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Theoretic Approach*

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Ruling out Speculative Hyperinflations: a Game Theoretic Approach *

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INTRODUCTION

An old problem faced by monetary economist was the integration of monetary theory with the theory of value. By this, I mean being able of proving the existence of an equilibrium with a strictly positive value for money (i.e., a finite price level). As it was noted by several authors (see Hahn 1966), a sufficient condition to obtain a positive value for any good is that the excess demand for that good be strictly positive when its price is zero. Thus, it should not be an inconvenient to show that an equilibrium exist with a positive value for currency in a commodity money standard. As long as there is non-satiation, the marginal utility will always be strictly positive, and a positive excess demand for the good will exist if the price is zero. A similar reasoning would show that a paper currency that is backed with a particular commodity will have a positive excess demand if the price is zero. However, there is absolutely no reason why the excess demand for worthless pieces of unbacked paper will be positive at a zero price. Even more, the only equilibrium price for unbacked pieces of paper in Arrow-Debreu economies is zero. Thus, the obvious question to ask is why worthless pieces of unbacked paper have positive value. Even though this is not a new question, it was only recently that it is presented in the first pages of almost every volume that is

concerned with the fundamentals of money.¹ The most probable reason for this fact is that during the last two decades, most monetary authorities suspended convertibility and there is no sign that they would restore it in the future. As Wallace(1980) states

. . . *inconvertibility* means that it is known with certainty that the issuer does not now and will never in the future stand ready to convert fiat money into a commodity. Viewed this way, fiat money systems have, I think, been rare. . . now, restoration of convertibility seems unlikely. So, and perhaps for the first time, theories of inconvertible money are of practical importance.

Given that standard economic models could not generate valued unbacked currency as an equilibrium outcome, monetary theorist followed two different paths. One was to exogenously impose some condition that would ensure that a money demand could be obtained from the first order conditions of optimizing agents, like money in the utility (production) function or cash-in-advance models.

The second, was to develop models with carefully described environments that could endogenously explain valued fiat money. Examples of these are the overlapping generations and borrowing constraints models.²

¹See Sargent(1987), Blanchard and Fischer(1989), Starr(1989), Kareken and Wallace(1980).

²A third approach was to develop models where the explicit pairing of agents implies that only bilateral exchange is possible, like Ostroy(1972) and Starr(1973), but those models are not really dynamic equilibrium theories.

All those models share two problems. The first one is that, for a general class of utility functions, they exhibit multiplicity of equilibria. Typically, there exist one stationary equilibrium where the price level responds only to fundamentals. But there also exists an infinite set of other equilibria, where the price level grows without bound; equilibria that we call (as many other do) speculative hyperinflations. We consider it a problem of those models because speculative hyperinflations seem to be empirically irrelevant.³ The existence of these speculative hyperinflations is due to the fact that even though these models generate a demand for money, they all share the property that the excess demand for money is zero when the price of money (the inverse of the price level) is zero. This implies that nothing prevents the price of money to converge to zero. And this is as it should be, because all these are models of pure inconvertible money. Obstfeld and Rogoff(1983) and Wallace (1981) have shown that speculative hyperinflations can be ruled out if the issuer guarantees a minimal redemption value for money. The redemption value is creating a positive excess demand for money when its price is zero. So, it prevents the value of money from being zero and it makes the economy work as a convertible money system or a commodity money system. But we do not observe the issuers of modern currencies guaranteeing any redemption value. So, neither Obstfeld and Rogoff nor Wallace results can be used to explain why speculative hyperinflations are empirically irrelevant in our modern unbacked currency economies. The second

³See Flood and Garber(1980), Hamilton and Whiteman(1985) and McCallum(1987)

problem of those models is the inability to explain which particular object will be used as money. But real world paper monies share a very well specified feature. All of them are issued by government institutions which introduce the currency into the economy.

In this paper, we develop a monetary model which attempts to give an answer to the two problems mentioned above. The most important feature of the model is the explicit introduction of the government with well specified utility function and strategy space. The assumed preferences of the government imply that it does not like inflation. We also assume that the government has the power to tax the agents and back the money stock at any time, using the tax revenues. Then, we show that the unique equilibrium which is sequentially rational in the sense of sub-game perfection is the stationary monetary equilibrium. In other words, speculative hyperinflations can occur only if the agents are following sub-optimal strategies. In addition, along the equilibrium path, convertibility is never observed for any time period. The strategy of the government is doing nothing if the economy follows the stationary equilibrium path, but convert the currency if the economy enters a speculative hyperinflation. So, the system behaves exactly as a convertible system.

The intuition of this result is the following. Assume that at some date in the future the US dollar (or the german mark, or the yen, or..) enters into a speculative hyperinflation. Would the FED just remain inoperative while the dollar loses its value forever?. No, because the

costs of announcing a backing scheme are probably small compared to the costs of the hyperinflation. But rational agents know that the FED would behave in this way. So a hyperinflation will never take place even though the FED will never back the money stock.

We do not want to argue that it is possible to construct very general models of fiat money where speculative hyperinflations cannot occur. Our interpretation is that a definition of convertibility based only on what happens along the equilibrium path is not appropriate. We do not provide a formal definition of convertibility but show that it should take into account the ability of the issuer to back the money stock in special circumstances, rather than the decision of actually backing it. In other words, the model suggests that out of equilibrium path strategies are important to decide whether the system is a pure inconvertible money one or not.

Adopting this view leads to important conclusions concerning the empirical relevance of models of pure inconvertible money. Is the 1990 US economy a pure inconvertible money system ?. If we assume (which seems most reasonable to do) that the FED does not like hyperinflations and that it has the ability to restore convertibility, we must conclude that the US behaves as a convertible currency system, even though convertibility might never be restored. Did a pure inconvertible money system ever exist?. This is a much harder question to answer. But if we accept a definition of convertibility based on out of equilibrium path

strategies we should agree that pure inconvertible money systems are less relevant than what the literature on monetary theory suggests. So, the answers we offer to the two problems stated above are based on the same fact: modern economies are essentially convertible systems. We should not observe speculative hyperinflations, because these are not equilibria outcomes in convertible systems. The object that will be used as currency is trivially determined by the will of the issuer in a convertible system.

The main difference between this approach and the one in O&R or Wallace is that the decision of converting the money stock is made endogenous by describing the decision problem of the government. This allows us to evaluate alternative policies from the point of view of the government. Also, we show that the optimal policy rules out speculative hyperinflations and is consistent with the observable fact that along the equilibrium path, governments are not converting the money stock.

Arguing that modern economies are convertible systems explains why hyperinflations are empirically irrelevant and explains why the value of money cannot be zero. However, it does not solve the much more interesting problem of explaining which should be the equilibrium price of money. We still need models with frictions (like Sargent and Wallace (1983)) which explain the equilibrium value of a convertible currency. The fact that it is convertible guarantees that the value cannot be zero, but does not explain why it is used as a mean of transaction or

which must be its equilibrium value (which will clearly depend on the demand for transaction purposes). So, we do not offer an explanation to the puzzle of the value of money stated at the beginning . We argue that the puzzle resides on the particular value money has, rather than on money having positive value, as it is often found in the literature.

The paper proceeds as follows. In section 1, we describe the model which introduces money through a cash-in-advance constraint. We solve for competitive equilibria and show how speculative hyperinflations can be solutions of this model. In section 2 we define the "convertibility game" by describing the strategies and payoffs of every agent and show how different government strategies can rule out speculative hyperinflations. In section 3 we define the equilibrium concept that will be used and state the main proposition. Section 4 analyzes the same problem using an alternative government payoff function. Section 5 contains the conclusions of the chapter.

THE MODEL

In this section, we develop the model, find the set of competitive equilibria and describe some features of the mapping from the set of equilibria to the space of agents' utilities. This is a two good endowment economy, inhabited by a large number of identical agents with utility function

$$W(x,y) = \sum_{t=0}^{\infty} \beta^t \{ U(x_t) + V(y_t) \}$$

$$\text{where } x = \{x_t\}_{t=1}^{\infty}, y = \{y_t\}_{t=0}^{\infty}$$

U and V are increasing, concave, differentiable and the limit of the first derivatives approaches infinity as the argument approaches zero and β is the discount factor. At every time period, the agents are endowed with one unit of productive time which they devote entirely to production (there is no labor-leisure choice in this model). The technology is

$$x_t + y_t = 1$$

To introduce money into the model, we assume that cash must be used to buy good y.⁴ Therefore, we impose the following Clower constraint

$$p_t \cdot y_t \leq M_t$$

where p_t is the money price of good y and M_t is the money held by the agent at time period t. Note that because of the linear technology, the money price of x must be the same as the money price of y.

If we let T_t be a lump-sum transfer in money that the government makes to the representative consumer at time t, the problem of the consumer is

$$\text{Max } W(x,y) = \sum_{t=0}^{\infty} \beta^t \{ U(x_t) + V(y_t) \}$$

$$\{x_t, y_t, M_{t+1}\}$$

$$\text{s.t. (1) } p_t(x_t + y_t) + M_{t+1} \leq p_t + M_t + T_t$$

$$(2) \quad p_t \cdot y_t \leq M_t$$

Let $M = \{M_t\}_{t=0}^{\infty}$ be the sequence of money demands, $M^s = \{M_t^s\}_{t=0}^{\infty}$ the

⁴For a motivation of this constraint see Lucas (1980)

sequence of money supplies and $p = \{p_t\}_{t=0}^{\infty}$ the sequence of money prices. Also, let $B(p) = \{(x,y,M) \text{ such that (1) and (2) above hold}\}$. The set B is the feasible set. Then, we can define a perfect foresight competitive equilibrium in the following way:

DEFINITION 1: The point $\{x,y,M,p\}$ is a perfect foresight competitive equilibrium (PFCE) given M^s if

$$i) (x,y,M) \in \operatorname{argmax} W(x,y) \text{ st } (x,y,M) \in B(p)$$

$$ii) x + y = 1 \quad \text{and} \quad M = M^s$$

Using the first order conditions of the representative consumer's problem plus market clearing conditions the following must hold in equilibrium

$$(3) \quad \beta \cdot V'(y_t)/p_t = U'(1-y_{t-1})/p_{t-1}$$

$$(4) \quad y_t = \min \{ m_t, y^* \}$$

where m_t is the real quantity of money and y^* is such that

$$V'(y^*) = U'(1-y^*)$$

In addition, in any equilibrium, the following transversality condition must be verified.⁵

$$(5) \quad \inf (M_T - p_T \cdot y_T) = 0 \quad \text{for all } t.$$

Now, let us characterize the set of PFCE under the assumption that the money supply grows at a constant rate. Then

$$M_t^s = (1+\pi) \cdot M_{t-1}^s \quad \text{and} \quad M_0^s$$

⁵It can be shown that if the condition does not hold, there exists an alternative allocation which attains a higher level of utility.

given where π is the rate of money growth which is assumed to be greater or equal to zero.⁶

Then, equilibrium conditions 3) and 4) become

$$(3') \quad m_t \cdot U'(1-y_t) = \{ \beta/1+\pi \} \cdot m_{t+1} \cdot V'(y_{t+1})$$

$$(4') \quad y_t = \min \{ m_t, y^* \}$$

which can be written as $m_{t+1} = F(m_t)$

The behavior of this equation depends crucially on the value of

$$\sigma(m_{t+1}) = [m_{t+1} \cdot V''(m_{t+1})] / V'(m_{t+1})$$

If σ is greater than 1, then $F'(m_t)$ is negative; if σ is less than 1, then $F'(m_t)$ is positive. So, F is monotonic only if σ is always greater than one or always lower than one. Also, given the concavity assumptions on U and V there exists a unique stationary monetary equilibrium. Two examples are plotted in figure 1. Figure 1.a corresponds to the case where σ is always greater than one, so F is strictly decreasing. Figure 1.b corresponds to the case where σ is lower than one so F is increasing.

Consider figure 1.b. The point m^* corresponds to the unique stationary monetary equilibrium. Also the point $(0,0)$ is a stationary solution; it corresponds to the nonmonetary equilibrium. Now, consider any $m_0 \in (0, m^*)$. It is possible to find an infinite decreasing sequence $\{m_t\}$, which is also a solution of the difference equation, as illustrated in figure 1.b. In any of these equilibria, the real value of the money

⁶This condition rules out hyperdeflations. We assume it to concentrate on hyperinflations

stock goes to zero because the price level grows faster than the money supply. This is what we mean by a speculative hyperinflation.

Note that no speculative hyperinflation can exist in the case of figure 1. However, it is possible for another type of nonstationary solutions to exist in the case of figure 1 or for more general cases

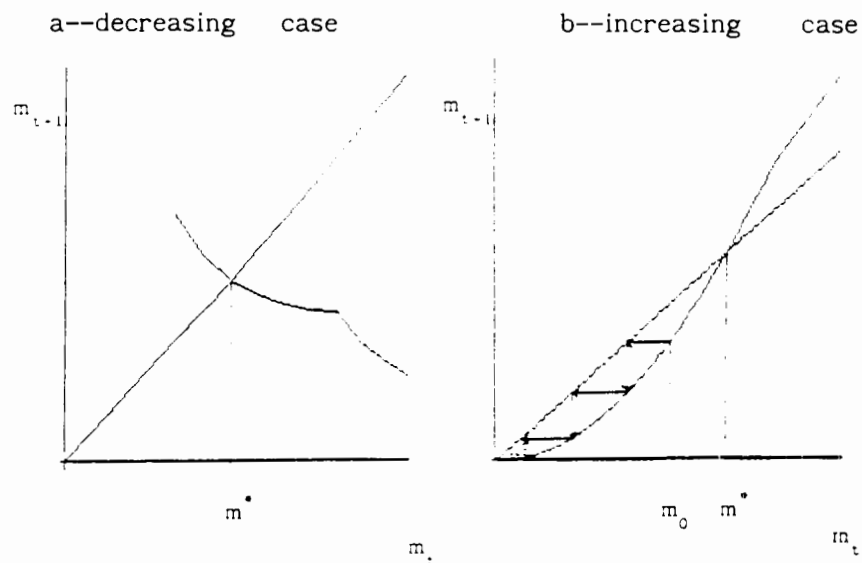


Fig. 1.--alternative shapes for the difference equation F

where F is not monotonic. As we are mainly interested on the case of speculative hyperinflations, we will assume that the utility function is such that F behaves as in figure 2. The conditions under which the analysis extends to other non-stationary solutions will be considered in the future. All these results can be summarized in the following proposition

PROPOSITION 1: Let $\pi \geq 0$, and let the difference equation $F(.)$ behave as in figure 1.b . Then, there exists a continuum of PFCE. In all of

these equilibria but one (the stationary one where $m_t = m^*$ for all t) the value of the money stock converges to zero as the time period goes to infinity. In addition, there exists a one-to-one correspondence between the set of PFCE and the set of initial conditions $[0, m^*]$.

Pf: See appendix.

Next, we derive an important result concerning the welfare level of the representative agent at different PFCE. This result will be used in the following sections.

PROPOSITION 2: If the assumptions of proposition 1 hold, the utility level attained by the representative agent at the stationary monetary equilibrium is higher than at any other PFCE.

Pf: See appendix.

The proof of proposition 2 is based on the fact that inflation reduces welfare because of the cash-in-advance constraint. At any speculative hyperinflation, the inflation rate is higher than the inflation rate at the stationary monetary equilibrium for all time periods. Therefore, the distortion is higher at all time periods.

THE GAME

In this section we model a game played by a sequence of

administrations, that constitute the government, and the agents. Each administration will be considered as a single player, with its own strategy space and payoff function and we assume that each of them lasts two periods in office.⁷ The main ingredient of the game is that at any time period, the current administration has the option of setting some price at which it will exchange the money stock for goods. In this section, we assume that each administration cares about the utility of the agents from the time they take office till infinity (in the next section we will see how the same results can be obtained with a very different assumption about governments preferences). Then, we show how different strategies used by the administrations induce the economy to be always at the stationary monetary equilibrium. One of such strategies is based on Obstfeld and Rogoff's result: back the money stock every period at a positive price no higher than the stationary price. But another strategy is just to back the currency at a price higher than the equilibrium price only if m_t is different from m^* . As in equilibrium $m_t = m^*$, convertibility is never announced and the system "looks like" a pure inconvertible money one. So, the existence or not of convertibility along the equilibrium path might be irrelevant for the determinacy of equilibrium in this framework. We start the description of the game by defining the strategies and payoffs of each administration.

⁷Modeling the government as a sequence of administrations is very important for this game. It allows for strategies which include punishments to past administrations and allows for the existence of a perfect equilibrium.

We will assume that at even time periods, a new administration (A_t , for t even) comes to office and stays for two periods. At the beginning of every time period, A_t chooses the rate at which it will exchange the money stock for goods: e_t . The rate e_t is expressed as goods per unit of currency (the same units as the inverse of the price level). In addition, each administration has to choose the rate of money growth. In this model, with lump-sum taxes, all administrations will optimally follow Friedman's rule, which means a negative rate of money growth. However, we want to analyze the case of positive money growth, to make it compatible with the results of proposition 1. Thus, we will assume that every administration is forced to raise a given amount of revenue, g^{\min} , through inflation tax, such that the rate of money growth cannot be lower than a positive number π^* , where π^* is the minimum rate of money growth such that the inflation tax revenue is equal to g^{\min} .⁸ However, to be sure that the restriction is feasible, we will assume that g^{\min} is small. Thus, we will not worry about the existence of a rate of money growth that satisfies the restriction. This assumption is formally stated in the appendix, where the feasibility of the optimal strategy is proved.

We will also assume that the convertibility operations are costly,

⁸It is possible to develop a model where lump-sum taxes are not available and the optimal policy for the government implies a positive rate of money growth (See Lucas and Stokey(1983)) but it would make the model unnecessarily complicated.

because of fixed expenses like setting an office to carry on the operations, paying the wage of the employees and the consulting fee of the economist that wisely chooses the value for π and e . We assume that there is a fixed cost, independent of the amount converted, if the government decides to establish convertibility. Thus,

$$C_t = \begin{cases} C > 0 & \text{if } e_t > 0 \\ 0 & \text{otherwise} \end{cases}$$

As A_t is not endowed with goods, it must raise taxes to be able to make the convertibility transactions. We will assume that if A_t is willing to convert, the revenue it raises must be enough to convert the whole money stock. We assume that A_t can resort to lump-sum taxation. Then, if the tax on goods is τ_t , we require

$$(6) \quad \tau_t \geq e_t \cdot M_t + C_t$$

Once the agents have chosen a value q_t^a to convert, we assume that A_t gives back any goods and currency it owns after the conversion. So

$$(7) \quad v_t^m = q_t^a + R_t(\pi)$$

$$(8) \quad v_t^g = \tau_t - q_t^a \cdot e_t$$

where v_t^i is the transfer at time t for $i=m,g$; and $R(\pi)$ is the revenue that the government gets from inflation.⁹ We will assume that τ and v 's evolve according to (6)-(8).

⁹We are assuming that the government transfers back to the consumers in a lump-sum fashion the revenue it gets from inflation. We could instead assume that the revenues are used to finance government expenditures, but it would not change the results of the chapter

So, the strategy of A_t at any time period consists on picking up the values for e_t and π_t , and the taxes and transfers follow (6)-(8).

Formally, if s_t is the strategy of the current administration

$$s_0 \in \mathbb{R}_+^2$$

$$s_t: H_{t-1} \rightarrow \mathbb{R}_+^2 \text{ such that (6) - (8) hold for every } t$$

where H_t is the set of possible histories up to period t , to be formally defined below.

Let S_t be the set of feasible s_t . The strategy for the administration that takes office at t , t even, is the pair $\{s_t \times s_{t+1}\}$

Regarding the payoff function, we assume that the government chooses strategies to maximize the utility function of the consumers from the time they take office till infinity.

We now turn to the problem of the consumer. Every period, consumers pick up $c_t = \{x_t, y_t, M_{t+1}, q_t^a\}$ subject to

$$(9) \quad p_t \cdot (x_t + y_t) + M_{t+1} \leq p_t \cdot (M_t - q_t^a) + q_t^a \cdot e_t \cdot p_t - \tau_t \cdot p_t + T'_t$$

$$(10) \quad y_t \cdot p_t \leq (M_t - q_t^a) + q_t^a \cdot e_t \cdot p_t$$

$$(11) \quad q_t^a \leq m_t$$

where T'_t is the sum of the transfers that G makes to the consumers (which include T_t, v_t^g, v_t^m) after the convertibility operations have been done. The first two conditions are just the budget constraint and the cash in advance constraint, after the conversion operations had taken place. Thus, consumers go to the market with some money and some cash goods, rather than with only money. Then,

$c_t: H_{t-1} \times S_t \times \mathbb{R} \rightarrow \mathbb{R}_+^4$ such that (9) - (11) hold for every t . where \times means cross product.

Let C_t be the set of all feasible c_t . Then, a particular history up to time t is defined as

$$h_t = \{c_i, p_i, s_i\}_{i=0}^t$$

and H_t is the set of all feasible histories.

The payoff function for the representative agent is the utility function described in section 2.

We assume that the convertibility game is played at the beginning of every period. For every time period, there are two relevant decision nodes. In the first one, the current administration plays by choosing a point in S_t , i.e. it decides (possibly contingent on the history) the price at which is willing to make the conversion operations, and the rate of money growth. In the second, agents play by choosing a point on C_t , i.e., they decide how much money to convert into goods to the government. Meanwhile, markets open and private agents make their desired transactions. At the beginning of next period the same process is repeated.

The timing of this extensive form game is depicted in figure 2. The symbol * indicates a decision node. Under each decision node it is indicated who plays, and above the figure the time period is indicated.

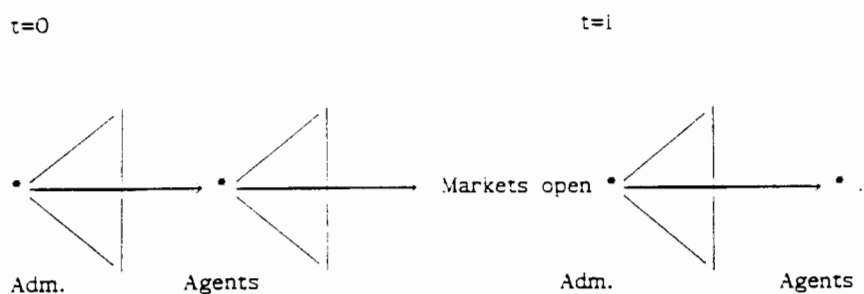


Fig. 2.--the game

Before going ahead, two interrelated assumptions which will play a very important role in the description of the equilibria should be explained. First, we will impose symmetric behavior on the atomic agents. Thus, we will not consider deviations of only a subset of the agents. We will only look at symmetric equilibria where all atomic agents behave as the representative agent. Second, we will impose competitive behavior on atomic agents, in the sense that they always assume that their actions do not affect the environment. So, they take prices, government strategy and all other agents' strategies as given when they solve their optimization problem. These assumptions imply that no Folk theorem type of equilibria can be obtained in which agents use "strategic type" of behavior, because they would never impose a punishment which hurts themselves. In this sense, the game we formulate is different than those of Barro and Gordon(1983), Rogoff(1989) and Stokey(1989). They show that given the preferences that the atomic agents have in their models, "collusion" type of equilibria can be obtained even though each agent maximizes its own utility function. That result depends on the preferences being a function of what all other agents do. However, in this convertibility game, you will convert all your money stock if the government offers a better deal than the market, independently of what the other agents are doing. We make these assumptions because we want to stay closer to the concept of a competitive equilibrium .

Next, we define a best response for the representative agent, given the prices and given the administrations strategies.

DEFINITION 2: The point $c = \{c_t\}_{t=0}^{\infty}$ is best response given $\langle p, s \rangle = \langle p_t, s_t \rangle_{t=0}^{\infty}$ if it maximizes $V(x, y)$ subject to (9)-(11).

Note that the novelty in the consumer's problem is the introduction of q_t^a as a choice variable. The solution will imply $q_t^a = m_t$ if $e_t \cdot p_t > 1$ and $q_t^a = 0$ if the inequality is reversed. The consumer is indifferent if $e_t \cdot p_t = 1$. Before defining the equilibrium concept, it is worth considering the effect on the set of PFCE of all administrations following arbitrary strategies $s = \{s_t\}_{t=0}^{\infty}$. Let us introduce a definition of a PFCE given that sequence of strategies.

DEFINITION 3: The point $\langle c, p \rangle$ is a PFCE given s (PFCE(s)) if

- i) c is best response given $\langle p, s \rangle$
- ii) $x + y = 1$, $M = M^s$.

The following proposition is useful in characterizing the set PFCE(s).

PROPOSITION 3: Let s be the sequence of strategies followed by the sequence of A 's, and let m^0, p^0, x^0, y^0 be a PFCE for this economy. This is also a PFCE(s) only if:

$$p_t^0 \cdot e_t \leq 1 \quad \text{for all } t$$

where e_t is the value corresponding to s .

Pf: Assume not. Then, agents will prefer to convert all their money

rather than buying goods. So, their relevant price at time t is $p_t = 1/e_t < p_t^0$.

But $U'(y_{t-1}^0)/p_{t-1}^0 = \beta \cdot U'(y_t^0)/p_t^0 < \beta \cdot U'(y_t^0)/(1/e_t)$

which violates FOC for the agent. This

implies that agents are not maximizing which violates condition ii) for PFCE(s) ■

The intuition is clear. For a PFCE the sequence of prices must fulfill the difference equation. By backing the stock of money at period t , the current administration imposes an upper bound on p_t . Any sequence which implies a price at t higher than that upper bound can not be equilibrium any more. This is the reason why Obstfeld and Rogoff(1983) get rid of all speculative inflations by guaranteeing a minimum positive value of the money stock at every period. Their result is equivalent to setting

$$e_t = \varepsilon \cdot (1+\pi)^t \quad \text{for all } t, \varepsilon > 0$$

As in a speculative hyperinflations the price level goes to infinity at a rate higher than $(1+\pi)$, there always exist a t such that $e_t \cdot p_t > 1$. By proposition 3, no speculative hyperinflation can be an equilibrium.

But consider a strategy which includes the following values for e :

$$e_0^M = 0$$

$$e_t^M = \begin{cases} 0 & \text{if } m_{t-1} = m^* \\ 1/p_{t-1} & \text{if } m_{t-1} \neq m^* \text{ or } t > 0 \end{cases}$$

In following this strategy, the government converts the currency only if the economy is in a hyperinflationary equilibria. If $m_t = m^*$ for all

t, the government will not back the currency. Can speculative hyperinflations be equilibria?. No, because at any of them, there will exist a time period t such that

$$e_t \cdot p_t = (1/p_{t-1}) \cdot p_t > 1$$

which contradicts the conditions of proposition 3. This discussion proves the following fact

FACT: Let $s^C = \{ e_t = \varepsilon (1+\pi)^t, \varepsilon > 0, \pi_t = \pi^* \}$ and $s^M = \{ e_t = e_t^M, \pi_t = \pi^* \}$, where e^M is the one described above. Then, the sets $\text{PFCE}(s^M)$ and $\text{PFCE}(s^C)$ have as unique element a monetary stationary equilibrium.

This fact just states that both O&R strategy where G converts the money stock every period and the strategy s^M where along the equilibrium path convertibility is never observed, induce a stationary monetary equilibrium as a unique outcome. This leads to

REMARK: The existence or not of convertibility along the equilibrium path is not the only relevant piece of information on the sequence of strategies $s = \{s_t\}_{t=0}^{\infty}$, for the determination of equilibria.

This remark shows that the definition of pure fiat money as opposed to convertible money, should not depend on the existence of convertibility along the equilibrium path (i.e., on the fact that we observe or not convertibility) but rather on the ability of the issuer to convert the money stock at any point in time and on its preferences.

EQUILIBRIA OF THE GAME

In this section, we introduce the equilibrium concept we will be dealing with, and state the main proposition. Given the dynamic structure of the game, we will require the agents to be sequentially rational. This means that they acknowledge that their actions might affect the strategies of the other players. So, the appropriate definition of equilibrium to use should be based on the concept of sub-game perfection.

A point worth looking at is that this game may lead to equilibrium allocations which are not PFCE. For example, it could be possible for the administrations to use specific strategies in order to support temporary deflations, to avoid or reduce the welfare cost of the cash-in-advance constraint. This sequence of strategies would require the whole money stock being retired from circulation and the entire product being taxed from time to time.

We will assume that this is not the case because we are interested on a decentralized economy. Otherwise, it does not make sense to consider a medium of exchange or to impose a cash-in-advance constraint. Then, the administrations cannot support a price level lower than the stationary one. Then, in any equilibrium of this game, the real value of money can never be higher than m^* . By proposition 2, the equilibrium that maximizes A's payoff is the stationary monetary equilibrium described in proposition 1.

We are ready to state the equilibrium concept.

DEFINITION 4: The point $\{c,p,s\}$ is a Subgame Perfect Competitive Equilibrium (SPCE) if

- i) $\{c\}$ induces a best response for every $H_{t-1}, s_t, p_t, \{s_i, p_i\}_{i=t+1}^{\infty}$ given, for all t .
- ii) $\{s_t, s_{t+1}\}$ maximize A_t 's payoff for every $H_{t-1}, \{a_i\}_{i=t}^{\infty}$ and s_{t+j} , $j \geq 2$ given, all t even, and $\{s_{t+1}\}$ maximizes A_t 's payoff for every H_t, s_{t+j} , $j \geq 2$, $\{a_i\}_{i=t+1}^{\infty}$ given, all t even.
- iii) $x_t + y_t = 1$, all t and $M^d = M^s$

The first two conditions are standard for a sub-game perfect equilibrium. They require that the equilibrium strategies induce a Nash for every node of the game, even out of equilibrium path. Note that while the agents consider the price sequence as given, the government does not. This is so, because agents are assumed to behave competitively, but the government knows that it can affect the price sequence through the conversion operations. The final condition requires market clearing on the goods and money markets.

Given that the administrations prefer the stationary equilibrium compared to the speculative equilibria, they will try to use the cheapest strategy to support the stationary equilibrium. Given that the convertibility operations are costly, they will try to use a strategy that prescribes convertibility only out of equilibrium path, i.e. one that includes a decision rule for e_t similar to e_t^M . However, for that

rule to be credible, possible punishments should be considered. To this end, we define a new state variable Ω_t , which will indicate to the future administrations whether a punishment should be imposed or not. Let d_t be 1 if the administration deviates at t , and equal to 0 if the administration does not deviate at t ; and let u_t be one if there is a punishment at t , and 0 if there is none. Then, we define

$$\Omega_t = \sum_{i=0}^t (d_i - u_i)$$

so, Ω is one when a punishment must be done.

The way a single administration punishes a past administration, is by increasing the rate of money growth as soon as they get to office. This implies a higher distortion faced by the past administration, while it does not affect the welfare of the representative agent from now to the future, leaving unchanged the payoff of the new administration.

Now, we are ready to state our main result

PROPOSITION 4: Let all administrations be playing π^* . Assume that the welfare of the representative agent is higher under Obstfeld and Rogoff's rule than under autarky. Then, the unique PFCE that can be sustained as a SPCE is the stationary monetary one.

Pf: First, we will prove that the stationary monetary one is a SPCE. Consider the following strategy s_t

$$\pi_t = \begin{cases} \pi^P & \text{if } \Omega_t = -1 \\ \pi^* & \text{otherwise} \end{cases}$$

$$e_t = \begin{cases} 1/p_{t-1} & \text{if } m_t \neq m^* \text{ for odd } t \\ 0 & \text{otherwise} \end{cases}$$

where $\pi^P > \pi^*$ is the punishment rate of money growth, that satisfies

$$\begin{aligned} & U(x^0) + V(1-x^0) - C + \beta \{U(x^*) + V(1-x^*)\} \geq \\ & \geq U(x'_1) + V(1-x'_1) + \beta \{U(x''_2) + V(1-x''_2)\} \end{aligned}$$

where x^* is the optimal choice of the credit good by the consumer that faces $\pi_t = \pi^*$ for ever; and x'_1 and x''_2 are the optimal choice of the credit good for the first and second period respectively by a consumer that faces π_1 , π_2 , and π^P for periods one, two and three, and π^* thereafter, where π_1 and π_2 are the minimal rates of money growth that raise a revenue equal to g^{\min} . The value x^0 is the optimal choice of the credit good if the consumer faces π^* forever but is being taxed by C , to finance the convertibility operations.

That condition implies that if the economy enters a speculative path, the current administration will establish convertibility, even though it costs C in resources, because if it does not, it will be punished, and the loss will be higher.

It is shown in the appendix that such a π^P exists.

Also, let $c_t = \{x^*, y^*, M^*, q^{*a}\}$ where x^*, y^* and M^* are the values for x, y and M in the stationary monetary equilibrium when the rate of money growth is always π^* and q^{*a} is the optimal response as defined in definition 2. By proposition 3, the only possible equilibrium is the stationary one. To prove that this is a sub-game perfect equilibrium we have to show that the strategies are optimal for any node of the game.

Consider any node the agents have to play. By definition of best response, their strategies are optimal. Consider a node where the government plays. If the economy is in the stationary equilibrium, the payoff is maximum for the government, so it is optimal. If the economy is not at the stationary equilibrium, it will convert, because if it does not, it will be punished and its payoff will be lower. Finally, once the past administration has deviated, you will impose the punishment, because it does not affect your payoff. So, the stationary equilibrium is a sub-game perfect equilibrium. Finally, assume that there exists $\{c', s', p'\}$ such that p' is the price sequence of a speculative hyperinflation and $\{c', s', p'\}$ is a SPCE. As in a speculative hyperinflation real balances approach zero, there exists a t such that the payoff of A_t is arbitrarily close to autarky. So, for any punishment you will always find some administration that will be better off by switching to the strategy described above. Then, s' is not maximizing strategy, so $\{c', s', p'\}$ cannot be SPCE. ■

So, the only equilibrium that is sequentially rational is the stationary equilibrium, where the price level responds only to fundamentals. In addition, unless the convertibility operations are costless (i.e., unless $C=0$), the optimal strategy implies no convertibility along the equilibrium path, because at every period in which convertibility is established, there is a loss C on resources.

A casual observer of this economy may erroneously believe that he has found a pure fiat money economy, because the government is not backing

the currency and he correctly perceives that it will not do it in the future. The general conclusion of this section is that an economy with the features described in this paper behaves exactly as a convertible economy. This means that the equilibrium outcome of both economies is the same.

AN ALTERNATIVE PAYOFF FUNCTION

Now, we want to modify the payoff function of the administrations. We keep the assumption that there is a sequence of administrations that last two periods in office, but we will assume that their aim is to maximize the revenue they can obtain in those two periods. We will consider two alternative ways of raising revenue. The first one is the inflation tax, and the second is a lump-sum tax, which, in order for a private sector to exist, is supposed to have an upper bound. We could interpret this tax as the compound effect of all other possible distorting taxes, and the upper bound would be the maximum of the Laffer Curve for all those taxes.

Then, if τ_t is the lump-sum tax at period t , the payoff function of the administration that takes office at period t is

$$W_t^g = \tau_t + \frac{\pi_t}{1+\mu_t} \cdot m_t + \tau_{t+1} + \frac{\pi_{t+1}}{1+\mu_{t+1}} \cdot m_{t+1}$$

where μ_t is the rate of inflation at period t , which may be different than the rate of money growth. For simplicity, we assume that there is no discount factor.

Each administration can choose, when it is at office, the rates of

money growth , the amount of lump-sum taxes and the price at which it will exchange money for goods. Then, at any node, the current administration chooses

$$s_t = \{ \pi_t, \tau_t, e_t \}$$

Restriction (6) must hold for τ_t , but now the transfers (T and ν 's) are zero.

A strategy for the government which takes office at t, is

$$s_t : H_{t-1} \longrightarrow \mathbb{R}^3$$

and a strategy for the agents is

$$c_t : H_{t-1} \times S_t \times \mathbb{R} \longrightarrow \mathbb{R}^4$$

where H_t is defined as before, and H_{-1} only contains the initial stock of money.

A result similar to the one stated in proposition 4 can be stated.

PROPOSITION 5: Let all administrations be playing a revenue maximizer rate of money growth π^* . Then, if the revenue that any administration collects from the inflation tax at any period is higher than C, the only PFCE that can be sustained as a SPCE is the stationary one.

Pf: As the utility functions are continuous, there exist a rate of money growth that maximizes the revenue, and given the specification of the administrations payoff function, that optimal rate is constant through time . We call it π^* . Now, consider the following strategy

$$\tau_t = \tau^{\text{MAX}} \quad \text{for all } t$$

$$\pi_t = \begin{cases} \pi^P & \text{if } \Omega_t = -1 \\ \pi^* & \text{otherwise} \end{cases}$$

$$e_t = \begin{cases} 1/p_{t-1} & \text{if } m_t \neq m^*, \tau^{\text{MAX}} \geq M_t/p_{t-1} \text{ and } t \text{ odd} \\ 0 & \text{otherwise} \end{cases}$$

where π^P satisfies

$$\text{a) } \frac{2 \cdot m^* \cdot \pi^*}{(1+\pi^*)} - C \geq \frac{m_1 \cdot \pi_1}{(1+\mu_1)} + \frac{m_2 \cdot \pi_2}{(1+\mu_2)}$$

$$\text{b) } \frac{m^P \cdot \pi^P}{(1+\mu^P)} + \frac{m^* \cdot \pi^*}{(1+\pi^*)} \geq \frac{m_1 \cdot \pi_1}{(1+\mu_1)} + \frac{m_2 \cdot \pi_2}{(1+\mu_2)}$$

where m^* is the demand for real balances if the rate of money growth is always π^* , m^P is the demand for real balances if the rate of money growth is π^P the first period and π^* from there on, and m_1 and m_2 are the demand for real balances for the first and second periods if the rates of money growth are π_1 , π_2 , π^P for the first three periods and π^* thereafter. The rates π_1 and π_2 are the inflation tax revenue maximizers if the rates of money growth at period three is π^P and π^* thereafter. Finally, μ^P is the rate of inflation if the consumers face π^P for the next period and π^* thereafter, and μ_1 and μ_2 are the inflation rates at periods one and two if consumers face π_1 , π_2 , π^P and π^* thereafter.

The first condition guarantees that the punishment is strong enough to make the convertibility threats credible, and the second guarantees

that the punishments are credible. Note that the same rate of money growth is used to punish deviations from the prescribed strategies and deviations from punishments.

We show in the appendix that such a π^P exists.

Also, let the strategy of the representative consumer be the same as in proposition 4. By definition of best response, the strategy of the consumers is optimal at any node. Now, consider a node where the government plays. If the economy is at the stationary equilibrium, the current administration is maximizing the revenue, so it follows the optimal strategy. If the economy enters a speculative path, then it is optimal for the current administration to establish convertibility even though it loses C , because otherwise it will be punished, and the loss will be higher. Finally, if one administration deviates, it is optimal for the following one to punish it, otherwise it will receive a punishment and the loss will be higher.

Now, assume that there exists a point $\{c', s', p'\}$ such that it is a SPCE and the sequence p' corresponds to a speculative hyperinflation equilibrium. As real money balances approach zero, there exists some administration that is collecting an arbitrarily small amount of revenues through inflation tax. Then, given the assumption stated in the definition, it will be in the interest of that administration to establish convertibility at the last period in office, because the benefits of so doing will be higher than the costs. But then, that administration was not following the optimal strategy, so it could not be a sub-game perfect competitive equilibrium. ■

Now, note that in the particular case where $C=0$, the strategy proposed by O&R would also be a subgame perfect equilibrium strategy, because as there is no cost associated with the convertibility operations, there is no advantage in suspending convertibility out of equilibrium path. In addition, the equilibrium strategies described in proposition 4 could be simplified, because there is no need to enforce the convertibility out of equilibrium path.

It is important to note that the uniqueness result of propositions 4 and 5 both refer to equilibria for a given path of money growth. The results do not say that the equilibrium is unique in rates of money growth. It is possible to find other equilibria, where the rates of money growth differ from the equilibria described in the propositions. However, given the assumptions of the paper, it is not possible to find any equilibrium where the price level sequence exhibits a speculative path.

CONCLUSIONS

The main objective of the chapter is to show how the explicit introduction of the government into a monetary model forces us to reconsider the concept of "pure inconvertible system". We conclude that if the government does not like speculative hyperinflations and has the ability to tax the agents, speculative hyperinflations cannot be equilibria. The system behaves as a convertible one even though along the unique equilibrium path convertibility might never be announced.

This suggests that the definition of a convertible money system should not be based on the existence or not of convertibility along the equilibrium path but rather on the existence of an agent which does not like speculative hyperinflations and that has the ability to convert the stock of money.

This paper offers an explanation for the empirical irrelevance of speculative hyperinflations by arguing that pure fiat money economies are empirically irrelevant. This statement has nontrivial implications for monetary theory. Monetary economists have long been puzzled by the fact that intrinsically useless pieces of paper have positive value. In a convertible system, the reason is obvious: just because the government wants it so, and has power enough to induce it. But in recent times, convertibility has been suspended and we still observe worthless pieces of paper having value. One interpretation is that these are different systems and we need different answers. The view adopted in this paper is that these are essentially the same system and hence we can offer the same answer : the US dollar and the german mark have positive value because the Federal Reserve and the Bundesbanke want it that way.

However, this does not solve the puzzle of the value of money, because the puzzle remains even if you consider convertible or commodity money. The real puzzle(which cannot be explained in Arrow-Debreu economies) is why the value of money (convertible or not) is the one we observe. There is neither role for unbacked nor for backed currency in Arrow-Debreu economies. So, we still need models with frictions that

imply a role for a medium of exchange, not to explain why money has a positive value, but to explain why money is used in transactions and which should be its equilibrium price.

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