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The Algebraic Equality of Two Asymptotic Tests for the Hypothesis that a Normal Distribution has a Specified Correlation Matrix[*]

Heinz Neudecker[†]
and
Albert Satorra[‡]

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[†] Universiteit van Amsterdam.

[‡] Universitat Pompeu Fabra.

Abstract

It is proved the algebraic equality between Jennrich's (1970) asymptotic X^2 test for equality of correlation matrices, and a Wald test statistic derived from Neudecker and Wesselman's (1990) expression of the asymptotic variance matrix of the sample correlation matrix.

1 Introduction

Let $R = (r_{ij})$ be the sample correlation matrix of a random sample of size n from a $p-$variate normal distribution with correlation matrix $P = (\rho_{ij})$. Jennrich [1] presented a test statistic for the hypotheis that the sampled distribution has correlation matrix P, viz

$$\varphi := \frac{1}{2}\mathrm{tr}\mathcal{Z}^2 - (\mathrm{dg}\mathcal{Z})'T^{-1}\mathrm{dg}\mathcal{Z} \tag{1}$$

where $\mathcal{Z} := n^{1/2}P^{-1}(R-P)$ and $T := I + P \times P^{-1}$, $\mathrm{dg}\,\mathcal{Z}$ denoting the diagonal of $\mathcal{Z}$ written as a column vector, "$\times$" standing for Hadamard multiplication. All matrices have order p. Jennrich stated that φ has an asymptotic χ^2- distribution with $\frac{1}{2}p(p-1)$ degrees of freedom. Large observed values of φ suggest rejection of the hypothesis that the sampled distribution has correlation matrix P.

The derivation was based on the asymptotic covariance matrix Γ for the $\frac{1}{2}p(p-1)$-dimensional vector of maximum-likelihood estimates r_{ij} of the correlation coefficients ρ_{ij}, arranged in a prescribed order. Jennrich found (in our notation)

$$\Gamma^{-1} = \frac{1}{2}\tilde{D}'(P^{-1}\otimes P^{-1})\tilde{D} - \tilde{D}'(I\otimes P^{-1})JT^{-1}J'(P^{-1}\otimes I)\tilde{D}. \tag{2}$$

Our notation will be explained below.

Neudecker and Wesselman [3] presented the following convergence result under normality:

$$n^{\frac{1}{2}}\ \mathrm{vec}(R-P) \xrightarrow{D} \mathcal{N}(0,\Omega_N), \tag{3}$$

where

$$\Omega_N := \frac{1}{2}(I+K)\{I-(I\otimes P)K_d\}(P\otimes P)\{I-K_d(I\otimes P)\}(I+K) \tag{4}$$

and "$\xrightarrow{D}$" denotes convrgence in distribution. Here K is the appropriate commutation matrix and $A_d := I \times A$ generically.

The convergence result (3) can be used to derive a test statistic for the same purpose as Jennrich's. This will be done in the following section. Jennrich's φ will be written in the same notation. Subsequently the equality of the two test statistics will be proved.

2 The two test statistics

We shall start with a quick overview of the notation that we propose to use. First define a $\frac{1}{2}p(p-1)$ column vector $w(A)$ for symmetric zero-axial matrix A (this means : $A' = A$ and $A_d = 0$), which lists the infradiagonal elements of A. Then introduce the $p^2 \times \frac{1}{2}p(p-1)$ operator $\tilde{D}$ with properties

$$\text{vec}A = \tilde{D}w(A) \quad \text{and} \quad w(A) = \frac{1}{2}\tilde{D}'\text{vec}A \tag{5}$$

for A as defined above. Useful additional properties are reported in the Appendix.

It is now possible to present the more amenable convergence result based on (3) and (5):

$$2n^{\frac{1}{2}}w(R-P) \xrightarrow{D} \mathcal{N}(0, \tilde{D}'\Omega_N\tilde{D}), \tag{6}$$

where obviously

$$\tilde{D}'\Omega_N\tilde{D} = 2\tilde{D}'\{I-(I\otimes P)K_d\}(P\otimes P)\{I-K_d(I\otimes P)\}\tilde{D} \tag{7}$$

by virtue of A1 of the Appendix.

As $\{I-K_d(I\otimes P)\}\tilde{D}$ has full column rank (see A2 and A3), $\tilde{D}'\Omega_N\tilde{D}$ can be inverted. (The matrix Ω_N was clearly singular.). This yields the convergence result

$$4\,n\,\omega'(\tilde{D}'\Omega_N\tilde{D})^{-1}\omega \xrightarrow{D} \chi^2_{\frac{1}{2}p(p-1)} \tag{8}$$

where $\omega := w(R-P)$.

The statistic $4\,n\,\omega'(\tilde{D}'\Omega_N\tilde{D})^{-1}\omega$ is suitable for testing the above mentioned hypothesis about P, being then an alternative to Jennrich's statistic φ. In fact, we will see that the two statistics are the same.

We shall rewrite Jennrich's φ to make it comparable to the test statistic of (8). Clearly

$$\varphi = \frac{1}{2}\mathrm{tr}\mathcal{Z}^2 - (\mathrm{dg}\mathcal{Z})'T^{-1}\mathrm{dg}\mathcal{Z} =$$

$$\frac{1}{2}(\mathrm{vec}\mathcal{Z})'\mathrm{vec}\mathcal{Z} - (\mathrm{vec}\mathcal{Z})'JT^{-1}J'(\mathrm{vec}\mathcal{Z}) =$$

$$\frac{1}{2}n\{\mathrm{vec}(R-P)\}'(P^{-1}\otimes P^{-1})\mathrm{vec}\,(R-P) - n\{\mathrm{vec}(R-P)\}'(I\otimes P^{-1})JT^{-1}J'(P^{-1}\otimes I)\mathrm{vec}(R-P) =$$

$$\frac{1}{2}n\omega'\tilde{D}'(P^{-1}\otimes P^{-1})\tilde{D}\omega - n\omega'\tilde{D}'(I\otimes P^{-1})JT^{-1}J'(P^{-1}\otimes I)\tilde{D}\omega, \qquad (9)$$

where $\mathrm{vec}\,\mathcal{Z} = n^{1/2}(I\otimes P)\ \mathrm{vec}(R-P)$, $\mathrm{vec}\,\mathcal{Z}' = n^{1/2}(P^{-1}\otimes I)\ \mathrm{vec}(R-P)$, $\mathrm{vec}(R-P) = \tilde{D}\omega$. We define J' to be the $p\times p^2$ matrix that converts $\mathrm{vec}\mathcal{Z}$ to $\mathrm{dg}\mathcal{Z}$, viz

$$J'\mathrm{vec}\mathcal{Z} = \mathrm{dg}\mathcal{Z} \qquad (10)$$

(For further properties of J see the Appendix.)

We see that φ is a quadratic form in w with weight matrix the matrix Γ^{-1} of (2) as discussed by Jennrich.

As the following convergence

$$2n^{\frac{1}{2}}w(R-P) \xrightarrow{D} \mathcal{N}(0, 4\Gamma) \qquad (11)$$

also holds, we clearly have $4\Gamma = \tilde{D}'\Omega_N\tilde{D}$, or equivalently, using (2),

$$(\tilde{D}'\Omega_N\tilde{D})^{-1} = \frac{1}{4}\tilde{D}'\{\frac{1}{2}(P^{-1}\otimes P^{-1}) - (I\otimes P^{-1})JT^{-1}J'(P^{-1}\otimes I)\}\tilde{D} \qquad (12)$$

This result which is based on statistical arguments and relied on the correctness of Jennrich's and Neudecker and Wesselman's results, gives an expression for the inverse of the matrix $\tilde{D}'\Omega_N\tilde{D}$ of (7). It is tempting, however, to prove equality (12) in an algebraic way. This will be pursued in the next section.

3 The algebraic equality

THEOREM 1.

$$\tilde{D}'\{I-(I\otimes P)K_d\}(P\otimes P)\{I-K_d(I\otimes P)\}\tilde{D}\tilde{D}'\{P^{-1}\otimes P^{-1}-2(I\otimes P^{-1})JT^{-1}J'(P^{-1}\otimes I)\}\tilde{D}=4I,$$

where all matrices have been defined above.

PROOF. Using A5, A6 and A7 we can replace $\tilde{D}\tilde{D}'$ by $(I+K)$. Then

$$\{I-K_d(I\otimes P)\}(I+K)(I\otimes P^{-1})JT^{-1}J'=$$

$$\{I-K_d(I\otimes P)\}(I\otimes P^{-1})JT^{-1}J'=$$

$$\{I-K_d(I\otimes P)\}K(I\otimes P^{-1})JT^{-1}J'=$$

$$(I\otimes P^{-1})JT^{-1}J'-JT^{-1}J'+K\{I-K_d(P\otimes I)\}(I\otimes P^{-1})JT^{-1}J'=$$

$$(I\otimes P^{-1})JT^{-1}J'-JT^{-1}J'+K(I\otimes P^{-1})JT^{-1}J'-K_d(P\otimes P^{-1})JT^{-1}J'=$$

$$(I+K)(I\otimes P^{-1})JT^{-1}J'-K_d$$

by virtue of A6, A7, A9, A8 and A10.

Further

$$\{I-K_d(I\otimes P)\}(I+K)(P^{-1}\otimes P^{-1})=$$

$$(I+K)(P^{-1}\otimes P^{-1})-K_d(I\otimes P)(P^{-1}\otimes P^{-1})(I+K)=$$

$$(I+K)(P^{-1}\otimes P^{-1}) - K_d(P^{-1}\otimes I)(I+K).$$

Hence

$$(P\otimes P)\{I-K_d(I\otimes P)\}\tilde{D}\tilde{D}'\{P^{-1}\otimes P^{-1}-2(I\otimes P^{-1})JT^{-1}J'(P^{-1}\otimes I)\}\tilde{D}$$

$$=(P\otimes P)(I+K)(P^{-1}\otimes P^{-1})\tilde{D}-(P\otimes P)K_d(P^{-1}\otimes I)(I+K)\tilde{D}$$

$$-2(P\otimes P)(I+K)(I\otimes P^{-1})JT^{-1}J'(P^{-1}\otimes I)\tilde{D}+(P\otimes P)K_d(P^{-1}\otimes I)(I+K)\tilde{D}=$$

$$2\tilde{D}-2(P\otimes P)K_d(P^{-1}\otimes I)\tilde{D}-2(I+K)(P\otimes I)JT^{-1}J'(P^{-1}\otimes I)\tilde{D}+2(P\otimes P)K_d(P^{-1}\otimes I)\tilde{D}=$$

$$\tilde{D}-2(I+K)(P\otimes I)JT^{-1}J'(P^{-1}\otimes I)\tilde{D},$$

by virtue of A1, and finally

$$\tilde{D}'\{I-(I\otimes P)K_d\}(P\otimes P)\{I-K_d(I\otimes P)\}\tilde{D}\tilde{D}'\{P^{-1}\otimes P^{-1}-2(I\otimes P^{-1})JT^{-1}J'(P^{-1}\otimes I)\}\tilde{D}=$$

$$2\tilde{D}'\{I-(I\otimes P)K_d\}\tilde{D}-2\tilde{D}'\{I-(I\otimes P)K_d\}(I+K)(P\otimes I)JT^{-1}J'(P^{-1}\otimes I)\tilde{D}=$$

$$4I-4\tilde{D}'(P\otimes I)JT^{-1}J'(P^{-1}\otimes I)\tilde{D}+4\tilde{D}'(I\otimes P)K_d(P\otimes I)JT^{-1}J'(P^{-1}\otimes I)\tilde{D}=4I$$

by virtue of A1, A3, A2, A8, A7, A6 and A8.

Clearly $\tilde{D}'(P\otimes I)J=\tilde{D}'(I\otimes P)J$. This establishes the algebraic equality.

4 Appendix

4.1 Properties of $\tilde{D}$:

A1 $K\tilde{D}=\tilde{D}$

A2 $K_d\tilde{D}=0$

A3 $\tilde{D}'\tilde{D}=2I$

A4 $\tilde{D}\tilde{D}'\text{vec}A = 2\text{vec}A \quad \text{for } A : A = A', \quad A_d = 0$

A5 $\tilde{D}\tilde{D}' = I + K - 2K_d$

In the following A will always be symmetric zero-axial.

PROOF.

1. $K\tilde{D}w(A) = K\text{vec}A = \text{vec}A = \tilde{D}w(A), \quad \forall w(A)$ hence $K\tilde{D} = \tilde{D}$
2. $K_d\tilde{D}w(A) = K_d\text{vec}A = \text{vec}A_d = 0, \quad \forall w(A)$, hence $K_d\tilde{D} = 0$
3. $\tilde{D}'\tilde{D}w(A) = \tilde{D}'\text{vec}A = 2w(A), \quad \forall w(A)$ hence $\tilde{D}'\tilde{D} = 2I$.
4. $\tilde{D}'\tilde{D}\text{vec}A = 2\tilde{D}w(A) = \text{vec}A$
5. $(I + K - 2K_d)\text{vec}B = \text{vec}(B + B' - 2B_d) =$

$$\frac{1}{2}\tilde{D}\tilde{D}'\text{vec}(B + B' - 2B_d) =$$

$$\frac{1}{2}\tilde{D}\tilde{D}'\text{vec}B + \frac{1}{2}\tilde{D}\tilde{D}'K\text{vec}B - \tilde{D}\tilde{D}'K_d\text{vec}B = \tilde{D}\tilde{D}'\text{vec}B, \quad \forall\, \text{vec}B$$

hence $I + K - 2K_d = \tilde{D}\tilde{D}'$. We used the defining equations (5) in this proof.

4.2 Properties of J and K_d

A6 $J'(B \otimes C)J = B \times C$

A7 $JJ' = K_d$

A8 $J'\tilde{D} = 0$

A8 $KJ = J$

A9 $J'J = I$

For proofs see Kollo and Neudecker [2].

4.3 Property of T^{-1}

A10 $JT^{-1}J'(P \otimes P^{-1})K_d + JT^{-1}J' = K_d$

PROOF. $T = I + P \times P^{-1} = J'(I + P \otimes P^{-1})J$ by A6 and A9. Clearly $T^{-1}(T - I) + T^{-1} = I$, hence $JT^{-1}J'(P \otimes P^{-1})K_d + JT^{-1}J' = K_d$, by A6 and A7.

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