CHOICE BY SEQUENTIAL PROCEDURES*

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Abstract. We propose a rule of decision-making, the sequential procedure guided by routes, and show that three influential boundedly rational choice models can be equivalently understood as special cases of this rule. In addition, the sequential procedure guided by routes is instrumental in showing that the three models are intimately related. We show that choice with a status-quo bias is a refinement of rationalizability by game trees, which, in turn, is also a refinement of sequential rationalizability. Thus, we provide a sharp taxonomy of these choice models, and show that they all can be understood as choice by sequential procedures.

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1. Introduction

There is evidence suggesting that individuals make use of operational procedures to analyze complex choice situations (see, e.g., Gigerenzer, et al, 1999; Hogarth and Karelaia, 2005; Manrai and Prabhakant, 1989). In this line, suppose a decision-maker (DM) that, when confronting the problem of choosing an alternative from a menu, first identifies a pair of alternatives. In that pair the DM has no difficulty in discarding the alternative that she dislikes. Then, in the remaining menu of alternatives she identifies a new pair and discards the dominated alternative in the pair. The DM repeats this process until only one alternative remains, representing the choice. That is, the DM reaches the chosen alternative by discarding alternatives in binary comparisons in a certain order, say a route. For example, the DM may be more used to comparing some pairs of alternatives than others and find that her familiarity suggests a particular route through all the pairs of alternatives. We refer to this behavioral heuristic as a *sequential procedure guided by routes.*

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To illustrate, consider the set of alternatives $\{1,2,3\}$ and let the choices of the DM be alternative 2 from $\{1,2\}$, 3 from $\{2,3\}$, and 1 from $\{1,3\}$ and from $\{1,2,3\}$. The questions arise as to whether this choice behavior can be understood as a sequential procedure guided by routes, and if so, by which routes. Clearly, we only need to evaluate the choice from the grand menu $\{1,2,3\}$. If the DM focuses first on the pair $\{2,3\}$, she immediately discards alternative 2. She then considers the surviving menu $\{1,3\}$, where she discards option 3, leading to option 1 which is precisely the observed choice. Hence, the answer to the first question above is yes, the behavior of the DM can be understood by way of sequential procedures guided by routes. Now, it is easy to see that any route starting with a pair of alternatives different to $\{2,3\}$ does not lead to the choice of 1 from $\{1,2,3\}$. Hence, the behavior of our DM is consistent with only a few routes. Intuitively, in general, the more routes that are consistent with the behavior of a DM the more structured her behavior will be. This is so because behavior consistent with a route immediately implies that her choice from any menu of alternatives does not depend on the presence or absence of the dominated alternative in the first pair in the menu, dictated by the route. Clearly, then, the more routes consistent with choice, the more congruent choice behavior is.

In this paper we make use of the notion of sequential procedures guided by routes to gain a deeper understanding of a number of key boundedly rational choice models in the literature:

- Status-quo bias (Masatlioglu and Ok, 2005, 2010): the DM typically values an alternative more highly when it is regarded as the status-quo, than she would otherwise.
- Rationalizability by game trees (Xu and Zhou, 2007): the choices of the DM are the equilibrium outcome of the conflict between different binary relations, modeled as a game.
- Sequential rationalizability (Manzini and Mariotti, 2007): the DM sequentially applies a collection of binary relations in a fixed order of priority.

First, we show how these three models, as well as the classical rational choice model, can all be described in terms of sequential procedures guided by routes. We characterize the exact nature of the routes consistent with each one of these choice models. This contributes to gain a deep understanding of the kind of behavior involved in the different models. Clearly, the classical rational choice behavior is consistent with every single possible route. Then, we show that the status-quo biased choice model can be equivalently represented by the sequential procedure where the routes have a very specific structure, in consonance with a certain perception of the alternatives. In a nutshell, the DM perceives one alternative as similar in some dimension to each one of the alternatives in a collection of alternatives, while the alternatives in the complementary set are perfectly perceived in isolation, without the need of relating them to each other. This perception of the alternatives predisposes the class of routes that the DM contemplates. Next, we prove that rationalizability by game trees can be equivalently represented by the sequential procedure where the routes are in consonance with the alternatives being perceived in nested classes of similarity. For example, the DM considers one binary attribute and divides the alternatives in two classes, one composed of the alternatives enjoying the attribute and the complementary one. Then, within one class of alternatives the DM considers another binary attribute and divides the class into several subclasses and so on. As in the former case, the routes contemplated by the DM follow from this perception of the alternatives. Finally, we show that sequential rationalizability is behaviorally equivalent to a minimal congruence property: the existence of one route and the DM following the sequential procedure for such a route.

Second, these behavioral characterizations based on the sequential procedure guided by routes facilitate the systematic comparison of the different models of choice, leading to what may be regarded as a priori unexpected implications. In particular, we show that choice with a status-quo bias presents more consistency with routes than choice rationalizable by game trees and the latter presents more consistency with routes than sequential rationalizable choice. That is, every rational choice is also status-quo biased choice, which in turn is also rationalizable by a game tree, which in turn is also sequentially rationalizable. Thus, the behavioral characterizations of the different choice models provide a sharp taxonomy.

2. Sequential Choice Procedures

2.1. **Preliminaries.** Let X be a finite set of alternatives. We denote by $\mathcal{X}$ (respectively, by $\mathcal{B}$) the collection of all non-empty subsets (resp., of all binary subsets) of X. A choice function is a mapping $c : \mathcal{X} \to X$ such that $c(A) \in A$. That is, a choice function c assigns to every non-empty set $A \subseteq X$ a unique element $c(A) \in A$.

We now formally introduce the notion of sequential procedures guided by routes. In so doing we define two concepts, routes over pairs of alternatives and a behavioral congruence condition based on them. A route is a linear order $<$ over $\mathcal{B}$.[1] We interpret a route as the way in which the alternatives are considered by the DM. That is, a DM contemplating a route perceives the alternatives with a certain structure, or in other words, with a certain frame. This means that when facing a menu A, the DM does not immediately scrutinize all the available alternatives at once, but follows an order, the route, by which she sequentially examines the alternatives in ordered pairs. The route contemplated by the DM may be dictated by experience, physical appearance of the alternatives, etc.

For any route $<$ and set A, denote by $A^<$ the first binary subset in A according to the route $<$.[2] This is the very first pair that the DM contemplates when, following the route $<$, evaluates the menu A. Given this perception of the choice situation, a minimal behavioral congruence property when contemplating the route $<$ entails considering the first pair of alternatives $A^<$ from menu A, evaluating which is the liked and which the disliked

[1]A linear order is a complete, transitive and antisymmetric binary relation.

[2]Formally, (i) $A^< \in \mathcal{B}$, (ii) $A^< \subseteq A$ and (iii) for any $B \in \mathcal{B}$ such that $B \subseteq A$ we have $A^< < B$.

alternative in $A^<$, and concluding that the choice from A should be independent of the availability of the disliked alternative in $A^<$. Formally, denote by $nc(B)$ the element that is not chosen from the binary menu $B \in \mathcal{B}$ given the choice function c. Then, we say that the choice function c is *$<$-consistent* for the route $<$ if for every set $A \in \mathcal{X}$ with at least two alternatives, $c(A) = c(A \setminus nc(A^<))$.

The behavior of the DM may be consistent with the sequential procedure for a number of routes. We interpret this case as if the DM might contemplate each one of them. In what follows we apply the sequential procedure guided by routes to a number of different choice models, where we only vary the structure of the class of routes that the DM contemplates.

2.2. **Rational Choice.** The most structured case of decision-making is when the choices from every single possible menu can be understood as the result of the maximization of a well-behaved preference relation. Formally, we say that a choice function c is a *rational choice* or RAT, if there exists a linear order P on X such that for all $A \in \mathcal{X}$, $c(A) = M(A, P)$.[3]

It is easy to see that rational choice can be equivalently described by the sequential procedure guided by routes whenever the DM considers all the possible routes. That is, an individual is rational if and only if her behavior is congruent with every route. This clearly represents the case where the framing of the menus of alternatives is inconsequential for the DM. The following property establishes the independence of the routes on behavior.

Strong Route Consistency (SRC). c is $<$-consistent for every route $<$.

We can now formally establish the characterization of rational choice by way of sequential procedures guided by routes.[4]

Theorem 1. *A choice function satisfies SRC if and only if it is a rational choice.*

2.3. **Endogenous Status-Quo Biased Choice.** There is a wide range of empirical literature supporting the view that DMs typically value an alternative more highly when it is regarded as the status-quo than they would otherwise. This is the so-called status-quo bias (see, e.g., Thaler, 1980 and Kahneman, Knetsch, and Thaler, 1991). Masatlioglu and Ok (2005, 2010) provide the first axiomatic studies of this choice behavior.[5] In Masatlioglu and Ok's setting, a choice problem is a pair (A, x) where A is the set of alternatives, and $x \in A$ or $x = \diamond$. When $x \in A$, the pair (A, x) represents a choice problem with the status quo x, while if $x = \diamond$ the choice problem is standard in the sense that it is without a status quo. Thus, choice is defined over the collection of all choice problems (A, x). Masatlioglu

[3]In general, for any binary relation P, $M(A, P) = \{x \in A : (y, x) \in P \text{ for no } y \in A\}$.

[4]All the proofs are contained in the Appendix.

[5]Other theoretical contributions to the status-quo bias literature are Mandler (2004), Koszegi and Rabin (2006), Dean (2008), Apesteguia and Ballester (2009), Bossert and Sprumont (2009), Giraud (2011), and Ok, Ortoleva and Riella (2011).

and Ok introduce a set of properties for choice behavior that are equivalent to the following status-quo biased choice model. The DM regards a subset Q of X as the alternatives that are clearly superior to the status-quo. Then, if the DM confronts a choice problem without a status quo, she simply maximizes a well-behaved preference relation. If there is a status quo the DM assesses whether any of the superior alternatives to the status-quo is available. If there are any she will maximize from among the superior alternatives to the status-quo, that is she will choose the optimal alternative from $Q \cap A$ according to her preference relation. If not, she will stay with the status quo.

In this paper we build on Masatlioglu and Ok's models to offer, for the first time, two versions of the status-quo biased phenomena where the status quo is *endogenous*. That is, we define status-quo dependence within the standard model of choice, by way of choice functions defined on $\mathcal{X}$ instead of over the collection of all choice problems (A, x). We entertain two different possible reactions to the case where there are attractive alternatives from the point of view of the status-quo. In one case, as in Masatlioglu and Ok, we restrict the DM to maximize within $Q \cap A$. This represents a marked status-quo bias, since the status-quo makes her forget about all the alternatives that are available in A, but are not in Q. Alternatively, we also consider the case that when $Q \cap A \neq \emptyset$, the DM abandons the status-quo, but otherwise chooses the optimal alternative in A. This represents the behavior of a DM that although biased by the status-quo, is less subject to it.

Formally, we say that a choice function c is an *extreme endogenous status-quo biased choice* if there exists a linear order P on X, an element $d \in X$ and a set $Q \subseteq X \setminus \{d\}$ such that: (i) for all $A \in \mathcal{X}$ where $d \notin A$, $c(A) = M(A, P)$, (ii) for all $A \in \mathcal{X}$ with $d \in A$ and $A \cap Q = \emptyset$, $c(A) = d$, and (iii) for all $A \in \mathcal{X}$ with $d \in A$ and $A \cap Q \neq \emptyset$, $c(A) = M(A \cap Q, P)$. We say that c is a *weak endogenous status-quo biased choice* if (iii) is replaced by (iii)' for all $A \in \mathcal{X}$ with $d \in A$ and $A \cap Q \neq \emptyset$, $c(A) = M(A \setminus \{d\}, P)$. We say that a choice function c is an *endogenous status-quo biased choice* or SQB if it is an extreme or weak endogenous status-quo biased choice.

The question arises as to whether endogenous status-quo biased choice can be understood as a sequential procedure for a certain class of routes. Here we show that this is in fact the case. To do so we first need to introduce the following general definitions, and then we will apply them to the case of the endogenous status-quo biased choice.

The DM may entertain a *similarity structure* over the alternatives. Namely, she may regard some alternatives as belonging to a given analogy class, in the sense of sharing some feature, being it physical or not. Then, the routes that the DM contemplates are in fact characterized by the similarity structure the DM considers, in the following sense. For any two disjoint classes of the similarity structure, the route will first place all the binary comparisons involving elements within a given class, and only later will she consider the crossed pairs formed by one element of each class. That is, the DM first evaluates the pairs formed by similar alternatives, and only after she assesses pairs of dissimilar alternatives, those belonging to different classes of similarity. Intuitively, the similarity structure biases the boundedly rational DM to contemplate particular orders over the pairs of alternatives. Formally, a similarity structure $\mathcal{S}$ is a collection of non-empty subsets of X, the union of which is X. We say that the route $<$ *respects* the similarity structure $\mathcal{S}$ if, for every $A, B \in \mathcal{S}$ with $A \cap B = \emptyset$, all binary subsets of A and all binary subsets of B precede all binary subsets conformed by one element of A and one element of B.

Clearly, a perfectly rational DM is not distorted by any particular similarity structure. That is, the rational DM is able to perfectly evaluate every single alternative on its own, and hence, considers every possible route over the binary problems, as Theorem 1 shows. This is not the case of the endogenous status quo biased DM. We say that a similarity structure is *centered* if: (i) all subsets of $\mathcal{S}$ have at most two elements, (ii) if there are more than two binary subsets in $\mathcal{S}$, they all intersect in a unique element s. A centered similarity structure can be understood as a star-type network, where there is an alternative, the center, connected to a number of alternatives, and there is no other link between alternatives. Then, the routes respecting a centered similarity structure consider all the pairs of alternatives containing the center and an alternative similar to the center, before the pairs containing the center (or a similar alternative to the center) and a dissimilar alternative to the center. In order to establish that centered similarity structures characterize the class of routes that leads to endogenous status-quo biased choice, consider first the following condition.

Route Consistency on a Centered Similarity Structure (RCCSS). There exists a centered similarity structure $\mathcal{S}$ such that c is $<$-consistent for every route $<$ that respects $\mathcal{S}$.

Theorem 2. *A choice function satisfies RCCSS if and only if it is an endogenous status-quo biased choice.*

2.4. **Rationalizability by Game Trees.** Consider a DM whose behavior can be understood as the result of a collection of selves in internal conflict. Xu and Zhou (2007) provide a model of this heuristic: the DM's choices are the equilibrium outcome of an extensive game with perfect information. The alternatives are the terminal nodes of the game and the criteria take the form of preference relations at the non-terminal nodes. The outcome is the subgame perfect Nash equilibrium of the game.[6]

Formally, a (binary) game tree $\Gamma = (G, \{P_i\}_{i=1}^k)$ is a pair where: (i) G is a binary tree (each node has either zero or two successors) with each alternative of X appearing as an end node of the tree once and only once and (ii) a collection of asymmetric binary relations, one for each non-terminal node i of the tree. Given Γ and any subset $A \in \mathcal{X}$, let $\Gamma|A$ be

[6]Other models that are consistent with this view of decision-making by way of multi-selves in internal conflict are Kalai, Rubinstein and Spiegler (2002), Horan (2009), de Clippel and Eliaz (2010), and Ambrus and Rozen (2011).

the reduced game that retains all the branches leading to terminal nodes in A. We then say that a choice function c is *rationalizable by game trees* or RGT if there exists a game tree Γ such that for all $A \in \mathcal{X}$, $c(A) = SPNE(\Gamma|A)$, where $SPNE(\Gamma|A)$ stands for the subgame perfect Nash equilibrium outcome of $\Gamma|A$.

We show that rationalizability by game trees is also a special case of sequential procedures guided by routes. To prove this claim, consider the following similarity structure. We say that a similarity structure $\mathcal{S}$ is *nested* if for every $A, B \in \mathcal{S}$ with $A \cap B \neq \emptyset$, $A \subset B$ or $B \subset A$. A DM entertaining a nested similarity structure perceives the alternatives as hierarchically ordered. That is, when considering a similarity class A, the DM wonders whether the alternatives in A are sufficiently homogenous for a good understanding of them, or can in fact be partitioned into different subclasses, each one composed of alternatives sharing some attribute that the DM regards as important. Then, any route respecting a nested similarity structure will first consider the pairs in each one of the subclasses, and only then will consider pairs with elements of different subclasses. The following property formally describes the class of sequential procedures guided by routes respecting a nested similarity structure, and Theorem 3 establishes the behavioral characterization of rationalizability by game trees based on this property.

Route Consistency on a Nested Similarity Structure (RCNSS). There exists a nested similarity structure $\mathcal{S}$ such that c is $<$-consistent for every route $<$ that respects $\mathcal{S}$.

Theorem 3. *A choice function satisfies RCNSS if and only if it is rationalizable by game trees.*

Remark 1. In our definition of rationalizability by game trees we have assumed that the tree G is binary, and that the binary relations in the non-terminal nodes are asymmetric. The original notion of Xu and Zhou involves game trees that are not necessarily binary and where the binary relations in the non-terminal nodes are assumed to be linear. Denote this case by RGT(LO). It is easy to see that the binariness of the game tree is without loss of generality. Every choice function rationalizable by a non-binary game tree with linear orders in the non-terminal nodes can be equivalently rationalized by a game tree where all the non-terminal nodes have two successors. It is also immediate that relaxing the assumption on the linearity of the binary relations to asymmetry signifies an extension. The following proposition, due to Horan (2009), establishes that the inclusion is strict. That is, there are choice patterns that are rationalizable by game trees with asymmetric relations but that are not with linear orders. Let $\mathcal{C}^{\omega}$ denote the class of choice functions that are ω, where ω may be RGT(LO) or RGT.

Proposition 1. $\mathcal{C}^{RGT(LO)} \subset \mathcal{C}^{RGT}$.

2.5. **Sequential Rationalizability.** In the model of sequentially rationalizable choice of Manzini and Mariotti (2007) a DM faced with a choice problem applies a number of binary relations in a fixed order of priority, gradually narrowing down the set of alternatives until one is identified as the choice.[7] Formally, let $M_i^j(A)$ with $i \leq j$ denote the set $M_i^j(A) = M(M(\ldots M(M(A, P_i), P_{i+1}), \ldots, P_{j-1}), P_j)$. That is $M_i^j(A)$ is the set of alternatives surviving from A the sequential application of binary relations $P_i, P_{i+1}, \ldots, P_{j-1}, P_j$. We then say that a choice function c is sequentially rationalizable or SR whenever a non-empty ordered list of acyclic binary relations on X exists $\{P_1, \ldots, P_K\}$, such that $c(A) = M_1^K(A)$ for all $A \subseteq X$. In that case we say that $\{P_1, \ldots, P_K\}$ sequentially rationalizes c.

We show here that the model of sequential rationalizability can be equivalently understood as the sequential procedure guided by one given route. This clearly represents the least behaviorally structured case of all those we have studied in this paper. We show in Theorem 4 that the following property completely characterizes sequential rationalizability.

Weak Route Consistency (WRC). There exists one route $<$ such that c is $<$-consistent.

Theorem 4. *A choice function satisfies WRC if and only if it is sequentially rationalizable.*

Remark 2. The proof of Theorem 4 shows that every acyclic rational can be split into a collection of ordered binary relations, each one containing a single binary comparison, with no effect on the sequential rationalization of the choice function c. Furthermore, since every binary relation comprising a single binary comparison is trivially acyclic, this naturally suggests an equivalent notion of sequential rationalization; one that uses binary relations comprising a single binary comparison. This equivalence can be immediately extended to cases where the structure of the binary relations is at least acyclic. Examples of such relations are orders or semi-orders (a special case of transitive and asymmetric binary relations; see Manzini and Mariotti, 2009). We summarize this result in Corollary 1. To this end, we say that P is α whenever the structure of P is at least acyclic.

Corollary 1. $\mathcal{C}^{SR(\alpha)} = \mathcal{C}^{SR}$.

Remark 3. While in this paper we favor acyclic binary relations, Manzini and Mariotti (2007) impose only asymmetry, and hence allow for cycles in the binary relations. The models share the same intuition in that they consider a DM who gradually narrows down the set of alternatives by applying a set of incomplete criteria in a fixed order. They nevertheless differ in terms of the structure imposed on the binary relations.[8]

[7]Papers contributing to the view of sequential decision-making are Cherepanov, Feddersen, and Sandroni (2010), Mandler, Manzini, and Mariotti (2011), Manzini and Mariotti (2011), Masatlioglu, Nakajima and Ozbay (2011), and Tyson (2011).

[8]See Bossert, Sprumont and Suzumura (2005) and Ehlers and Sprumont (2008) for a thorough discussion of models of rationalization by a single asymmetric or a single acyclic binary relation.

We now show that the use of acyclic or asymmetric binary relations makes a difference. That is, there are choice patterns that can be sequentially rationalized using asymmetric binary relations (SR(As)) that cannot be sequentially rationalized by means of acyclic binary relations. However, we also show that in the prominent case of two asymmetric binary relations, called Rational Shortlist Methods (RSM) (see Manzini and Mariotti, 2007), the two asymmetric relations can be transformed into a collection of acyclic relations, and hence every choice function that is a RSM, is also a SR.[9]

Proposition 2. $\mathcal{C}^{RSM} \subset \mathcal{C}^{SR} \subset \mathcal{C}^{SR(As)}$.

2.6. **Relation Between The Models.** We have shown that rational choice, endogenous status-quo biased choice, rationalizability by game trees and sequential rationalizability can all be understood by way of sequential procedures guided by routes. Importantly, we have established these results by delimiting the classes of routes for which these choice patterns are consistent. We have proved that rational choice is equivalent to sequential procedures for every single possible route, that endogenous status-quo biased choice and rationalizability by game trees are sequential procedures only for certain subsets of routes, and finally that sequential rationalizability is so for the extreme case of a route. This exercise makes plain most of the relations between the models. Namely, that rational choice is a refinement of all the rest of models of choice, and that both the endogenous status-quo biased choice and rationalizability by game trees are refinements of sequential rationalizability. Notice also that the characterization results immediately show that the endogenous status-quo biased choice is also a refinement of rationalizability by game trees. To see this, consider the centered similarity structure $\mathcal{S}$ associated to a status-quo biased choice c. Think of the nested similarity structure $\mathcal{S}'$ where we consider all singletons in $\mathcal{S}$ and the set formed by the union of all binary subsets in $\mathcal{S}$. Clearly, any route $<$ that respects $\mathcal{S}'$ also respects $\mathcal{S}$ and thus, $c(A) = c(A \setminus \{nc(A^<\})$. This proves that c is a rationalizable by game trees.

The following result establishes that all the described relationships are in fact strict.

Proposition 3. $\mathcal{C}^{RAT} \subset \mathcal{C}^{SQB} \subset \mathcal{C}^{RGT} \subset \mathcal{C}^{SR}$.

All the previous results establish that a variety of models can be understood as sequential procedures guided by routes. An immediate question is whether all sorts of choice patterns can be accommodated in this way. A simple argument shows that this is in fact not the case. Suppose that we observe that our DM chooses alternative 1 from $\{1, 2\}$ and from $\{1, 3\}$, but chooses 2 from $\{1, 2, 3\}$. It is impossible to reconcile this behavior with choice by sequential procedures. The intuition is very simple. Given the choices in the binary

[9] Salant and Rubinstein (2008) and García-Sanz and Alcantud (2010) also study the case of the sequential application of two binary relations.

problems, alternative 1 can never be rejected by the DM and hence, it should be the choice from the grand set.[10]

3. Final Remarks

In this paper we propose a procedural rule of decision-making, the sequential procedure guided by routes, and show that a number of relevant choice models in the literature can be equivalently understood as special cases. Further, we show that one can use the notion of sequential procedure guided by routes to give structure to the different boundedly rational models offered in the literature. We show that rational choice is a refinement of endogenous status-quo biased choice, which in turn is a refinement of rationalizability by game trees, which finally is a refinement of sequential rationalizability. We prove these claims by characterizing the nature of the classes of routes consistent with each case.

We see that the notion of sequential procedures guided by routes suggests promising new lines for future research. First, it might be particularly interesting to study the role of manipulating the routes considered by an agent by way of marketing strategies or of a principal in a market interaction environment. Second, the sequential procedures guided by routes can be used as a tool to learn to gain consistency in choice across menus of options. That is, making the DM to go through various routes may make her aware of the route dependency of her behavior, and hence may make her reconsider some binary comparisons that lead to violations of rationality. Finally, the size and composition of the class of routes consistent with observed behavior suggests itself as a measure of the rationality of the DM. Clearly, the more routes the DM observes, the less capricious her behavior is.[11]

Appendix A. Proofs

Proof of Theorem 1: A rational choice clearly satisfies SRC. We show that a choice function c that satisfies SRC is a rational choice. Since c is a single-valued function defined over the universal domain $\mathcal{X}$, the preference P revealed from the choices in the binary sets must be complete and asymmetric. We prove that it is also acyclic. Suppose by contradiction that P contains a cycle, that is, $x_1 P x_2 P \dots P x_k P x_1$, and suppose, w.l.o.g, that the cycle is composed of different alternatives. Consider a route $<$ such that $\{x_{k-1}, x_k\} < \{x_{k-2}, x_{k-1}\} < \dots < \{x_1, x_2\}$ are the first binary sets. By the recursive application of SRC it follows that $c(\{x_1, \dots, x_k\}) = x_1$. Consider now another route $<'$ where $\{x_k, x_1\}$ is the first binary set. SRC clearly implies that $c(\{x_1, \dots, x_k\}) = c(\{x_2, \dots, x_k\}) \neq x_1$, which is absurd. Therefore,

[10]Models that would account for this sort of behavior and then are not sequential procedures guided by routes are described in Cherepanov, Feddersen, and Sandroni (2010), de Clippel and Eliaz (2010), Manzini and Mariotti (2011), Masatlioglu, Nakajima and Ozbay (2011), and Ok, Ortoleva and Riella (2011).

[11]There is recent growing interest in the development of new tools to measure the rationality of individuals. See, e.g., Apesteguia and Ballester (2011), Beatty and Crawford (2010), Dean and Martin (2010), and Echenique, Lee and Shum (2010). See also Choi, Kariv, Muller, and Silverman (2011).

P is a linear order. Now, it can be easily shown that SRC implies that for all $A \in \mathcal{X}$, $c(A) = M(A, P)$.$\square$

Proof of Theorem 2: We first prove that if c is an extreme (respectively, weak) endogenous status-quo biased choice, it must satisfy RCCSS. Consider the status-quo d and the set Q. If $Q \cup \{d\} = X$ or $Q = \emptyset$ the choice function is rational and the result follows trivially for the centered similarity structure formed by all the singletons. Otherwise, construct $\mathcal{S}$ with sets $\{x, d\}$, where $x \notin Q \cup \{d\}$ (respect., $x \in Q$), and singletons $\{y\}$, with $y \in Q$ (respect., $y \notin Q \cup \{d\}$), which is clearly a centered similarity structure. Consider a route $<$ that respects $\mathcal{S}$. We now prove that c is $<$-consistent. Consider a set $A \in \mathcal{X}$ with at least two alternatives. We distinguish several cases depending on the composition of A. If $d \notin A$, or $d \in A \subseteq Q \cup \{d\}$, or $d \in A \subseteq X \setminus Q$, behavior is rational over all subsets of A, and thus $c(A) = c(A \setminus \{nc(A^<)\})$. Therefore, we are left to consider the case in which $d \in A$ and $A \setminus (Q \cup \{d\}) \neq \emptyset \neq A \cap Q$. Given that $<$ respects $\mathcal{S}$, $A^<$ is a binary set containing d and an element $x \in A \setminus (Q \cup \{d\})$ (respect., $x \in A \cap Q$). Since $x \notin Q$ (respect., $x \in Q$), $nc(A^<) = x$ (respect., $nc(A^<) = d$). Thus, $c(A) = M(A \cap Q, P) = M((A \setminus \{nc(A^<)\}) \cap Q, P) = c(A \setminus \{nc(A^<)\})$ (respect., $c(A) = M(A \setminus \{d\}, P) = c(A \setminus \{nc(A^<)\})$), and c satisfies RCCSS.

In the other direction, suppose now that c satisfies RCCSS. We now prove that c is an endogenous status-quo biased choice. Let $\mathcal{S}$ be a centered similarity structure such that c is $<$-consistent for every route $<$ that respects $\mathcal{S}$. If all the subsets in $\mathcal{S}$ are singletons, c must be $<$-consistent for every route, and given Theorem 1, choice behavior is rational. This is but a special case of endogenous status-quo biased choice. If some of the subsets in $\mathcal{S}$ are non-singletons, since $\mathcal{S}$ is centered, the intersection of all non-singleton sets in $\mathcal{S}$ is non-empty. This intersection must be a unique element or a doubleton. In the former case, denote the element in the intersection by d. In the latter case, it is easy to see that it does not matter which of these two elements is chosen to be the status-quo, so pick any of these two elements and denote it by d. Define $Q = \{x \in X : c(x, d) = x\}$ and the revealed preference $\hat{P}$ from the binary choices on $X \setminus \{d\}$. Since a route respecting $\mathcal{S}$ can order all binary sets in $X \setminus \{d\}$ in any way, $<$-consistency implies linearity of $\hat{P}$. We can extend $\hat{P}$ to a linear order P on X by placing d anywhere. We now show that c is either an extreme or a weak endogenous status-quo biased choice associated to d, Q and P.

Let U be the set of all elements $u \in X \setminus \{d\}$ such that $\{u\} \in \mathcal{S}$ and let V be the set of all elements $v \in X \setminus \{d\}$ such that $\{d, v\} \in \mathcal{S}$. Clearly, since $\mathcal{S}$ is centered, $U \cup V = X \setminus \{d\}$ and also, a route $<$ respects $\mathcal{S}$ if and only if $\{d, v\} < \{d, u\}$ and $\{d, v\} < \{u, v\}$ for all $u \in U$ and $v \in V$. Consider any set A with three alternatives conforming a choice cycle involving the binary menus. We prove that A is of the form $\{d, u, v\}$ where $u \in U$ and $v \in V$. If $d \notin A$ or the other two elements in A belong both to U, the routes respecting $\mathcal{S}$ can order the three binary sets in A in any way, and hence, $<$-consistency contradicts the assumed cycle. Similarly, if $u, v \in V$, then $\{d, v\}$ and $\{d, u\}$ can be ordered in any way, and the two precede $\{u, v\}$, which implies that there is no cycle in the binary menus.

Let T be the set of all alternatives different than d that conform cycles of three alternatives with d. Define the sets $T^+ = \{x \in T : c(x, d) = x\}$ and $T^- = \{x \in T : c(x, d) = d\}$. We now claim that $T^+ \subseteq U$ and $T^- \subseteq V$, or viceversa. Consider two sets of three alternatives conforming cycles, $A = \{d, a^+, a^-\}$, and $B = \{d, b^+, b^-\}$, where $a^+, b^+ \in T^+$ and $a^-, b^- \in T^-$. We now show that $a^+ \in U \Leftrightarrow b^+ \in U$ and $a^- \in U \Leftrightarrow b^- \in U$. If either $a^+ = b^+$ or $a^- = b^-$, the claim follows trivially. Suppose then that a^+, b^+, a^-, b^- are four different alternatives. If $a^- = c(\{a^-, b^+\})$ then we know that a^-, b^+ and d form a cycle and hence a^-, b^+ must be separated by U and V. Since a^-, a^+ (resp. b^-, b^+) are also separated by U and V, it immediately follows that a^+ and b^+ (resp. a^- and b^-) belong to the same set U or V. If $b^+ = c(\{a^-, b^+\})$, since P is linear on $\{a^+, b^+, a^-, b^-\}$, it must also be $b^- = c(\{a^+, b^-\})$. Then we know that a^+, b^- and d form a cycle and we have to separate a^+ and b^- through U and V. Since a^-, a^+ (resp. b^-, b^+) are also separated by U and V, it immediately follows again that a^- and b^- (resp. a^+ and b^+) belong to the same set U or V. The repeated application of this argument proves the claim.

Now consider any set A with at least two alternatives. If $d \notin A$, since $M(A, \hat{P})$ is chosen in all binary sets containing it and containing another element of A, the recursive application of RCCSS leads to $c(A) = M(A, \hat{P}) = M(A, P)$. If $d \in A$ and $A \cap Q = \emptyset$, d is chosen against any other element in A and thus, the repeated application of RCCSS leads to $c(A) = d$. Then let A be such that $d \in A$ and $A \cap Q \neq \emptyset$. We divide the rest of the proof in two cases, namely, $T^+ \subseteq U$ leading to an extreme endogenous status-quo biased choice, and $T^+ \subseteq V$ leading to a weak endogenous status-quo biased choice.

First, assume that $T^+ \subseteq U$ (and consequently, $T^- \subseteq V$). Since $T^- \subseteq V$, we can consider a route $<$ in which the initial binary sets are all of the form $\{x, d\}$, with $x \in T^-$. The repeated application of RCCSS shows that $c(A) = c(A \setminus T^-)$. Now we prove that $M(A \cap Q, P)$ is the element chosen in all binary sets containing it and containing another element of $A \setminus T^-$. This is obvious for elements in $A \cap Q$ and for d. It must be true also for any element $x \in A \setminus T^-$ such that $x \neq d$ and $x \notin Q$, because otherwise the elements x, d and $M(A \cap Q, P)$ would form a cycle and x would be a member of T^-, which is absurd. Hence, we can apply repeatedly RCCSS on $<$ to reach $c(A) = M(A \cap Q, P)$. This proves that c is an extreme endogenous status-quo biased choice associated to d, Q and P.

Second, assume that $T^+ \subseteq V$ (and consequently, $T^- \subseteq U$). If $M(A \setminus \{d\}, P) \in Q$, then clearly $M(A \setminus \{d\}, P)$ is the element chosen in all binary sets containing it and containing any other element of A. Hence, we can apply repeatedly RCCSS to reach $c(A) = M(A \setminus \{d\}, P)$. If $d = c(\{d, M(A \setminus \{d\}, P)\})$, since $A \cap Q \neq \emptyset$, there must exist a cycle in A involving d, $M(A \setminus \{d\}, P)$ and an alternative in $A \cap Q$. Hence $T^+ \cap A \neq \emptyset$. Consider a route $<$ respecting $\mathcal{S}$ starting with a first binary comparison containing an element of $T^+ \cap A$. Then, RCCSS implies $c(A) = c(A \setminus \{d\})$ and the argument on sets without status-quo leads to

$c(A \setminus \{d\}) = M(A \setminus \{d\}, P)$, as desired. This proves that c is a weak endogenous status-quo biased choice associated to d, Q and P.

Thus, in any case, c is an endogenous status-quo biased choice, concluding the proof.□

Proof of Theorem 3: We first prove that if c is rationalizable by game trees, it must satisfy RCNSS. Consider the game tree $\Gamma = (G, \{P_i\}_{i=1}^k)$ that rationalizes c. Define a collection of subsets of alternatives by considering, for any non-terminal node i, the set of all the alternatives S_i that directly and indirectly succeed the node i. Clearly, this collection is a well-defined nested similarity structure $\mathcal{S}$. Consider a route $<$ that respects $\mathcal{S}$ and take a set $A \in \mathcal{X}$ with at least two alternatives. Let $A^< = \{x, y\}$. Consider the node i_{xy} in the tree such that x and y directly or indirectly succeed it and such that no succesor of i_{xy} has the same property. We now claim that $S_{i_{xy}} \cap A = \{x, y\}$. Suppose otherwise. Then, a different element $z \in S_{i_{xy}} \cap A$ would exist. Without loss of generality, alternative z belongs to the same subtree that alternative x and thus, there exist a successor i' of i_{xy} such that $x, z \in S_{i'}$ (and $y \notin S_{i'}$). Since $<$ respects $\mathcal{S}$, it must be $\{x, z\} < \{x, y\}$ contradicting that $A^< = \{x, y\}$. We have proved that alternatives x and y are the only alternatives in $S_{i_{xy}} \cap A$. Backward induction guarantees that $SPNE(\Gamma|A) = SPNE(\Gamma|A \setminus \{l\})$ where l is the dominated alternative in $\{x, y\}$ according to the preference in node i_{xy}. But obviously, this is exactly $nc(A^<)$. Hence, $c(A) = SPNE(\Gamma|A) = SPNE(\Gamma|A \setminus \{nc(A^<)\}) = c(A \setminus nc(A^<))$.

In the other direction suppose that c satisfies RCNSS; we prove that c is rationalizable by game trees. Let $\mathcal{S}$ be a nested similarity structure such that for every route $<$ that respects it and for every set $A \in \mathcal{X}$ with at least two alternatives, $c(A) = c(A \setminus nc(A^<))$. We first construct a game tree $\Gamma = (G, \{P_i\}_{i=1}^k)$. Associate the first node n_1 to the set of all elements $X \equiv A_1$. Consider a maximal set in $\mathcal{S}$, according to set inclusion, which is a strict subset of X but not a singleton (if there is no such a maximal set, simply pick up any set of two alternatives from X). Denote this set by A_2. In this case, node n_1 points to two nodes, n_2 and n_3, associated respectively with sets A_2 and $X \setminus A_2 \equiv A_3$. Consider a node n_j and its associated set A_j. If A_j contains only one alternative, this node is a terminal one. If A_j contains two alternatives, build two terminal nodes that succeed n_j, each one associated to one of the alternatives in A_j. If A_j contains three or more alternatives, consider a maximal set in $\mathcal{S}$, according to set inclusion, which is a strict subset of A_j but it is not a singleton (if there is not such a maximal set, simply pick up any set of two alternatives from A_j). Let this set be B. In this case, node n_j points to two nodes, n_r and n_s, associated respectively with sets B and $A_j \setminus B$. Proceed in this way until all terminal nodes are reached. Now, define the asymmetric preference associated to non-terminal node j, with successors r and s, as follows. For every $x \in A_r$ and for every $y \in A_s$, let xP_jy if $x = c(\{x, y\})$ and yP_jx if $y = c(\{x, y\})$. We have constructed a binary tree Γ and we will prove now by induction that for all $A \in \mathcal{X}$: $c(A) = SPNE(\Gamma|A)$.

The claim is trivial for sets with two alternatives. Suppose it is true for sets with at most p alternatives and let A be a set with $p+1$ alternatives. Let $<$ be a route such that for any $a, b, c, d \in X$, $\{a, b\} < \{c, d\}$ whenever i_{ab} directly or indirectly succeed i_{cd}. By construction, $<$ respects the nested similarity structure $\mathcal{S}$. Now take $A^< = \{x, y\}$. An analogous argument to the one used above shows that $A_{i_{xy}} \cap A = \{x, y\}$. By RCNSS, it must be $c(A) = c(A \backslash nc(A^<))$. By the inductive hypothesis, $c(A \backslash nc(A^<)) = SPNE(\Gamma|(A \backslash nc(A^<)))$ and also, $c(A^<) = SPNE(\Gamma|A^<)$. Hence, the choice in $A^<$ is determined by preference $P_{i_{xy}}$ and since there is no other alternative in $A_{i_{xy}} \cap A$, it must be $SPNE(\Gamma|A) = SPNE(\Gamma|(A \setminus nc(A^<)))$. This shows that $c(A) = SPNE(\Gamma|A)$, as desired.□

Proof of Theorem 4: We first prove that if c is a sequentially rationalizable, it satisfies WRC. Let $\{P_1, \ldots, P_K\}$ sequentially rationalize c. Construct the collection of relations $\{P'_1, \ldots, P'_K\}$ from $\{P_1, \ldots, P_K\}$, as follows: for all $j = 1, \ldots, K$, $(x, y) \in P'_j$ if and only if $(x, y) \in P_j$ and there is no $i < j$ such that $(x, y) \in P_i$ or $(y, x) \in P_i$. Clearly, $\{P'_1, \ldots, P'_K\}$ is an ordered collection of acyclic relations that sequentially rationalizes c. Assume, without loss of generality, that the constructed collection $\{P'_1, \ldots, P'_K\}$ is composed of non-empty relations (otherwise, simply remove the empty ones and re-number them).

Now, consider a relation P'_j in the constructed collection $\{P'_1, \ldots, P'_K\}$, that contains more than one pair of alternatives. Since P'_j is acyclic, there is a pair of alternatives (a, b) in P'_j, such that $(b, d) \notin P'_j$ for every $d \in X$. We can split the relation P'_j into two relations, $\{(a, b)\}$ and $P'_j \setminus \{(a, b)\}$. We show that for every $A \subseteq X$, $M(A, P'_j) = M(M(A, \{(a, b)\}), P'_j \setminus \{(a, b)\})$. If either a or b is not in A, then clearly $M(A, P'_j) = M(A, P'_j \setminus \{(a, b)\})$, and since $M(A, \{(a, b)\}) = A$, it follows that $M(A, P'_j \setminus \{(a, b)\}) = M(M(A, \{(a, b)\}), P'_j \setminus \{(a, b)\})$. In the case of $a, b \in A$, it follows that $M(A, \{(a, b)\}) = A \setminus \{b\}$. Given that (a, b) in P'_j and that $(b, d) \notin P'_j$ for every $d \in X$, $M(A, P'_j) = M(A \setminus \{b\}, P'_j)$. Hence, $M(A, P'_j) = M(M(A, \{(a, b)\}), P'_j \setminus \{(a, b)\})$, as desired. It then follows that the ordered collection of acyclic relations $\{P'_1, \ldots, P'_{j-1}, \{(a, b)\}, P'_j \setminus \{(a, b)\}, P'_{j+1}, \ldots, P'_K\}$ also sequentially rationalizes c. By iterating this relation-splitting process, we end up with a collection of relations $\{P^*_1, \ldots, P^*_{n(n-1)/2}\}$, each of which contains one pair of alternatives and that sequentially rationalizes c (and obviously, all pairs of alternatives appear in the sequence). Now, the ordered collection of relations $\{P^*_1, \ldots, P^*_{n(n-1)/2}\}$ immediately induces a route $<$ over the binary problems such that $c(A) = c(A \setminus \{nc(A^<)\})$, and hence WRC holds.

In the other direction suppose that c satisfies WRC; we prove that c is sequentially rationalizable. Let $<$ be a route such that for every set $A \in \mathcal{X}$ with at least two alternatives, $c(A) = c(A \setminus nc(A^<))$. Denote the linearly ordered collection of binary problems according to $<$ by $\{a_i, b_i\}_{i=1}^{n(n-1)/2}$. Without loss of generality, let it be assumed that $a_i = c(\{a_i, b_i\})$, $i = 1, \ldots, n(n-1)/2$. Define $P_i = \{(a_i, b_i)\}$. Clearly, $\{P_i\}_{i=1}^{n(n-1)/2}$ is a collection of acyclic relations.

We prove, by induction over the cardinality of choice problems $A \subseteq X$, that $\{P_i\}_{i=1}^{n(n-1)/2}$ sequentially rationalizes c. It is obvious for the case of $|A| \leq 2$. Supposing that the claim is true for $|A| = t$, we show it to be true for $|A| = t+1$. By WRI, $c(A) = c(A \setminus \{nc(A^<)\})$. By the inductive hypothesis $c(A \setminus \{nc(A^<)\}) = M_1^{n(n-1)/2}(A \setminus \{nc(A^<)\})$, and by the definition of $nc(A^<)$ and the construction of the binary relations, $M_1^{n(n-1)/2}(A \setminus \{nc(A^<)\}) = M_1^{n(n-1)/2}(A)$. Therefore, $c(A) = M_1^{n(n-1)/2}(A)$, as desired.□

Proof of Proposition 2: We start by proving that $\mathcal{C}^{RSM} \subseteq \mathcal{C}^{SR}$. Suppose c is an RSM. Then, a pair of asymmetric relations $\{P_1, P_2\}$ that sequentially rationalizes c exists. We now construct a collection of acyclic relations $\{P'_1, \ldots, P'_K\}$ that also sequentially rationalizes c. Let $P'_1 = P_1$ and given $P_2 = \{(a_2^1, b_2^1), \ldots, (a_2^r, b_2^r)\}$, define the binary relations $P'_{j+1} = \{(a_2^j, b_2^j)\}$, $1 \leq j \leq r$. First, we prove that all relations are acyclic. This is immediate for the relations $P'_2, \ldots, P'_{r+1}$. Suppose, by way of contradiction, that $P'_1 = P_1$ is cyclic. Then, a collection $x_1, \ldots, x_r \in X$, with $r > 1$, such that $(x_i, x_{i+1}) \in P_1$, $i = 1, \ldots, r-1$, and $(x_r, x_1) \in P_1$ exists. Then $M(\{x_1, \ldots, x_r\}, P_1) = \emptyset$, contradicting the fact that $\{P_1, P_2\}$ sequentially rationalizes c. Therefore, all relations are acyclic.

We now show that the collection $\{P'_1, \ldots, P'_{r+1}\}$ sequentially rationalizes c. Take any A. Suppose that $M_1^{r+1}(A)$ contains two or more distinct elements. Take any two such elements $x, y \in M_1^{r+1}(A), x \neq y$. Then, for $j = 1, \ldots, r+1$, it is neither the case that $(x, y) \in P'_j$ nor that $(y, x) \in P'_j$. But then, for $i = 1, 2$ it is neither true that $(x, y) \in P_i$ nor that $(y, x) \in P_i$. This contradicts the fact that $\{P_1, P_2\}$ rationalizes the choice in $\{x, y\}$ and therefore $M_1^{r+1}(A)$ contains at most one element. We now prove that $c(A)$ belongs to $M_1^{r+1}(A)$. Given that c is sequentially rationalized by $\{P_1, P_2\}$, it follows that $c(A) \in M(A, P_1) = M(A, P'_1)$ and, for any $y \in M(A, P'_1)$, it cannot be the case that $(y, c(A)) \in P_2$. Therefore, there is no P'_j, $j = 2, \ldots, r+1$, such that $(y, c(A)) \in P'_j$, and then $c(A) \in M_1^{r+1}(A)$. Hence, $c(A) = M_1^{r+1}(A)$ and $\{P'_1, \ldots, P'_{r+1}\}$ is a collection of acyclic binary relations that rationalizes c.

We now show that the inclusion is strict. To do so, we define a choice function c_1 that is in $\mathcal{C}^{SR}$, but is not an RSM. Let $X = \{1, \ldots, 4\}$ and c_1 be such that: $c_1(\{1,3\}) = 1$, $c_1(\{1,2\}) = 2$, $c_1(\{1,2,3,4\}) = c_1(\{1,2,3\}) = c_1(\{2,3,4\}) = c_1(\{2,3\}) = c_1(\{3,4\}) = 3$, and $c_1(\{1,2,4\}) = c_1(\{1,3,4\}) = c_1(\{1,4\}) = c_1(\{2,4\}) = 4$. Consider the ordered collection of acyclic relations $\{P_1, \ldots, P_6\} = \{\{(2,1)\}, \{(1,3)\}, \{(4,1)\}, \{(3,2)\}, \{(4,2)\}, \{(3,4)\}\}$. It is easy to check that this collection of relations sequentially rationalizes c_1. Note, however, that $4 = c_1(\{1,2,4\}) = c_1(\{1,3,4\})$ but $c_1(\{1,2,3,4\}) = 3$. This means that c_1 violates the classic Expansion property, and then by Theorem 1 in Manzini and Mariotti (2007), c_1 is not an RSM.[12] Therefore, $\mathcal{C}^{RSM} \subset \mathcal{C}^{SR}$.

We now show that $\mathcal{C}^{SR} \subset \mathcal{C}^{SR(As)}$. The fact that $\mathcal{C}^{SR} \subseteq \mathcal{C}^{SR}$ is obvious, so we prove that the inclusion is strict. Consider the following example. Let $X = \{1, \ldots, 6\}$ and c_2 be such that:

[12]**Expansion**: For all $A, B \subseteq X$, $x = c(A) = c(B) \Rightarrow x = c(A \cup B)$.

$c_2(\{1,3,4,5\}) = c_2(\{1,3,4\}) = c_2(\{1,3,5\}) = c_2(\{1,4,5\}) = c_2(\{1,3\}) = c_2(\{1,5\}) = 1$, $c_2(\{2,3,6\}) = c_2(\{2,5,6\}) = c_2(\{2,5\}) = c_2(\{2,6\}) = 2$, $c_2(\{2,3,4,5,6\}) = c_2(\{2,3,4,5\}) = c_2(\{2,3,5,6\}) = c_2(\{3,4,5,6\}) = c_2(\{2,3,4\}) = c_2(\{2,3,5\}) = c_2(\{3,4,5\}) = c_2(\{3,5,6\}) = c_2(\{2,3\}) = c_2(\{3,4\}) = c_2(\{3,5\}) = 3$, $c_2(\{2,4,5,6\}) = c_2(\{1,4,6\}) = c_2(\{2,4,5\}) = c_2(\{2,4,6\}) = c_2(\{1,4\}) = c_2(\{2,4\}) = c_2(\{4,6\}) = 4$, $c_2(\{1,3,4,5,6\}) = c_2(\{1,3,5,6\}) = c_2(\{1,4,5,6\}) = c_2(\{1,5,6\}) = c_2(\{4,5,6\}) = c_2(\{4,5\}) = c_2(\{5,6\}) = 5$, $c_2(\{1,3,4,6\}) = c_2(\{2,3,4,6\}) = c_2(\{1,3,6\}) = c_2(\{3,4,6\}) = c_2(\{1,6\}) = c_2(\{3,6\}) = 6$, and $c_2(A) = c_2(A \setminus \{2\})$ whenever $\{1,2\} \subseteq A$.

We first show that $c_2 \in \mathcal{C}^{SR(As)}$. Consider for instance $P_1 = \{(1,2)\}$, $P_2 = \{(1,3),(3,4),(4,2),(2,5),(5,6),(6,1)\}$, $P_3 = \{(5,4),(1,5),(2,6),(6,3),(3,5),(4,6)\}$ and $P_4 = \{(4,1),(3,2)\}$. One can verify that all choices are sequentially rationalized by this ordered collection of asymmetric relations.

Suppose now that c_2 is sequentially rationalized by the ordered collection of acyclic relations $\{P'_1, \ldots, P'_K\}$ with $P'_i = \{(a_i, b_i)\}$, $i = 1, \ldots, K$. Let T be the smallest positive integer such that $P'_T \neq \{(1,2)\}$ and $P'_T \neq \{(2,1)\}$. T is well-defined since the collection of acyclic relations must rationalize the choice in set $\{1,3\}$, for instance.

We now show that for every A with $\{a_T, b_T\} \subseteq A$ and $\{1,2\} \not\subseteq A$, it must be that $c_2(A) = c_2(A \setminus \{b_T\})$. If $T = 1$, it is immediate that $c_2(A) = M_1^K(A) = M_1^K(A \setminus \{b_1\}) = c_2(A \setminus \{b_1\})$. If $T > 1$, $\cup_{i<T}\{a_i, b_i\} = \{1,2\}$, and then it follows that $c_2(A) = M_1^K(A) = M_T^K(A) = M_T^K(A \setminus \{b_T\}) = M_1^K(A \setminus \{b_T\}) = c_2(A \setminus \{b_T\})$. We now show that $(a_T, b_T) \notin \{(1,2),(1,5),(3,5),(4,1),(6,1),(4,6),(5,6),(1,3),(6,3),(2,6),(3,4),(2,5),(3,2),(4,2),(5,4)\}$. To see this, simply notice that $c_2(\{1,3,4,5,6\}) \notin \{c_2(\{1,3,4,6\}), c_2(\{3,4,5,6\}), c_2(\{1,3,4,5\})\}$ implies that $(a_T, b_T) \notin \{(1,5),(3,5),(4,1),(6,1),(4,6),(5,6)\}$. Analogously, $c_2(\{2,3,4,6\}) \notin \{c_2(\{2,3,4\}), c_2(\{2,3,6\})\}$ implies that $(a_T, b_T) \notin \{(2,6),(3,4)\}$, and that $c_2(\{2,4,5\}) \notin \{c_2(\{4,5\}), c_2(\{2,5\})\}$ implies that $(a_T, b_T) \notin \{(4,2),(5,4)\}$. Finally, from $c_2(\{1,3,4,6\}) \neq c_2(\{1,4,6\})$, $c_2(\{2,3,4,5,6\}) \neq c_2(\{2,3,4,6\})$ and $c_2(\{2,3,6\}) \neq c_2(\{3,6\})$ we have that $(a_T, b_T) \notin \{(1,3),(6,3),(2,5),(3,2)\}$.

Given the binary choices in c_2, (a_T, b_T) is then a pair such that $b_T = c(\{a_T, b_T\})$. However, $M_1^K(a_T, b_T) = M_T^K(a_T, b_T) = a_T$ leading to a contradiction with the fact that the acyclic relations sequentially rationalize c_2.□

Proof of Proposition 3: That $\mathcal{C}^{RAT} \subset \mathcal{C}^{SQB}$ follows trivially. Let us show that $\mathcal{C}^{SQB} \subset \mathcal{C}^{RGT}$. Let $X = \{1,2,3,4\}$ and the choice function c_3 defined by $c_3(\{1,2\}) = c_3(\{1,3\}) = c_3(\{1,2,3\}) = c_3(\{1,2,4\}) = c_3(X) = 1$, $c_3(\{2,3\}) = c_3(\{2,4\}) = c_3(\{2,3,4\}) = 2$, $c_3(\{3,4\}) = c_3(\{1,3,4\}) = 3$ and $c_3(\{1,4\}) = 4$. It is immediate that $c_3 \in \mathcal{C}^{RGT}$ with the following binary tree: node 1 points to alternatives 2 and 4, node 2 points to node 1 and alternative 1, and node 3 points to node 2 and alternative 3, and where the binary relations in the three nodes are constructed from the choices in the binary problems involving the alternatives that are successors of them. To see that c_3 is not extreme endogenous status-quo biased choice note that from the cycle involving alternatives $1, 2, 4$ (respectively $1, 3, 4$) we can immediately deduce that the status quo can only be alternative 2 (respectively 4), which is absurd. Finally, to see that c_3 is not weak endogenous status-quo biased choice note that from the cycle involving alternatives $1, 2, 4$ (respectively $1, 3, 4$) the status quo can only be alternative 4 (respectively 1), which is absurd. Hence, $c_3 \notin \mathcal{C}^{SQB}$.

We now show that $\mathcal{C}^{RGT} \subset \mathcal{C}^{SR}$. Consider the choice function c_4 such that $c_4(A) = c_3(A)$ for every A in X, except for $c_4(\{1,3,4\}) = 1$ and $c_4(\{1,2,4\}) = c_4(X) = 4$. To see that c_4 is not RGT notice that the cycles involving alternatives $\{4,1,2\}$ and $\{1,3,4\}$ imply that the only possible game tree that explains choices has two branches; one comprising alternatives 1 and 2, and the other comprising alternatives 3 and 4. However, under this game tree, we should have $4 = c_4(X) = c_4(\{c_4(\{1,2\}), c_4(\{3,4\})\}) = c_4(\{1,3\}) = 1$, a contradiction. Finally, to show that $c_4 \in \mathcal{C}^{SR}$ consider the sequential procedure for the route: $\{2,3\} < \{1,2\} < \{3,4\} < \{2,4\} < \{4,1\} < \{1,3\}$. It follows immediately that the former rationalizes the choices of c_4, and hence the claim follows.□

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