chi(k_0) = \sum_s q(s)b_1(s) > (a(l) + gA)k_0$, it follows that k_0 is greater than $\bar{k} > 0$ such that $\chi(\bar{k}) = (a(l) + gA)\bar{k}$. Thus,

$$\begin{aligned} \chi'(\bar{k}) &< q(h)(a(h) + p(h)) + (1 - q(h))(a(l) + gA) \\ &< \bar{q}(a(h) + A) + (1 - \bar{q})(a(l) + gA) \\ &\leq \chi'(\bar{k}), \end{aligned} \tag{47}$$

where the last inequality follows from the definition of $\bar{q}$ in (23), a contradiction.

Next, we show that $b_1(l) > a(l)k_0$ and capital gets liquidated in state l. Suppose to the contrary that $b_1(l) \leq a(l)k_0$. Because the claims are non-contingent, $b_1(h) = b_1(l) \leq a(l)k_0$, it must be that $p(h) = A$ and $p(l) = \frac{q(h)}{\pi(h)} \frac{1-\pi(h)}{1-q(h)} A$ (Corollary 2 and Lemma 3), and also that the investment scale satisfies $\chi(k_0) = b_1(l) \leq a(l)k_0$ (Lemma 2). But, by Assumption 2(ii),

$$\begin{aligned}\chi'(k_0) &< \frac{\pi(h)g}{\pi(h)g + 1 - \pi(h)}(a(h) + A) + \left(1 - \frac{\pi(h)g}{\pi(h)g + 1 - \pi(h)}\right)(a(l) + gA) \\ &\leq q(h)(a(h) + A) + (1 - q(h))(a(l) + p(l)) \end{aligned} \tag{48}$$

where the second inequality follows from the fact that $p(l) = \frac{q(h)}{\pi(h)}\frac{1-\pi(h)}{1-q(h)}A$ and that $p(l) \geq gA$. But then the investment scale k_0 is sub-optimal (Lemma 4), a contradiction.

Next, we show that $q(h) = \bar{q}$. Since $b_1(l) > a(l)k_0$, we have $p(l) = gA$. Since the claims are non-contingent, $b_1(h) = b_1(l)$, and since $a(l) + gA < a(h)$ by Assumption 2(iii), it must be that $b_1(h) < a(h)k_0$ and, thus, $p(h) = A$. By Lemma 3, the entrepreneurs' optimal financing decision implies that $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)}\frac{A}{p(l)}$, where if $b_1(l) < (a(l) + gA)k_0$, then $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} = \frac{1-\pi(h)}{1-q(h)}\frac{A}{p(l)}$. Using the fact that $p(l) = gA$ and $p(h) = A$, we have that $q(h) = \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$. Since $\chi(k_0) = \sum_s q(s)b_1(s) < (a(l) + gA)k_0$, it must be that $k_0 < \bar{k}$. By Lemma 4, this is an equilibrium if and only if:

$$\chi'(\bar{k}) > \frac{\pi(h)g}{\pi(h)g + 1 - \pi(h)}(a(h) + A) + \left(1 - \frac{\pi(h)g}{\pi(h)g + 1 - \pi(h)}\right)(a(l) + gA), \tag{49}$$

which holds if and only if also $\bar{q} = \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$ (see definition of $\bar{q}$); thus, $q(h) = \bar{q}$. On the other hand, if $b_1(l) = (a(l) + gA)k_0$, then $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)}\frac{A}{p(l)}$. Using the fact that $p(l) = gA$ and $p(h) = A$, we have $q(h) \leq \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$. We also have that $k_0 = \bar{k}$ and, therefore, by Lemma 4 it must be that:

$$\chi'(\bar{k}) = q(h)(a(h) + A) + (1 - q(h))(a(l) + gA), \tag{50}$$

which holds if and only if $q(h) = \bar{q}$ (see definition of $\bar{q}$).

Finally, (i) $p(l) = gA$ since capital gets liquidated in state l, and (ii) $p(h) = A$ since $b_1(h) = b_1(l) \leq (a(l) + gA)k_0$ and since $a(l) + gA < a(h)$ (Assumption 2(iii)); thus, $b_1(h) < a(h)k_0$. ∎

Proof of Proposition 3. Assume that $\bar{q} < \pi(h) - \lambda\zeta^+$.

We first show that $b_1(h) > b_1(l)$ (recall $b_1(h) \geq b_1(l)$ by Lemma A.4). Suppose to the contrary that $b_1(h) = b_1(l)$. If $b_1(l) < (a(l) + gA)k_0$, then by Assumption 2(iii) we also have $b_1(h) < a(h)k_0$ and, thus, $p(h) = A$. Following the same arguments as in the proof of Proposition 2, we can show that then $b_1(l) > a(l)k_0$, capital gets liquidated in state l and, thus, $p(l) = gA$. The entrepreneurs' optimal financing decision implies $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} = \frac{1-\pi(h)}{1-q(h)}\frac{A}{p(l)}$, which in turn implies that $q(h) = \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$. Since $k_0 < \bar{k}$ defined by $\chi(\bar{k}) = (a(l) + gA)k_0$, using Lemma 4 and the definition of $\bar{q}$ we have that $\bar{q} = \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$ and, thus, $q(h) < \pi(h) - \lambda\zeta^+$, which contradicts Corollary 1. Instead, if $b_1(l) = (a(l) + gA)k_0$, then again $p(l) = gA < A = p(h)$ but $k_0 = \bar{k}$. By Lemma 3, $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)}\frac{A}{p(l)}$ and, thus, $q(h) \leq \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$. By Lemma 4, we must have $q(h) = \frac{\chi'(\bar{k})-(a(l)+gA)}{(a(h)+A)-(a(l)+gA)} = \bar{q} < \pi(h) - \lambda\zeta^+$, again contradicting Corollary 1.

Since $b_1(h) > b_1(l)$, we have $q(h) = \pi(h) - \lambda\zeta^+$.

Since $b_1(h) \leq a(h)k_0$ (Lemma A.5), it must be that $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)}\frac{A}{p(l)}$ or, equivalently, that $\gamma p(h) \leq p(l)$, where the inequality is an equality if the borrowing constraint in the low state is slack. That $p(l) < p(h)$ is also shown in Lemma A.5.

Finally, we find the necessary and sufficient conditions for capital to be liquidated in the low state. Note that, by Lemma A.5, $b_1(h) \leq a(h)k_0$ and, thus, capital is not liquidated in state h. If capital is to be liquidated in the low state, then it must be that $b_1(l) > a(l)k_0$ and $p(l) = gA$. By Lemmas 3 and A.5, and the assumption that $\bar{q} < \pi(h) - \lambda\zeta^+$, it must be that $b_1(h) = a(h)k_0$ and $p(h) < A$; otherwise, the entrepreneurs would strictly prefer to borrow against the high state. By Lemma 4, the optimal investment scale satisfies:

$$\chi'(k_0) = q(h)(a(h) + p(h)) + (1 - q(h))\,(a(l) + gA), \tag{51}$$

where $p(h) \leq \gamma^{-1}p(l) = \gamma^{-1}gA$, with equality if $b_1(l) < (a(l) + gA)k_0$. Thus, capital gets liquidated in the low state if and only if at the capital price $p(h) = \gamma^{-1}gA$, the investment scale satisfies $\chi(k_0) > \sum_s q(s)a(s)k_0$, i.e. at the lowest possible equilibrium prices of capital, the entrepreneurs still want to borrow more than their intermediate cashflows. ■

Proof of Proposition 4. In equilibrium, the investors hold the claims issued by the entrepreneurs, i.e. $v(s) = b_1(s)$ for all s. Since in equilibrium $b_1(h) \geq b_1(l)$ (see Propositions 2 and 3), using Proposition 1 and Corollary 1, the investors' lifetime welfare is:

$$\begin{aligned} U^I &= e - \sum_s q(s)b_1(s) + \lambda\left(\sum_s \pi(s)b_1(s) - \zeta^+\left(b_1(h) - b_1(l)\right)\right) + (1-\lambda)\left(W + \sum_s \pi(s)b_1(s)\right) \\ &= e + (1-\lambda)W \\ &= e + \lambda n W \\ &\leq e + \lambda\zeta^+\left(b_1(h) - b_1(l)\right), \end{aligned} \tag{52}$$

with equality if and only if the trading arrangements allocate the claims efficiently. The second equality follows from the fact that $b_1(h) > b_1(l)$ implies that $q(h) = \pi(h) - \lambda\zeta^+$ (Corollary 1). The third equality follows from $n\lambda = 1 - \lambda$. The last inequality follows from the fact that the traders' rents are bounded by the losses of the mechanism (Proposition 1). Also, since $W \geq 0$, it must be that $U^I \geq e$. ■

Proof of Lemmas 5-6. We first prove Lemma 6. Let U denote the investors' lifetime welfare in the laissez-faire equilibrium. By inspection of problem (P3), it is weakly optimal for the planner to set $T_0 = e - \chi(k_0)$ and $T_2(s) = gA(k_0 - k_1(s))\ \forall s$, as this minimizes the transfers $\{T_1(s)\}$ and, thus, the inefficient capital liquidations. The planner's problem therefore reduces to:

$$\max_{k_0,\{k_1(s),T_1(s)\}} \sum_s \pi(s)\left[(a(s)k_0 - T_1(s) + Ak_1(s)\right]$$

subject to

$$0 \le T_1(s) \le a(s)k_0, \tag{53}$$

$$0 \le k_1(s) \le k_0, \tag{54}$$

$$U \le e - \chi(k_0) + \sum_s \pi(s)\left(T_1(s) + gA(k_0 - k_1(s))\right). \tag{55}$$

It is without loss of generality to assume that (i) $k_1(s) = k_1$ for all s, and (ii) $T_1(h) \ge T_1(l)$ with equality if $T_1(l) < a(l)k_0$. Furthermore, it must be that $k_1 = k_0$ if $T_1(h) < a(h)k_0$, since it is cheaper for the planner to finance investment with intermediate cashflows than by liquidating capital inefficiently. Finally, constraint (55) must hold with equality, as otherwise the planner can reduce transfers to the investors and increase the entrepreneurs' welfare.

We now show that it is optimal to set $T_1(l) = a(l)k_0$. Suppose to the contrary that $T_1(s) = T_1 < a(l)k_0$. Then, since $k_1 = k_0$, from constraint (55) the planner's investment scale is $\chi(k_0) = e - U + T_1 < a(l)k_0$, as $U \ge e$ by Proposition 4. Consider next a small increase in the transfer of dT_1 and the investment scale of $dk_0 = \frac{dT_1}{\chi'(k_0)}$, so that constraint (55) is still satisfied. Then, the change in the entrepreneurs' welfare is:

$$\sum_s \pi(s)(a(s) + A)dk_0 - dT_1 = \left(\frac{\sum_s \pi(s)(a(s) + A)}{\chi'(k_0)} - 1\right) dT_1, \tag{56}$$

which is positive by Assumption 2(ii) as $\chi(k_0) < a(l)k_0$, a contradiction.

We now use the above results to show that k_0 and k_1 are given by equations (33) and (34).

Suppose that $p_U^{SP} = A$, as defined in equation (32). It is sufficient to show that the first-best investment scale is indeed feasible without inefficient liquidations. But this follows immediately since $k_0^{FB} \le \widetilde{k}_U$ and, thus, the first-best scale can be implemented with some transfer $a(l)k_0^{FB} < T_1(h) \le a(h)k_0^{FB}$.

Suppose that $gA < p_U^{SP} < A$. It is straightforward that $T_1(h) = a(h)k_0$, as otherwise the planner can always increase the entrepreneurs' welfare by increasing the investment scale and the transfer in the high state at the same time. Assume to the contrary that $k_1 < k_0$, and consider a small increase in the continuation scale of dk_1 and a reduction in the investment scale of $dk_0 = \frac{gA}{\sum \pi(s)a(s)+gA-\chi'(k_0)} dk_1 < 0$, so that constraint (55) is still satisfied. Observe that this is feasible: because $T_1(s) = a(s)k_0$ and $k_1 < k_0$, it follows that $k_0 > \widetilde{k}_U$, which together with $p_U^{SP} > gA$ implies that $\chi'(k_0) > \chi'(\widetilde{k}_U) > \sum \pi(s)(a(s) + gA)$. The entrepreneurs' welfare clearly increases, since it is equal to Ak_1 and k_1 has increased, a contradiction. Since we have established that $T_1(h) = a(h)k_0$ and $k_1 = k_0$, constraint (55) implies that $k_0 = \widetilde{k}_U$.

Suppose that $p_U^{SP} = gA$. It is again straightforward to show that $T_1(h) = a(h)k_0$ by the same reasoning as above. Assume to the contrary that $k_1 = k_0$. From constraint (55), it follows that $k_0 = \widetilde{k}_U$, which together with $p_U^{SP} = gA$ implies $\chi'(k_0) < \sum_s \pi(s)(a(s) + gA)$. Consider a small increase in the investment scale of dk_0 and an increase in the continuation scale of $dk_1 = \frac{\sum_s \pi(s)a(s)+gA-\chi'(k_0)}{gA} dk_0 > 0$, so that constraint (55) is still satisfied. Since the entrepreneurs' welfare is Ak_1, it increases, a contradiction. Therefore, we have established that $T_1(h) = a(h)k_0$ and $k_1 < k_0$. Maximization of the entrepreneurs' welfare w.r.t. k_0 subject to constraint (55) implies $\chi'(k_0) = \sum_s \pi(s)a(s) + gA$. Given k_0, the expression for k_1 follows directly from constraint (55).

We will now prove Lemma 5. Consider the transfers of consumption goods that the planner allocates to investors, as computed above, i.e. $T_0 = e - \chi(k_0)$, $T_1(l) = a(l)k_0 < T_1(h) \le a(h)k_0$, and $T_2(s) = gA(k_1 - k_0)\ \forall s$. Suppose that the planner lets each investor design the trading arrangement on his own and lets them trade the claims to the transfers $\{T_t(s)\}$ in these arrangements, except that she dictates to the investor that the feasibility constraint (12) in the mechanism design problem must hold with equality. Given the transfers, it is straightforward to show that Proposition 1 continues to hold, albeit the distortion to the claims valuation $\widehat{\zeta}$ must now be (weakly) larger than ζ. Because the investors are ex-ante identical and the claims to the transfers are allocated efficiently in the new trading arrangement, the investors' lifetime welfare must equal $U^I = T_0 + \sum_s \pi(s)(T_1(s) + T_2(s))$. Formally, let $v(s) = T_1(s) + T_2(s)\ \forall s$ and $\widehat{\mu}^*$ be the optimal trading arrangement subject to feasibility constraint (12) holding with equality. Then, the investors' lifetime welfare satisfies:

$$
\begin{aligned}
U^I &= T_0 + \lambda V\left(\{v(s)\}, \widehat{\mu}^*\right) + (1-\lambda)\left(W + \sum_s \pi(s)v(s)\right) \\
&= T_0 + \lambda\left(\sum_s \pi(s)v(s) - \lambda\widehat{\zeta}|v(h) - v(l)|\right) + (1-\lambda)\left(W + \sum_s \pi(s)v(s)\right) \\
&= T_0 + \sum_s \pi(s)v(s) - \lambda\widehat{\zeta}|v(h) - v(l)| + (1-\lambda)W,
\end{aligned}
$$

where $(1-\lambda)W = \lambda n W = \lambda\widehat{\zeta}|v(h) - v(l)|$ because $\widehat{\mu}^*$ allocates the claims to the transfers efficiently. Since this trading arrangement achieves the maximal feasible investor welfare for a given set of transfers, it must be optimal for the planner. ■

Proof of Proposition 5. First, clearly the equilibrium is constrained inefficient if the trading arrangements in it allocate the claims inefficiently among investors (see explicit conditions in Lemma A.3). Second, if the equilibrium features no risk-sharing, then $U^I = e$ by Proposition 4. By inspection of the planner's problem, setting $U = e$, we see that her investment and continuation scales coincide with those of the benchmark economy, which are clearly different from equilibrium. Thus, the equilibrium is again constrained inefficient. Finally, suppose that in equilibrium both the claims are allocated efficiently and there is partial risk-sharing. In this case, we know that in equilibrium $b_1(l) \geq a(l)k_0$ and $b_1(h) \in (a(l)k_0, a(h)k_0]$ (see Proposition 3 and Lemmas A.4 and A.5). There are thus three cases to consider.

Case 1. Suppose that in equilibrium $b_1(l) = a(l)k_0 < b_1(h) < a(h)k_0$, and thus there are no capital liquidations. Since the allocation of claims is efficient, from Proposition 4, it follows that $U = e + \lambda\zeta^+(b_1(h) - b_1(l))$ and, since $q(h) = \pi(h) - \lambda\zeta^+$, we have:

$$\chi(k_0) = \sum_s q(s)b_1(s)k_0 = \sum_s \pi(s)b_1(s) + e - U. \tag{57}$$

Therefore, $k_0 < \widetilde{k}_U$. The only scenario in which the planner chooses scale below $\widetilde{k}_U$ is when $k_0^{SP} = k^{FB}$ (see Lemma 6). But the scale in the laissez-faire equilibrium is below the first-best scale k^{FB} by Lemma 4 and Proposition 3 (see discussion following equation (24)). Thus, in this case, the equilibrium is constrained inefficient.

Case 2. Suppose that in equilibrium $b_1(s) = a(s)k_0 \ \forall s$, and thus there are no capital liquidations. By reasoning analogous to Case 1, it must be that:

$$\chi(k_0) = \sum_s q(s)b_1(s)k_0 = \sum_s \pi(s)a(s)k_0 + e - U, \tag{58}$$

which implies that $k_0 = \widetilde{k}_U$. For this to be consistent with equilibrium, it must be that $p(l) = \gamma p(h)$, $p(h) \in [\gamma^{-1}gA, A]$, and using the expression for γ:

$$\begin{aligned}\chi'(\widetilde{k}_U) &= \sum_s q(s)(a(s) + p(s)) \\ &= \sum_s \pi(s)a(s) - \lambda\zeta^+(a(h) - a(l)) + \frac{\pi(h) - \lambda\zeta^+}{\pi(h)}p(h).\end{aligned} \tag{59}$$

Plugging the bounds for $p(h)$, we conclude that this is an equilibrium if and only if:

$$\frac{1-\pi(h)+\lambda\zeta^{+}}{1-\pi(h)}gA-\lambda\zeta^{+}(a(h)-a(l)) \leq \chi'(\widetilde{k}_U)-\sum_s \pi(s)a(s) \leq \frac{\pi(h)-\lambda\zeta^{+}}{\pi(h)}A-\lambda\zeta^{+}(a(h)-a(l)). \tag{60}$$

On the other hand, from Lemma 6, the planner sets the investment scale to $\widetilde{k}_U$ and does not liquidate capital if and only if:

$$gA \leq \chi'(\widetilde{k}_U) - \sum_s \pi(s)a(s) \leq A. \tag{61}$$

By inspection of (60) and (61), we conclude that, in this case, the equilibrium may or may not be constrained efficient.

Case 3. Suppose that in equilibrium $b_1(l) > a(l)k_0$ and $b_1(h) = a(h)k_0$, and thus capital is liquidated in the low state. By reasoning analogous to Case 1, it must be that:

$$\chi(k_0) = \sum_s q(s)b_1(s)k_0 > \sum_s \pi(s)a(s)k_0 + e - U, \tag{62}$$

which implies that $k_0 > \widetilde{k}_U$. For this to be consistent with equilibrium, it must be that $p(l) = gA$, $p(h) = \gamma^{-1}gA$, and

$$\begin{aligned}\chi'(k_0) &= \sum_s q(s)\left(a(s)+p(s)\right) \\ &= \sum_s \pi(s)a(s) - \lambda\zeta^{+}(a(h)-a(l)) + \frac{1-\pi(h)+\lambda\zeta^{+}}{1-\pi(h)}gA \\ &> \chi'(\widetilde{k}_U).\end{aligned} \tag{63}$$

On the other hand, from Lemma 6, the planner would choose the same investment scale, i.e. $k_0^{SP} = k_0$ if and only if:

$$\chi'(k_0) = \sum_s \pi(s)a(s) + gA. \tag{64}$$

Combining equations (63) and (64) implies $(1-\pi(h))(a(h)-a(l)) = gA$. Thus, in this case, the equilibrium is generically constrained inefficient. ■

Proof of Proposition 6. See text. ■

B Verification of consumption non-negativity

In Section 2.1, we conjectured that the claims prices $q_t(s) = q(s)$ and $\sum_s q(s) = 1$ are part of equilibrium. We now verify these conjectures. To do so, it suffices to show that, at the conjectured prices, every investor's consumption is non-negative in every period.

We begin with $t = 0$. We need to show that all investors' consumptions are non-negative after they have purchased the claims from the entrepreneurs and participated in the trading arrangements. In equilibrium, the entrepreneurs' claims satisfy $b_1(s) < (a(h) + A)k$ for all s, for k such that $\chi(k) = (a(h) + A)k$. Therefore, the consumption of an impatient investor at $t = 0$ is:

$$c_0^{I,\beta=0} = e - \sum_s q(s)b_1(s) + \sum \omega, \tag{65}$$

where $\sum \omega$ denote the (possibly random) payments that he receives in the trading arrangement. Since $\omega \geq 0$ w.p.1 (Lemma A.3) and $e > 2(a(h) + A)k$ (Assumption 1), it follows that $c_0^{I,\beta=0} > 0$. On the other hand, the consumption of a patient investor at $t = 0$ is:

$$c_0^{I,\beta=1} = e - \sum_s q(s)b_1(s) - \omega, \tag{66}$$

where ω denotes the (possibly random) payment that he makes in the trading arrangement that he participates in. From Lemma A.3 and the traders' participation constraints (PC), we have that $\omega \leq b_1(h) < (a(h) + A)k$ w.p.1. Thus, $c_0^{I,\beta=1} > 0$ since $e > 2(a(h) + A)k$.

We next consider $t = 1$. Let us first look at the investors' resources prior to financing the operations of the traditional sector firms. The impatient investors' resources at $t = 1$ and state s are given by $b_1(s) - \sum v_s$, where $\sum v_s$ denotes the (possibly random) allocation of claims of the mechanism. From the feasibility constraint (FC), we have $b_1(s) \geq \sum v_s$. The patient investors' resources at $t = 1$ and state s are given by $b_1(s) + v_s$, where v_s is the (possibly random) allocation of claims that this investor receives in the mechanism he participates in. Since by Lemma A.3, $v_s \geq 0$ w.p., we must also have $b_1(s) + v_s \geq 0$. Next, observe that the aggregate resources of the investors at $t = 1$ are given by $b_1(s)$. There are two scenarios in equilibrium. Either (i) there are no capital liquidations, or (ii) there are liquidations and the resources required to operate the traditional sector firms are: $p(s)\widehat{k}(s) = b_1(s) - a(s)k_0$. But, in both cases, the investors' resources $b_1(s)$ are more than sufficient to finance these firms' operations. Moreover, every investor (impatient or patient) is willing to do so at an interest rate of one, since the investors do not discount consumption between $t = 1$ and $t = 2$.

The investors' consumption at $t = 2$ is trivially non-negative, since they (if they finance these firms) receive positive repayments from the traditional sector firms.

Finally, it is straightforward to extend the results in Lemma A.3, with regards to non-negativity of allocations and payments, to trading arrangements that are required to be efficient. Then, by arguments analogous to those for the equilibrium, we can show that the investors' consumption is also non-negative at the planner's allocation.

C Sorting in secondary markets

We now provide a microfoundation for the sorting of investors in secondary markets; namely, that an investor posts his claims in a trading arrangement if and only if he is impatient. To do so, we will generate common knowledge of gains from trade within each trading arrangements; that is, we suppose that the traders within a given trading arrangement observe the preference type β of the investor who posts his claims for sale in that arrangement. We then use arguments akin to Milgrom and Stokey (1982) to show that there cannot be trade between investors based solely on information heterogeneity.

For concreteness, we specify the following timing for the sorting decision. First, each investor purchases the claims $\{d_t(s)\}$ and designs the trading arrangement μ in which to sell them. Second, he learns his type (β, x). Finally, he decides whether to post his claims in the arrangement μ, and whether to become a trader in the trading arrangements of designed by other investors. The timing assumption that the mechanism is designed before the types are learned is inessential for the arguments that follow.

We make the following indifference-breaking assumptions. First, an investor posts his claims in a trading arrangement only if his net expected payoff from doing so is strictly positive. Second, an investor becomes a trader if and only if he is willing to make a strictly positive payment for some portfolio of claims. This has two immediate implications: (i) an investor is a trader if and only if he is patient, and (ii) an investor posts his claims in the trading arrangements if he is impatient. Therefore, we are only left to prove that in equilibrium no patient investor would post his claims in a trading arrangement.

Let $\widetilde{V}(x)$ denote the net expected payoff to a patient investor from posting his claims in the trading arrangement if he has received signal x, then

$$\widetilde{V}(x) = \mathbb{E}\left\{\sum_{i \in I} p^i - \sum_{i \in I} v_s^i | x\right\}, \tag{67}$$

where $\{p^i, v_h^i, v_l^i\}_{i \in I}$ denotes the allocations of trader $i \in I$ in the arrangement and I is the (possibly random) set of traders participating in the trading arrangement. Thus, this investor's net expected payoff from posting his claims is given by the expected payments he receives from the traders minus the expected value of the claims that he transfers to them. For now, we leave the matching process by which traders are matched with trading arrangements unspecified, as it is not essential for our arguments.

Let $P = \{x : \widetilde{V}(x) > 0\}$ denote the equilibrium set of patient investors who post their claims for sale. Thus, in equilibrium, if the traders are matched with a mechanism of a patient investor, they know that this investor has signal $x \in P$. As a result, the traders' participation constraints imply that the traders' (aggregate) expected payoff from participating in the trading arrangement of a patient investor must be non-negative, i.e.

$$\mathbb{E}\left\{\sum_{i\in I} v_s^i - \sum_{i\in I} p^i | x \in P\right\} \geq 0. \tag{68}$$

But then $P = \emptyset$, since otherwise the payoff in (68) would be strictly negative, a contradiction.

Thus, we have shown that a fraction λ of investors post their claims in trading arrangements, whereas a fraction $1-\lambda$ of investors becomes traders in those trading arrangements. In the text, we had assumed that parameters satisfy $\lambda n = 1-\lambda$ and that there is an exact matching of traders with arrangements: each trading arrangement is matched with n randomly selected traders. For the comparative statics illustrated in Figure 2, it is useful to generalize this matching process slightly in order to avoid the integer problem. To this end, let $\underline{n} \equiv \lfloor \frac{1-\lambda}{\lambda} \rfloor$, $\bar{n} \equiv \lceil \frac{1-\lambda}{\lambda} \rceil$ and $\alpha \equiv \frac{\frac{1-\lambda}{\lambda} - \underline{n}}{\bar{n} - \underline{n}}$, and suppose that a fraction α of randomly selected trading arrangements is matched with $\bar{n}$ randomly selected traders and the rest with $\underline{n}$ traders. This matching process ensures that all trading arrangements and traders are matched and, when $\underline{n} = \bar{n}$, then as in our baseline case each trading arrangement is matched with exactly $n = \frac{1-\lambda}{\lambda}$ traders. Furthermore, as λ increases continuously from 0 to $\frac{1}{2}$, the number of traders in each trading arrangement decreases and $\lambda\zeta^{+}$ increases continuously to $\frac{1}{2}\zeta^{\max}$.