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**Option Pricing under Stochastic Volatility
and Stochastic Interest Rate in the
Spanish Case**

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Abstract

Among the underlying assumptions of the Black-Scholes option pricing model, those of a fixed volatility of the underlying asset and of a constant short-term riskless interest rate, cause the largest empirical biases. Only recently has attention been paid to the simultaneous effects of the stochastic nature of both variables on the pricing of options. This paper has tried to estimate the effects of a stochastic volatility and a stochastic interest rate in the Spanish option market. A discrete approach was used. Symmetric and asymmetric GARCH models were tried. The presence of in-the-mean and seasonality effects was allowed. The stochastic processes of the MIBOR90, a Spanish short-term interest rate, from March 19, 1990 to May 31, 1994 and of the volatility of the returns of the most important Spanish stock index (IBEX-35) from October 1, 1987 to January 20, 1994, were estimated. These estimators were used on pricing Call options on the stock index, from November 30, 1993 to May 30, 1994. Hull-White and Amin-Ng pricing formulas were used. These prices were compared with actual prices and with those derived from the Black-Scholes formula, trying to detect the biases reported previously in the literature. Whereas the conditional variance of the MIBOR90 interest rate seemed to be free of ARCH effects, an asymmetric GARCH with in-the-mean and seasonality effects and some evidence of persistence in variance (IEGARCH(1,2)-M-S) was found to be the model that best represent the behavior of the stochastic volatility of the IBEX-35 stock returns. All the biases reported previously in the literature were found. All the formulas overpriced the options in Near-the-Money case and underpriced the options otherwise. Furthermore, in most option trading, Black-Scholes overpriced the options and, because of the time-to-maturity effect, implied volatility computed from the Black-Scholes formula, underestimated the actual volatility.

1.- INTRODUCTION

Since its apparition in mid seventies the Black-Scholes formula (**Black and Scholes, 1973**) has become the most popular one in the valuation of options. It is known that several of the underlying assumptions necessary to develop the implied differential equation that the option must satisfy are extremely restrictive. Among them, the hypotheses of a fixed volatility of the underlying asset and of a constant short-term riskless rate of interest, cause the largest empirical biases on the pricing of the options. Although the consequences of the failure to fulfil of these hypotheses has been studied ever since the formulation of the Black-Scholes model, the simultaneous effects of both a stochastic interest rate and a stochastic volatility process on the valuation of options have received attention only recently (**Amin and Ng, 1993**). Even though the empirical evidence is still scarce, it seems that in the at-the-money case the stochastic nature of the volatility causes the Black-Scholes formula to overprice the Call options (**Hull and White, 1987**). On the contrary, when the interest rate is stochastic the formula underprices the options (**Merton, 1973**). The combined effect of both depends on the relative variability of the two processes (**Amin and Ng, 1993**).

From a statistical point of view, in order to provide an estimation of the riskless interest rate and of the volatility of the underlying asset, one needs to model the respective stochastic processes. The main problem is that volatility cannot be directly observed. This drawback can be solved (at least in part, see **Andersen, 1992**) if one assumes that the volatility of the underlying asset is measured by the variance of its

corresponding stochastic process. From a discrete approach, the modelling of this time-varying second moment can be achieved from Engle's autoregressive conditional heteroscedasticity (ARCH) models (Engle, 1982)¹. The dynamic behaviour of the volatility is modelled, allowing a time-varying conditional variance while leaving constant the unconditional variance. Furthermore these models are empirical, in the sense that they were created specifically in order to capture the stylized facts of short run financial dynamics, to wit the clustering of volatility and the existence of leptokurtic unconditional distributions of returns.

In this paper, the stochastic behaviour of a Spanish short-term interest rate and the stochastic volatility of the returns of a Spanish stock exchange index were estimated. A discrete approach, generalized autoregressive conditional heteroscedastic models (GARCH), was used. The estimators of the stochastic processes were used on pricing Call options on the Spanish index IBEX-35, bearing in mind the stochastic nature of the variables involved. These prices were compared with actual prices and with those derived directly from the Black-Scholes formula, trying to detect the empirical biases suggested in the literature. After the above introduction, the different formulas of valuation of the options, depending on the underlying hypotheses one assumes, are described in section 2. The statistical models are presented in section 3. The data and the empirical results are discussed in section 4. Section 5 finishes with some concluding remarks.

2.- SEVERAL FORMULAS FOR THE PRICING OF THE OPTIONS

According to Whaley (1986) the required assumptions necessary to price an European call option from the Black-Scholes formula can be formulated as follows: there are no transaction costs; markets are free of costless arbitrage opportunities; the short-term riskless rate of interest is constant through time and the instantaneous options price change is described by the stochastic differential equation,

$$dS/S = \mu dt + \sigma dW \quad (1)$$

where S is the price of the underlying asset, W is a Wiener process², μ is a constant³, and σ is the instantaneous standard deviation. The assumption of a constant short-term interest rate implies that the interest rate uncertainty does not have any effect on the volatility of the underlying asset price. Likewise, it is assumed that the volatility, i.e. σ , is constant through time. These two assumptions hardly ever hold.

Under these assumptions Black (1976a) (and Black and Scholes, 1973) derived the partial differential equation governing the movements of the call option price (C) through time,

$$\frac{1}{2} \sigma^2 S^2 C_{SS} - rC + C_t = 0 \quad (2)$$

where r is the riskless rate of interest. Under the boundary condition that the terminal call price is equal to the maximum value of 0 or the in-the-money amount of the option, the value of a European call option is,

where CBS is the European call option price, S is the price of the underlying asset,

$$CBS = SN(z_1) - Ee^{-rt}N(z_2)$$

$$z_1 = \frac{\ln(S/E) + (r + \sigma^2/2)t}{\sigma\sqrt{t}} \quad (3a)$$

$$z_2 = z_1 - \sigma\sqrt{t}$$

E is the strike price, r is a riskless interest rate (annual rate), t is the time to expiration (fraction of a year), σ is the volatility (annualized) and $N()$ is the standard normal cumulative distribution.

Under the put-call parity the European put option price, PBS , can be obtained from,

$$PBS = CBS - S + Ee^{-rt} \quad (3b)$$

These formulas must be adjusted for dividends when the stocks that compose the index yield them (see for instance Bookstaber, 1987). Let V_d be the present value of all known dividends paid during the option's time remaining until expiration, and let $\gamma = -(1/t)\ln(1 - V_d/S)$,

$$CBS' = Se^{-\gamma t}N(z'_1) - Ee^{-rt}N(z'_2)$$

$$z'_1 = z_1 - \frac{\gamma\sqrt{t}}{\sigma} \quad (3a')$$

$$z'_2 = z_2 - \frac{\gamma\sqrt{t}}{\sigma}$$

$$PBS' = CBS' + (Ee^{-rt} - Se^{-\gamma t}) \quad (3b')$$

The volatility is not usually constant over time, changing in response to shocks and also systematically. If the stock had a stochastic volatility, the price process,

$$dS/S = \mu dt + \sigma dW \quad (1)$$

would be augmented by an independent specification of the volatility process as a geometric Wiener process,

$$d\sigma/\sigma = \lambda dt + \gamma dV \quad (4a)$$

λ and γ are constants and V is a standard Brownian motion; or as a Ornstein-Uhlenbeck process (ξ is a constant),

$$d\sigma/\sigma = \lambda (\xi - \sigma) dt + \gamma dV \quad (4b)$$

However, because volatility is non-observable and non-traded it becomes impossible to construct a perfect-hedge portfolio and, therefore, arbitrage arguments cannot be used. In this case, it is not possible to find a closed form solution to the partial differential equation that the option price must satisfy (see for instance **Johnson, 1979**; and **Johnson and Shanno, 1985**).

Hull and White (1987) circumvent this problem by assuming that the volatility risk can be diversified. In particular they assume that volatility is uncorrelated with aggregate consumption, that is to say volatility has zero systematic risk. A second assumption states that volatility is uncorrelated with the stock price. This is equivalent to assuming no leverage and a constant volatility of firm value. Finally they do not assume that the volatility is a traded asset. These assumptions allow the use of risk-neutral valuation and give an option value in form of a stochastic integral. Under the above assumptions the solution of the integral can be derived by means of Taylor series expansion or by numerical procedures, via Monte Carlo simulation. Specially, for the case when volatility is uncorrelated with stock price⁴

$$CHW_{t+1, t+1+\tau} = \frac{1}{N} \sum_{j=1}^N CBS'_j (S_{t+1}, E, \bar{\sigma}_j) \quad (5)$$

$$\text{where } \bar{\sigma}_j = (1/\tau) \sum_{i=2}^{t+1+\tau} h_{t, t+i}^{(1/2), j}$$

CBS' is the call option price with dividend yield adjustment derived from (3a'); N is the number of replications for random variables $\eta_{i+1, j} = \epsilon_{i+1, j} / \sqrt{h_{i+1, j}}$, $i = 1, \dots, \tau-1$ and $j = 1, \dots, N$, which are drawn from the standard normal distribution with mean 0 and standard deviation of 1; and $h_{i+1, j}$ is an estimation of the stochastic standard deviation. In this case the estimation was derived from the estimated asymmetric GARCH's explained in section 3.

There is empirical evidence suggesting that the volatility of stock prices is not only stochastic, but also has a systematic component which is related to the stochastic volatility of the consumption growth (Amin and Ng, 1993). Black and Scholes (1972) and Whaley (1982), among others, point out that options on high and low risk stocks are associated with different biases in the prices obtained from the Black-Scholes formula. Low risk stocks generally have a stochastic return volatility highly correlated with the return volatility of the market. On the contrary, high risk stocks have a small systematic volatility component. Since the interest rate is determined by the consumption volatility in equilibrium, and this latter is stochastic, the former is also stochastic.

When the mean and the variance of consumption growth, the mean of the stock returns, and the covariance of the stock returns and consumption growth are

predictable, **Amin and Ng (1993)** derive an option-pricing formula independent of the preference parameters, even if the individual stock returns volatility is related to the consumption volatility. When these processes are not predictable the option valuation formula generally depends on preference parameters except when the unpredictable component of the volatility of individual stock returns is not related to the consumption volatility.

$$CAN = E_0[S_0 N(d_1(G_T)) - E e^{-R(G_T)t} N(d_2(G_T))]$$

$$d_1(G_T) = \frac{\ln(S_0 / (E e^{-R(G_T)t})) + \frac{1}{2} \bar{h}_s(G_T) t}{[\bar{h}_s(G_T) t]^{1/2}}$$

$$d_2(G_T) = d_1(G_T) - \bar{h}_s(G_T) t \tag{6}$$

$$R(G_T) = \sum_{t=0}^{T-1} \frac{r_t}{T}$$

$$\bar{h}_s(G_T) = \sum_{t=1}^T \frac{h_{s,t}}{T}$$

The expectation, E_0 , is taken with respect to the state variables that govern the evolution of the stock returns variance process and the interest rate. The variance $h_{s,t}$ can be also estimated from the asymmetric GARCH models explained below. In addition, as in **Hull and White (1987)**, option prices can be computed using Monte Carlo simulation.

Amin and Ng (1993) point out that all the dynamics of the consumption process relevant to option valuation are embodied in the interest rate process. The formula is preference free as it does not explicitly include the preference parameters other than through the interest rate. The above formula is valid if the processes are defined

to be continuous time diffusion processes. Nelson (1990) shows that the discrete time GARCH model converges to a continuous time diffusion model as the sampling interval gets arbitrarily small (say daily or less). Hence, the application of that type of discrete model in the present situation is justified. Also, in this case the mean, variance and covariance processes need not to be predictable, since the definition of Ito integrals guarantees that integrands be predictable (Amin and Ng, 1993). Finally, Amin and Ng (1993) point out that several option pricing formulas in the literature are special cases of (6)⁵. This is the case, in particular, for the Bailey-Stultz (1989) stochastic variance formula for index options when one assumes logarithmic preferences and perfect correlation between the idiosyncratic components of the stock returns and the consumption growth. In this case, the covariance between the stock returns and consumption growth is then equal to the variance of the consumption growth.

3.- GARCH MODELLING OF THE STOCHASTIC VOLATILITY

As was mentioned above, the short run dynamics of the returns of financial assets is characterized by the clustering of their volatility and by the existence of leptokurtic unconditional distributions. These facts are well fitted by Bollerslev's (1986) GARCH(p,q) model, a generalization of Engle's (1982) ARCH(q) model. In this paper it was assumed that the non-linear dependency characteristic of stock-market returns was completely described by GARCH models. In particular, both the stochastic returns process of the index and the short-term interest rate, say r_t , can be described with the following ARMA(p',q')-GARH(p,q) model,

$$\begin{aligned}
r_t &= \phi_0 + \sum_{i=1}^{p'} \phi_i r_{t-i} + \sum_{i=1}^{q'} \theta_i \varepsilon_t \\
h_t &= \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j}
\end{aligned} \tag{7}$$

$$\begin{aligned}
\varepsilon_t &= \xi_t h_t^{\frac{1}{2}} \\
\xi &\sim i.i.d. N(0, 1)
\end{aligned}$$

In this model ξ_t was assumed to be normally distributed. Therefore, ε_t were generated by a conditional normal distribution with variance h_t ,

$$\varepsilon_t / \Omega_{t-1} \sim N(0, h_t)$$

where Ω_{t-1} was the information set at time t .

In order to ensure the non-negativity of the conditional variances it was assumed that $\omega \geq 0$, $\alpha_i \geq 0$ and $\beta_i \geq 0$. In addition, the following restriction was imposed in order to guarantee the stationarity,

$$\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1$$

Since it was assumed that the model was conditional normally distributed, the parameters of the GARCH(p,q) model were estimated by maximum likelihood. This method ensures the consistency of the estimators and, if the distribution assumption is right, their asymptotic efficiency (Weiss, 1986; Lee and Hansen, 1994). The log-likelihood can be written as,

$$LnL = \sum_{t=1}^T Ln l_t = -.5 \sum_{t=1}^T \left[\ln h_t + \frac{\varepsilon_t^2}{h_t} \right]$$

The BHHH maximization algorithm was used (Berndt, Hall, Hall, Hausman, 1974).

Starting from the most parsimonious model, i.e. a GARCH(1,1), different specifications were tried. If the models were nested, the alternative models would be tested by means of a likelihood ratio test, $-2(\text{Ln}L_0 - \text{Ln}L_A)$, distributed as a χ^2 . For instance, a GARCH(1,1), $H_0: p=1, q=1$, vs a GARCH(1,2), $H_A: p=1, q=2$, could be tested through this test distributed as χ^2_1 .

It was possible that the stock returns were a function of the conditional variance of the model, namely a GARCH-in-Mean or GARCH-M model, (Engle, Liliien and Robins, 1987).

$$r_t = g(r_{t-1}, h_t; b) + e_t$$

If it was the case an increase in the conditional variance would be associated with a change in the conditional mean, depending on the sign of the partial derivative of g_0 with respect to h_t . This fact would imply a tradeoff between the risk and the expected returns. To test this possibility the squared standardized residuals, that is a proxy of the stochastic variance, were introduced in the models⁶ as another explanatory variable in the stock returns regression. If the associated parameter was statistically significant that ARMA(p',q')-GARCH-M model would be estimated.

It was also likely that there would exist 'leverage' effects (Black, 1976b; French, Schwert and Stambaugh, 1987; Schwert, 1990 and Nelson, 1991). In this case the volatility could be affected differently by positive or negative innovations to returns. In financial markets a negative correlation between current returns and future volatility is usually observed. That is to say, whereas negative excess-returns (bad news) lead to a large temporary increase in future volatility, positive returns

(good news) have no (very small) effects. As Black (1976b) point out, a reduction in equity value could raise the debt-to-equity ratio and hence increase the riskiness of the firm, i.e. the future volatility.

These effects are gathered by the 'asymmetric' GARCH models which allow innovations of different sign to have different impacts on volatility. To test for the presence of asymmetric effects in the estimated GARCH models, the tests proposed by Engle and Ng (1993) were used. They are based in the news impact curve which displays the implied relation between ϵ_{t-1} and h_t holding constant the information dated $t-2$ and earlier. The curve measures how new information is incorporated into volatility estimates. In a GARCH model the curve is quadratic and centered on $\epsilon_{t-1} = 0$. For other specifications of the variance the curve might be asymmetric or centered on other values of ϵ_{t-1} . One might test if there was information in ϵ_{t-1} which could predict the squared standardized residuals, in which case the variance would be misspecified.

Let S_{t-1} be a dummy variable which takes the value 1 if $\epsilon_{t-1} < 0$ and 0 otherwise, and $z_t = \epsilon_t \cdot h_t^{-1/2}$ the standardized residuals. The Sign-Bias test statistic is the t-statistic of b in the regression,

$$z_t^2 = a + b S_{t-1} + u_t$$

This test examines the different impacts that positive and negative innovations have on volatility and that have not been predicted by the model. The Positive (Negative) Size-Bias test examines the different impacts that large and small positive (negative) innovations will have on volatility.

$$z_t^2 = a + b S_{t-1}^- \epsilon_{t-1} + u_t$$

$$z_t^2 = a + b S_{t-1}^+ \epsilon_{t-1} + u_t$$

The Joint test statistic is the F-statistic in the following regression,

$$z_t^2 = a + b_1 S_{t-1}^- \epsilon_{t-1} + b_2 S_{t-1}^+ \epsilon_{t-1} + u_t$$

If any of these tests failed to accept the null hypothesis there would be evidences of asymmetric effects in the GARCH models previously estimated. In this case several asymmetric models would be tried.

In particular, Nelson's (1991) exponential GARCH model (EGARCH):

$$\ln h_t = \omega + \sum_{j=1}^p \beta_j \ln h_{t-j} + \sum_{j=1}^q \alpha_j [\theta \xi_t + \gamma (|\xi_t| - \sqrt{2/\pi})] \quad (8)$$

In this model the variance is always positive, that is, no restrictions on the parameters are required. The news impact curve of the EGARCH, as in the GARCH models, has its minimum on $\epsilon_{t-1} = 0$ and is exponentially increasing in both directions but with different parameters. In particular, note that if $|\theta| < \gamma$ and $\theta < 0$, the conditional variance will increase more for negative innovations than for positive. Furthermore, the EGARCH model allows big innovations to have a greater impact on volatility than the standard GARCH model.

Glosten, Jagannathan and Runkle's (1989) model (GJR) was also tried:

$$h_t = \omega + \beta h_{t-1} + \alpha \epsilon_{t-1}^2 + \gamma S_{t-1}^- \epsilon_{t-1}^2 \quad (9)$$

where $S_t^- = 1$ if $\epsilon_t < 0$, $S_t^- = 0$ otherwise

The news impact curve for this model is centered on $\epsilon_{t-1} = 0$, but has different slopes for its positive and negative sides.

Finally some of the models proposed in Engle and Ng (1993) were estimated:

Asymmetric GARCH model (AGARCH):

$$h_t = \omega + \beta h_{t-1} + \alpha (\epsilon_{t-1} + \gamma)^2 \quad (10)$$

The news impact curve is asymmetric and centered at $\epsilon_{t-1} = -\gamma$ which is to the right of the origin when $\gamma < 0$.

Nonlinear asymmetric GARCH model (NGARCH):

$$h_t = \omega + \beta h_{t-1} + \alpha (\epsilon_{t-1} + \gamma \sqrt{h_{t-1}})^2 \quad (11)$$

VGARCH model:

$$h_t = \omega + \beta h_{t-1} + \alpha (\epsilon_{t-1} / \sqrt{h_{t-1}} + \gamma)^2 \quad (12)$$

The news impact curves for these two latter models are symmetric and centered at $\epsilon_{t-1} = (-\gamma)\sqrt{h_{t-1}}$. The slope of the two upward-sloping portions of the VGARCH is steeper than that of the NGARCH.

And a partially nonparametric GARCH model (PNP):

$$h_t = \omega + \beta h_{t-1} + \sum_{i=0}^4 \phi_i P_{it-1} (\epsilon_{t-1} - i\sigma) + \sum_{i=0}^4 \delta_i N_{it-1} (\epsilon_{t-1} + i\sigma)$$

$$P_{it} = 1 \text{ if } \epsilon_t > i\sigma, 0 \text{ otherwise}$$

$$N_{it} = 1 \text{ if } \epsilon_t < -i\sigma, 0 \text{ otherwise}$$

(13)

where σ is the unconditional standard deviation of the dependent variable. This model allows consistent estimation of the news impact curve. The long memory in the variance equation is given by the parametric component, h_{t-1} .

In the presence of in-the-mean or asymmetric effects the information matrix obtained under the assumption of conditional normality is not block diagonal between the parameters in the conditional mean and the conditional variance. Therefore, unlike the GARCH(p,q) model, consistent estimation of the GARCH-M and the asymmetric models requires that the full model be correctly specified.

In addition, in many stock markets the Monday returns are significantly lower than those of other days. Likewise, French and Roll (1986) point out that the variance rate of the returns slows down significantly in days when the market is closed, causing that the returns variance on Mondays to be higher than on the rest of the days. In this sense Engle, Kane and Noh (1993)'s multiplicative model was tried. For instance a seasonal GARCH (GARCH-S hereinafter) may be expressed as:

$$h_t = \alpha_t^\delta \left[\omega + \alpha_{t-1}^{-\delta} \left(\sum_{i=1}^q \alpha_i e_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j} \right) \right] \quad (14)$$

where d_t was a calendar days variable equal to the number of calendar days between the close of the previous trading day (t-1) and the close of trading day t; δ measured the average speed of the variance rate over the d_t calendar days of trading day t. For instance, if t=Monday and the previous trading day was Friday, then $d_t=3$ and h_{t-1} was increased by d_t^δ .

It is possible that both stock returns and interest rate series, present outliers that could cause an increase in the kurtosis and hence induce spurious ARCH effects. The possible outliers present in the time series were detected following sequential likelihood ratio tests proposed by Tiao (1985).

Each day, t_0 , the following tests were performed:

H_0 : Y_{t_0} is neither an 'innovational'(IO) nor a 'level shift' (LS) outlier.

H_1 : Y_{t_0} is an 'innovational' (IO) outlier in t_0 .

H_2 : Y_{t_0} is a 'level shift' (LS) outlier in t_0 .

An 'innovational' outlier may be interpreted as a change in the slope of the time series while the 'level shift' outlier is a change in its y-coordinate, i.e. in the level of the series. There also 'additive' outliers in which only the level of the t_0 observation is affected. The different outliers were defined as follows,

$$(IO) \quad r_t = \frac{\Theta(B)}{\Phi(B)} (e_t + \omega e_t^{(t_0)})$$

$$(LS) \quad r_t = \frac{\Theta(B)}{\Phi(B)} e_t + \omega a_t^{(t_0)}$$

$$e_t^{(t_0)} = 1, t=t_0 \quad 0, \text{otherwise}$$

$$a_t^{(t_0)} = 1, t \geq t_0 \quad 0, \text{otherwise}$$

where r_t was the observable time series. 'Additive' outliers were not detected since they do not mean actual structural breaks but only transient deviations. The likelihood ratio test statistics were defined as follows, H_1 vs H_0 $\lambda_1 = w_1/\sigma_e$; H_2 vs H_0 $\lambda_2 = w_2/(\rho\sigma_e)$. Under the null hypothesis H_0 , λ_1 and λ_2 were distributed as $N(0,1)$.

Since t_0 as well as the other time series parameters are not known, Tiao (1985) proposes an iterative procedure to detect of outliers: model the time series by means of an ARIMA model, compute the likelihood ratio tests, eliminate the effect of the outliers, if any, and repeat the process until all outliers are identified and their

effects simultaneously estimated. These tests must be taken with caution, since, as Tsay (1988) points out, they are carried out under the extremely restrictive hypothesis that the variance of the time series is constant through time.

4.- SOME EMPIRICAL RESULTS

The data consisted of daily price observations for the most important Spanish stock index, IBEX-35, and a daily short-term interest rate, MIBOR90 (Table 1 and Figure 1). The analyzed period comprised from October 1, 1987 to January 20, 1994 for the stock index (1557 observations) and from March 19, 1990 to May 31, 1994 for the interest rate (815 observations). The data corresponding to the period October 29 to November 30, 1987 was excluded from the analysis, leaving finally 1518 observations. The effects of the market crash on October 19 were maximum during this period (see Figure 1), implying a very large reduction of the returns and a dramatic increase of the variance.

The MIBOR90 is the most representative proxy of the short-term riskless interest rate in the Spanish case with sufficient span of data. The Spanish stock exchange index IBEX-35 is a value-weighted index composed of the top thirty-five stocks in terms of volume of contracting in the Spanish stock market. It does not adjust for dividends but adjusts for share issues. The daily stock index returns series and the annualized historical standard deviations were computed as shown in Table 1. Options on the stock index are European, that is to say they cannot be exercised early. All the option contracts mature the third friday of each month. The market for

the options on the index began in January of 1992, implying very few data and, what is more important, a reduced volume of contracting (Viñolas and Moro, 1993). For this reason, and in order to avoid possible start up effects, instead of using the whole available period, options were priced in the period November 30, 1993-May 30, 1994. Only the results of pricing Call options are shown in this paper, although both Call and Put options were actually priced⁷. Non-synchronism of option and stock prices⁸ was mitigated eliminating options quotations with daily volume of less than 100 contracts and those quotations whose closing prices were lower than 10% of the average of the closing prices of that day (Day and Lewis, 1992).

The stylized facts of interest rate and daily stock returns dynamics are shown in **Table 2** and in **Figure 2**. Clustering of the volatilities, the high leptokurtosis and the negative skewness in the unconditional distributions of both stock returns and interest rate can be clearly seen. Wednesday returns were significantly lower than those of other days, being significantly negative. However, there was no evidence of Monday effect in stock returns (compare with Peiró, 1994). The volatility of the returns measured as the standard deviation was maximum on Monday and minimum on Wednesday and Friday. These figures were greater and lower than the overall, respectively. Concerning the interest rate, there was no day where it was statistically different than any other. The standard deviations of the interest rate also showed a different behaviour through the week (maximum Friday, minimum Thursday), although unlike the returns there was no systematic pattern. The equality of variances was tested, as a first approximation, through the usual F-test. However, this test is not robust under non-normality. For this reason, the degrees of freedom

of the test were corrected as suggested by Box (1953) using the sample kurtosis. Only in the case of stock returns were the differences statistically significant.

The autocorrelation functions for both, the returns and the interest rate time series, are shown in Table 3. Since in the presence of ARCH effects the confidence intervals of the autocorrelation functions are wider than usual (Millhøj, 1985) Table 3 should be interpreted with caution. The correlation structures for the 'mean' of both time series were quite simple. An AR(2) and certain evidence of 'seasonality' (lag seven statistically significant) for the stock return; a MA(1) for the interest rate series once differenced. Once several ARMA(p',q') had been estimated for both time series, possible outliers were identified following Tiao's procedure. Several level step and innovational outliers were detected, although only in the interest rate time series. Table 3 also shows the autocorrelation functions of the squared residuals of the best ARMA(p',q') models: an AR(1,2)MA(7) for the stock returns and a MA(1) for the differenced interest rate (see Table 3). In the case of the interest rate the estimated models incorporated the outliers as shown above, i.e. intervention analysis models (Tiao, 1985). Squared residuals were also correlated only in the stock returns case. There were more than three lags statistically significant. This fact suggested an ARMA structure, i.e. GARCH models. There were symptoms of 'seasonality' in the squared residuals (lags four and five). Since an extreme degree of kurtosis, as in this case, might reduce the power of the Ljung-Box test (Baillie and Bollerslev, 1989) the existence of ARCH effects in the residuals series was tested following the proposals of Engle (1982). Specifically, let e_t be the residuals from the ARMA(p',q') models, the statistic is a Lagrange Multiplier test for

$\alpha_1 = \alpha_2 = \dots = \alpha_q = 0$, calculated as TR^2 from the regression of e_t^2 on $e_{t-1}^2, e_{t-2}^2, \dots, e_{t-q}^2$, where T denotes the sample size. Results are also shown in Table 3. ARCH effects only in the stock returns series seemed to be present.

When modelling the conditional heteroskedasticity of the stock returns, a multiplicative seasonal GARCH(1,1) model was first tried. Starting from this parsimonious model, other alternative GARCH's were tested by means of a likelihood ratio test. Then, the standardized residuals ($e_t \cdot h_t^{-1/2}$) from the 'best' GARCH models were computed. These residuals were used for testing the presence of in-the-mean and asymmetric effects, as was explained in section 3 (Table 4). The standardized residuals of the mean adjustment regression, i.e. ARIMA(0,1,1), were used in the interest rate case. Only in the stock returns case there was some evidence of both in-the-mean and asymmetric effects. Several asymmetric GARCH-M models were tried in this case. The results of the estimation of the ARMA(p',q')-GARCH(p,q) models are shown in Table 5⁹.

Since there was no coefficient statistically significant, the interest rate seemed to be a random walk without drift. The dynamic structure for the mean of the stock returns was very simple in all the cases, an AR(1). Although the parameters corresponding to the AR(2) and the 'seasonal' (MA(7)) component were never statistically significant, note that the standard errors of the coefficients were lower than the value of the coefficients, that is, the t-ratios were greater than one in most of the cases. As the efficiency of the estimates was not guaranteed by the estimation procedure (the residuals were not normally distributed, see Table 6) it was possible

that the parameters were actually significant. With the exception of the AGARCH and NGARCH cases, the in-the-mean effect was always significant (with a range from .17 to .23).

Concerning the GARCH part (in the stock returns case) the seasonality in the conditional variance was corroborated. The average speed of the variance rate over the d_t calendar days of trading day t , δ , was always significant (range equal to .57-.89). There was evidence of persistence (integration) in variance in all the cases ($\Sigma_t \alpha + \Sigma_t \beta = 1$). In an integrated GARCH model (IGARCH) the unconditional variance does not exist. Mandelbrot (1963) and Fama (1965) point out that the distribution of speculative prices are characterized by an infinite unconditional variances. Nelson (1992) suggests that the persistence in variance is a consequence of the high frequency in the observation of financial data. Lamoreaux and Lastrapes (1990) point out that the apparent integration in variance could be caused by structural breaks. However, neither level step nor innovational outliers were detected in the stock returns time series. The most reasonable explanation for the integration in variance in the present case could be the speculative nature of the analyzed variables. In addition, when the same series were observed in lower frequencies (say weekly and monthly) the evidence of a unit root, i.e. IGARCH effects, disappeared.

With the exception of the AR(1,2)MA(7)-GARCH(1,2)-M-S in the stock returns case, all the models passed the misspecification and diagnostic tests (Table 6). There were symptoms of asymmetry only in the AR(1,2)MA(7)GARCH(1,2)-M-S model.

In addition, the excess of kurtosis in the 'raw' residuals (in the sense of not standardized) exceeded the excess kurtosis in the standardized residuals. The opposite occurred for the rest of the asymmetric GARCH's. Therefore, the conditional variances seemed to be correctly specified in these models (Hsieh, 1989). As suggested in Engle and Ng (1994), the unconditional variance of the GARCH's ($\text{Var}(h_t)$) and of the squared residuals ($\text{Var}(\epsilon_t^2)$) were compared. If $\text{Var}(\epsilon_t^2) \geq \text{Var}(h_t)$ (and the unconditional variance existed) the conditional variance would be correctly specified. This was the case in all the estimated models (see Table 6). Note that as in many financial series the standardized residuals appeared to be leptokurtic in all cases. As the distribution assumption (normality) was not met the asymptotic efficiency of the estimation procedure is not guaranteed.

Once the AR(1,2)MA(7)-GARCH(1,2)-M-S was discarded, several tests were used to choose the best model from among the remaining asymmetric GARCH's. First of all, only the EGARCH model had a higher log-likelihood than the symmetric GARCH. However, since not all the models were nested, log-likelihood could not be used in the comparison of alternative models. Instead, the Akaike information criterion (AIC) was computed. The EGARCH model was the model with the lowest AIC. Furthermore, the EGARCH conditional variance had the least skewed and fattest tailed distribution of all the conditional variance series. In addition, the forecasting performance of the different models was computed (Hamilton and Susmel, 1994). If the specification were correct and the parameters were known with certainty, then h_t would be the conditional expectation of ϵ_t^2 . The following loss functions were considered:

$$MSE = \frac{\sum_{t=1}^T (e_t^2 - h_t)^2}{T}$$

$$MAE = \frac{\sum_{t=1}^T |e_t^2 - h_t|}{T}$$

$$LE2 = \frac{\sum_{t=1}^T (\ln(e_t^2) - \ln(h_t))^2}{T}$$

$$LE = \frac{\sum_{t=1}^T |\ln(e_t^2) - \ln(h_t)|}{T}$$

One should be cautious with the MSE. This standard is based on unconditional fourth moments of the stock returns, and these failed to exist because the processes were IGARCH. At any rate the EGARCH model was superior to others. Therefore, the AR(1,2)MA(7)-EGARCH(1,2)-M-S model was finally chosen.

Summing up, the random walk for the interest rate and the AR(1,2)MA(7)-IEGARCH(1,2)-M-S for the stock returns, were finally chosen. The presence of in-the-mean effects in this latter, implied that an increase in the volatility was associated with an increase in the stock returns (close to 20%). This effect, however, was not symmetric. Since $\gamma < 0$, the volatility increased more for bad (and big) news than for good (and small) news. There was also seasonality in the conditional variance of the stock returns. The volatility on Mondays was 30% greater than on Fridays.

Once those models which better represented the stochastic behaviour of both interest rate and stock returns had been chosen, Call and Put options on the index were priced for the period November 30, 1993 - May 30, 1994, using Hull-White, Amin-Ng and Black-Scholes formulas. Near-the-Money (Stock price \approx Strike), In-the-Money (Stock price $>$ Strike) and Out-the-Money (Stock price $<$ Strike) situations were considered. The range for Near-the-Money was taken as five percent

of the stock price. In order to compute Black-Scholes prices, the historical volatility (computed as indicated in Table 1) and the observed interest rate were fixed each expiration day (the third Friday of each month) for the whole life of the option. The Monte Carlo simulations involved in the Hull and White (1987) and the Amin and Ng (1993) formulas were carried out independently in each one of all the different contracts. Specifically, the residuals of the mean adjustment regression were simulated¹⁰. Then, the stochastic volatility and the stochastic interest rate were computed using the parameters estimated in the final models. These variables were introduced in the corresponding formulas in order to obtain Hull-White and Amin-
Ng prices. Final estimates of the prices were computed as the mean values over all the simulations.

Amin and Ng's (1994) preference-free formula was used in this paper. The Amin and Ng's (1994) option valuation formula does not depend on preference parameters if the unpredictable component of the volatility of stock returns is not related to the consumption volatility, as in the present case¹¹. Furthermore, as was explained above, the diffusion limits of the EGARCH are special cases of a continuous time specification (Nelson, 1990). Since the definition of Ito integrals requires that integrands be predictable, one does not need to impose predictability of the mean, variance and covariance processes as long as the stock returns and consumption processes are defined as diffusions.

Since the price series were not independent identically distributed, comparison of the different option prices was carried out by looking at the statistical significance

of the constant in an ARIMA model,

$$(CBS_t - CHW_t) = \alpha + \frac{\Theta(B)}{\Phi(B)} \epsilon_t$$

where, for instance, CBS was Black and Scholes prices and CHW denoted Hull and White's. The results for Call options are summarized in Table 7 and in Figure 3.

All the formulas overpriced the option in the Near-the-Money case and underpriced otherwise. There were no significant differences between actual and the rest of the prices in Out-the-Money case. The Black-Scholes prices were greater than both Hull-White and Amin-Ng prices. The differences were only statistically significant in the Near-the-Money case. It is likely that the lack of significance was due to the small number of observations, i.e. reduced statistical power (at least in the In-the-Money case). There were no significant differences between Hull-White and Amin-Ng prices. Note however, that Hull-White prices were slightly lower than Amin-Ng, perhaps with the exception of Out-the-Money case.

The biases found in this paper were fully coincident with those reported previously¹². Hull and White (1987) point out that when the stochastic volatility was negatively correlated with the stock prices, as in this case, Black-Scholes overprices Near-the-Money options and underprices deep Out-the-Money and deep In-the-Money options. Furthermore, the biases became more exaggerated as the volatility and time to maturity increased (see Figure 3) as Hull and White (1987) suggest. Amin and Ng (1993) point out that since the Black-Scholes call price is a concave function of the variance for At-the-Money options, Jensen's inequality

implies that the stochastic nature of the variance reduces the call price relative to the Black-Scholes price. This fact can be seen in Figure 3. Contrarily, the stochastic nature of the interest rate implies that Black-Scholes underprices the options. Note that in the present case the stochastic volatility bias seemed to dominate the stochastic interest rate bias, since under the combined effect Black-Scholes also overpriced the options. At any rate, certain underpricing effects seemed to be present, since Hull-White prices were slightly lower than Amin-Ng prices (not significantly). All these biases were reinforced by the extreme mean reversion in volatility, that is the existence of IGARCH processes (Amin and Ng, 1993).

The results also showed the time-to-maturity effect (Hull and White, 1987). Longer term options had lower implied volatilities, as calculated by the Black-Scholes formula, than did shorter term options wherever the Black-Scholes overprices the option (see Figure 4). Since the implied volatility is a linear function of the Black-Scholes price and this latter overpriced the option this effect seems to be counterintuitive¹³ (the same paradoxical situation was found in Hull and White, 1987). In fact, it was expected that longer times to maturity would increase uncertainty about both volatility and stock price, hence increasing the option price. Note that in contrast to Hull and White (1987) in the present case this bias was found for shorter times to maturity, say 11-13 days.

5.- CONCLUDING REMARKS

Whereas no ARCH effects seemed to be present in the conditional variance of the MIBOR90 interest rate, an asymmetric GARCH (IEGARCH(1,2)-M-S) has been found to be the model that better represented the behaviour of the stochastic volatility of the IBEX-35 stock returns. In the latter in-the-mean, asymmetric effects and seasonality were found. There was also evidence of persistence in variance. It is likely that this could be caused by the speculative nature of the variable and by the high frequency in the observation of the data.

The ARMA(p',q')-GARCH(p,q) models for both, the interest rate and the stock returns, were used in the simulation of the stochastic processes. In particular final estimates of the Hull-White and Amin-Ng prices were computed as the mean values over all the simulations. Bearing in mind the stochastic nature of both the interest rate and the volatility of the stock returns, the results reported in this paper confirmed all the empirical biases inherent to the Black-Scholes formula described in the literature. All the formulas overpriced the options in the Near-the-Money case, underpricing otherwise. Likewise, in Near-the-Money and In-the-Money situations the Black-Scholes formula overpriced the options. The stochastic volatility bias seemed to dominate the stochastic interest rate bias, since under the combined effect Black-Scholes also overpriced the options. It was not clear that there would exist actual differences between the Amin-Ng and the Hull-White's formulas. However, because the low power of the tests (due to the small number of observations) it was possible that Hull-White prices were slightly lower than Amin-

Ng prices. All these facts are reinforced if one bears in mind that most option trading takes place within the five percent range which was chosen for the Near-the-Money case.

Because of the time-to-maturity effect, it was likely that in most situations (Near-the-Money and time to maturity longer than 11-13 days) implied volatility from the Black-Scholes formula underestimated the actual volatility. As **Hull and White (1987)** point out this could be due to the local concavity of the Black-Scholes price with respect to the volatility. Because of this concavity, increases in volatility do not increase the option price as much as decreases in volatility decrease the price. As the time to maturity increases, the variance of the stochastic volatility increases, exacerbating the effect of the curvature of the option price with respect to volatility. In addition, this effect could be explained by the asymmetric relationship between innovations and volatility found in this paper. In this sense **Day and Lewis (1992)** show that implied volatility from the Black-Scholes formula cannot capture the entire predictable part of future volatility relative to some GARCH and EGARCH models.

At least two problems have not been fully solved yet. First of all, the GARCH approach adopted in this paper to model the stochastic volatility, is only one of several approaches one could have been used (**Jarrow and Wiggins, 1989**). In this sense, a continuous-time-finance approach based on continuous time stochastic differential equations could be used (see **Kaehler and Marnet, 1993**). The discrete time GARCH model converges to a continuous time diffusion model as the sampling interval gets arbitrarily small (**Nelson, 1990**). Furthermore, the EGARCH is closest

in construction to the Stochastic Volatility Models (SVM), and it is possible to write the SVM such that EGARCH is nested within it (Danielsson, 1994). However, the relationship between what GARCH considers volatility and what volatility really is remains unclear¹⁴. Nor is it clear that discrete models fully match with option pricing formulas. Amin and Ng (1994) point out that it is possible, in most cases, to construct a volatility process with the same spectrum as the discrete models and, hence, empirically indistinguishable from them. However, they acknowledge that 'strictly speaking, except in the continuous time limit, GARCH and EGARCH processes are not consistent with Assumption 2' (Amin and Ng, 1994, footnote 6, pp. 887).

Secondly, as was mentioned above, the GARCH approach starts from an empirical perspective, trying to find a specification of the volatility process which adequately represents the stylized facts of the short run dynamics. However, it is likely that not all the stylized facts would have been captured by the models. For instance, certain excess of kurtosis, i.e. non normality, remains in all the models. Several implications could be derived, the most important being that estimates were no longer efficient. In addition, since the simulation procedures were based on standard normal random variables the results of such simulations should be interpreted cautiously. At any rate, because the biases described in this paper fully coincided with those reported previously in the literature, this latter seemed to be only a minor problem.

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TABLE 1. DATA DESCRIPTION

Daily stock prices:

IBEX-35 stock index

October 1, 1987 - January 20, 1994 (1557 obs)⁽¹⁾

The daily stock returns were computed using,

$$r_t = \text{Ln} \left[\frac{P_t}{P_{t-1}} \right]$$

where P_t was the stock price

The annualized historical standard deviation was computed using a moving average of twenty terms,

$$\sigma = \sqrt{\left[\frac{1}{20} \sum_{t=t-9}^{t+10} (r_t - \bar{r})^2 \right]} \cdot \sqrt{252}$$

r_t was the daily stock return (or the daily interest rate)

\bar{r} denoted the average of the stock returns (or the interest rate

252 were the total number of trading days in a year

Daily short-term riskless rate of interest:

MIBOR90 Madrid Interbanking Official Rate (3 months)⁽²⁾

March 19, 1990 - May 31, 1994 (814 obs)

- (1) The period October 29, 1987 - November 30, 1987 was excluded of the analysis (leaving 1518 obs)
(2) Annualized

TABLE 2. UNCONDITIONAL DISTRIBUTIONS OF THE DAILY STOCK INDEX RETURNS AND OF THE DAILY INTEREST RATE

IBEX-35

	Overall	Monday	Tuesday	Wednesday	Thursday	Friday
Mean	8.4936E-04	.9091E-04	1.1685E-04	-9.3720E-04	5.5332E-04	4.42696E-04
Standard Deviation (Annualized)	.190	.238	.190	.159	.190	.159
Skewness	-.74361(.000)					
Kurtosis	11.89989(.000)					
Observations	1557	313	316	320	311	297

MIBOR90

	Overall	Monday	Tuesday	Wednesday	Thursday	Friday
Mean (Annualized)	12.054	12.055	12.059	12.071	12.090	11.994
Standard Deviation (Annualized)	2.141	2.144	2.131	2.158	2.122	2.177
Skewness	-9.9830(.000)					
Kurtosis	185.5971(.000)					
Observations	815	163	164	167	164	157

Within brackets the p-values of marginal significance for the null hypothesis of zero skewness and a kurtosis of 3 (Normality)

TABLE 3. AUTOCORRELATION FUNCTIONS

IBEX-35 Returns

1- 12	.15	.10	.07	.06	.03	.02	.07	.03	.02	-.00	-.05	-.01
ST.E.	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
θ	33.1	48.3	50.1	55.9	56.9	57.7	65.3	66.3	67.0	67.0	71.2	71.5
13- 24	-.01	-.05	-.02	-.01	-.01	.02	-.05	-.03	-.02	.04	.03	-.04
ST.E.	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
θ	71.7	76.0	76.5	76.7	76.8	77.6	82.1	83.2	83.7	85.7	86.8	89.1

Squared residuals from the following ARMA(p',q'):

$$(1 - \phi_1 B - \phi_2 B^2) r_t = (1 - \theta_1 B) \varepsilon_t$$

B is the backshift operator

1- 12	.19	.14	.08	.14	.10	.05	.05	.06	.05	.07	.04	.09
ST.E.	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
θ	58.0	86.8	96.5	126	142	146	150	156	161	169	171	183
13- 24	.07	.07	.05	.04	.06	.05	.02	.02	.01	.08	.02	.03
ST.E.	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
θ	190	197	201	203	209	213	214	215	215	225	226	227

Test for ARCH of Order 1 $TR^2 = 108.9729(.000)$

MIBOR90

Interest rate_t

1- 12	.99	.99	.98	.97	.96	.95	.94	.93	.93	.92	.91	.90
ST.E.	.04	.06	.08	.09	.10	.11	.12	.13	.14	.15	.15	.16
θ	807	1602	2385	3155	3912	4656	5387	6107	6815	7512	8198	8873
13- 24	.89	.89	.88	.87	.87	.86	.85	.85	.84	.84	.83	.83
ST.E.	.17	.17	.18	.18	.19	.19	.20	.20	.21	.21	.21	.22
θ	9538	10193	10839	11475	12101	12719	13329	13932	14527	15115	15697	16273

Interest rate-Interest rate_{t-1} (differenced series)

1- 12	.19	.05	.01	.06	-.00	-.05	-.04	-.06	-.03	.03	-.04	-.04
ST.E.	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04
θ	29.9	31.7	31.8	34.4	34.4	36.6	37.7	40.9	41.5	42.0	43.1	44.6
13- 24	-.06	.01	.01	.02	-.06	-.11	-.07	.00	-.02	-.06	-.05	-.04
ST.E.	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04
θ	48.0	48.1	48.2	48.4	51.4	61.4	65.9	65.9	66.2	69.7	72.1	73.6

Squared residuals from the following MA(q') of the differenced series with intervention analysis^(*):

$$\Delta r_t = (1 - \theta_1 B) \varepsilon_t$$

1- 12	-.00	-.00	-.00	.00	-.00	.00	-.00	.00	-.00	-.00	.00	-.00
ST.E.	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04
θ	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.1
13- 24	.00	.00	.00	.00	.00	.01	.01	-.00	.00	.00	.00	.00
ST.E.	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04
θ	.1	.1	.1	.1	.1	.2	.3	.3	.3	.3	.3	.3

Test for ARCH of Order 1 $TR^2 = 0.0049(.944)$

ST.E. Standard Error, θ Ljung-Box Statistic
ARCH test of order one (p-values of marginal significance)

(*) Intervention analysis of the outliers detected: 10(910213), LS(910318), LS(910322), LS(921214), 10(930223), 10(930617), LS(930730); where LS(i) denotes an level step outlier, IOi an innovational outlier and (i) the date of occurrence.

TABLE 4. RESULTS OF THE TESTS FOR THE PRESENCE OF IN-THE-MEAN AND ASYMMETRIC EFFECTS

Test	IBEX-35	MIBOR90
Sign-Bias	1.4967	0.3445
Neg. Size-Bias	-6.0469	-0.3755
Pos. Size-Bias	1.5694	-0.2096
Joint, F (p-value)	16.0030 (.00000)	0.2745 (.84379)
GARCH-M*	-7.5661	-0.5194

All of the tests were t-Student with the exception of 'Joint' which was a F

* t-student of the significance of the variance (squared standardized residuals) in the ARMA(p',q') adjustment 'mean' regressions

TABLE 5. RESULTS OF THE ESTIMATION OF THE GARCH MODELS FOR THE STOCHASTIC VOLATILITY AND THE STOCHASTIC INTEREST RATES

	IBEX-35				MIBOR30			
	EGARCH(1,2)-H-S	GARCH(1,2)-H-S	PNP-H-S	NGARCH(1,1)-H-S	GJR(1,1)-H-S	AGARCH(1,2)-H-S	VGARCH(1,1)-H-S	No ARCH
N(1)	1518	1518	1518	1518	1518	1518	1518	815
Log-L(2)	6314.8297	6310.4133	6299.9490	6289.2602	6289.1895	6280.6486	6255.6886	1029.1531
AIC	-8.09612	-8.05706	-8.06929	-8.06584	-8.06575	-8.05221	-8.02243	
Mean(3)	0.1042284E-01 ⁰⁰	-0.78327230E-03	-0.1879265E-02 ⁰⁰	-0.1073776E-02	-0.1472735E-02 ⁰⁰	-0.2725374E-03	-0.1614372E-02 ⁰⁰	0.3765665E-01
AR(2,1)	(.4181042E-02)	(.8752289E-03)	(.8398709E-03)	(.8268891E-03)	(.8438451E-03)	(.8227911E-03)	(.1055857E-02)	(.7111639E-01)
AR(2,2)	(.2657431E-01)	(.2852650E-01)	(.2838792E-01)	(.2806125E-01)	(.2815782E-01)	(.3020190E-01)	(.2692418E-01)	(.11636310000)
MA(7)	(.2824556E-01)	(.2700029E-01)	(.2989059E-01)	(.3042639E-01)	(.3043407E-01)	(.3065831E-01)	(.2828653E-01)	
H-effect	(.2644621E-01)	(.2667316E-01)	(.1952129E-01)	(.2341643E-01)	(.2352759E-01)	(.2498789E-01)	(.2236010E-01)	
	(.2370120E-01)	(.2113940E-01)	(.2291883 ⁰⁰)	(.1322661)	(.1784270 ⁰⁰)	(.5525339E-01)	(.1895359 ⁰⁰)	
	(.8315004E-01)	(.8889611E-01)	(.9953447E-01)	(.9655757E-01)	(.9833447E-01)	(.9150928E-01)	(.1152835)	
GARCH(4)								
δ	.7178285 ⁰⁰	.5751573 ⁰⁰	0.6763678 ⁰⁰	0.6993261 ⁰⁰	0.7051745 ⁰⁰	0.6936158 ⁰⁰	0.6433267 ⁰⁰	
H	(.4095184E-01)	(.3631360E-01)	(.4162703E-01)	(.4028551E-01)	(.4036422E-01)	(.5149024E-01)	(.4545836E-01)	
β	(.5071396E-01)	(.9921617E-06)	(.1284404E-05)	(.9875262E-06)	(.8931993E-06)	(.1006773E-03 ⁰⁰)	(.9076174E-05 ⁰⁰)	
α1	.9382517 ⁰⁰	.9440784 ⁰⁰	0.6823169 ⁰⁰	0.6881359 ⁰⁰	0.6898849 ⁰⁰	0.2215900 ⁰⁰	0.7402707 ⁰⁰	
α2	(.5520444E-02)	(.6595237E-01)	(.1687833E-01)	(.1919089E-01)	(.1752330E-01)	(.2633717E-01)	(.1040527E-01)	
γ1	(.3934569E-01)	(.1708088 ⁰⁰)	0.8408865E-03 ⁰⁰	0.1955706 ⁰⁰	0.1408134 ⁰⁰	0.5806984 ⁰⁰	0.1459292E-04 ⁰⁰	
γ2	(.3712922E-01)	(.2363977E-01)	(.3744765E-03)	(.1931524E-01)	(.2397781E-01)	(.2288319E-01)	(.1563836E-05)	
	(.7654539E-01 ⁰⁰)	(.2804660E-01)	(.252339E-02)	(.252339E-02)	(.252339E-02)	(.2651822E-03)	(.1179142)	
	(.2885590E-01)	(.3023621E-01)	(.252339E-01)	(.1318615E-01)	(.3592676E-01)	(.7371876E-03)	(.7967809E-01)	
						1.397410		
						(3.820011)		

(1) Number of observations
 (2) Log-likelihood
 (3) Mean adjustment regression (of the differenced basis for the interest rate case)
 (4) Seasonal GARCH part. For the interpretation of the coefficients see text
 Standard errors within brackets
⁰⁰ p < .1 ⁰⁰⁰ p < .05

TABLE 6. MISSPECIFICATION AND DIAGNOSTIC TEST FOR THE GARCH MODELS IN TABLE 5.

IBEX-35

	EGARCH(1,2)-M-S	GARCH(1,2)-M-S	GJR(1,1)-M-S	NGARCH(1,1)-M-S	AGARCH(1,2)-M-S	VGARCH(1,1)-M-S
ARCH(1)	-6153(-4327)	.0251(-.8741)	0.2096(0.6471)	0.4062(-.5239)	0.3487(-.5548)	1.4349(.2309)
Q(12) Q(24) Q(36)	1.7 4.4 7.0	1.9 4.9 7.0	1.8 5.5 8.2	2.0 5.9 8.6	10.3 18.4 29.7	5.1 8.8 12.5
Standardized resid.						
Skewness	-0.5427702(.000)	-1.7365260(.000)	-0.8116316(.000)	-0.7643892(.000)	-0.5571382(.000)	-0.6402767(.000)
Kurtosis	9.828029(.000)	16.910000(.000)	11.199470(.000)	11.664330(.000)	10.2426530(.000)	10.579250(.000)
Raw residuals						
Skewness	-0.7247026(.000)	-.8342356(.000)	-5.192891(.000)	-2.399614(.000)	-0.7317487(.000)	-4.262066(.000)
Kurtosis	10.72396(.000)	12.981890(.000)	62.570180(.000)	24.047650(.000)	10.613800(.000)	46.407530(.000)
Sign-bias	1.0468	1.9471	1.3954	1.0259	0.6112	1.1538
Neg. Size	0.2182	0.4007	0.3792	0.4489	-0.0962	0.0448
Pos. Size	-0.3708	-0.7044	-0.0604	-0.1097	0.0917	0.4325
Joint, F	0.8399(.4719)	1.2813(.2791)	1.20440(.3068)	0.7858(.5018)	0.2347(.8722)	1.0858(.3539)
Var(ϵ_t^2)	1.501000E-07	1.501000E-07	0.1438585E-06	0.1445766E-06	0.1476424E-06	0.1428076E-06
Var(h_t)	4.3024000E-09	3.0893000E-09	0.4699910E-08	0.4617363E-08	0.4532899E-08	0.2501005E-08
MSE	1.3980E-07	1.4670E-07	1.4550E-07	1.4150E-07	1.4390E-07	1.4130E-07
MAE	1.1061E-04	1.1118E-04	1.1341E-04	1.1243E-04	1.2027E-04	1.1133E-04
LE2	7.3995	8.0213	8.2255	8.0187	8.2419	8.4421
LE	1.9623	2.0865	1.9899	1.9980	2.0831	2.0525

p-values within brackets

TABLE 7. COMPARISON OF THE PRICES OF THE OPTIONS USING DIFFERENT FORMULAS (EUROPEAN CALL OPTIONS ON THE IBEX-35)

Near-the-Money (Stock price \approx Strike)¹

Variable	Mean	Std Dev	N
OPTION PRICE	71.414	50.368	674
BSCALL	175.210	83.979	674
HWCALL *	164.882	89.602	674
ANCALL *	165.631	89.996	674

In-the-Money (Stock price > Strike)

Variable	Mean	Std Dev	N
OPTION PRICE	241.33	86.774	43
BSCALL	185.01	71.610	43
HWCALL	172.81	80.990	43
ANCALL	174.70	83.250	43

Out-the-Money (Stock price < Strike)

Variable	Mean	Std Dev	N
OPTION PRICE	29.183	31.179	189
BSCALL	24.583	7.452	189
HWCALL	24.185	7.817	189
ANCALL	24.242	7.804	189

BS Black-Scholes (dividend yields adjusted)
 HW Hull-White (dividend yields adjusted)
 AN Amin-Ng (dividend yields adjusted)

* Statistically different from Black-Scholes (p<.05)

¹ The range for near-the-money was taken at the $\pm 5\%$ of the Stock price

FIGURE 1. IBEX-35, MIBOR90 AND HISTORICAL VOLATILITY OF THE STOCK RETURNS

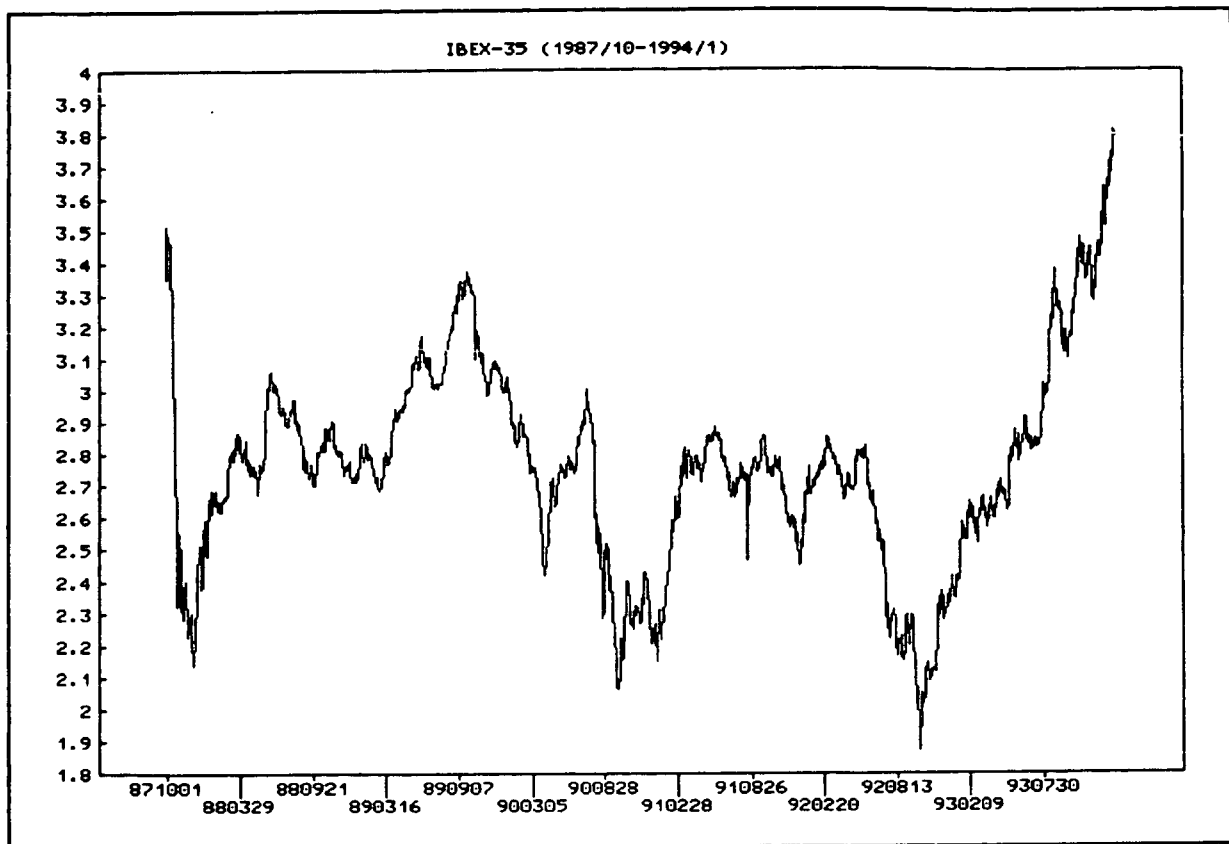


FIGURE 1 (cont). IBEX-35, MIBOR90 AND HISTORICAL VOLATILITY OF THE STOCK RETURNS

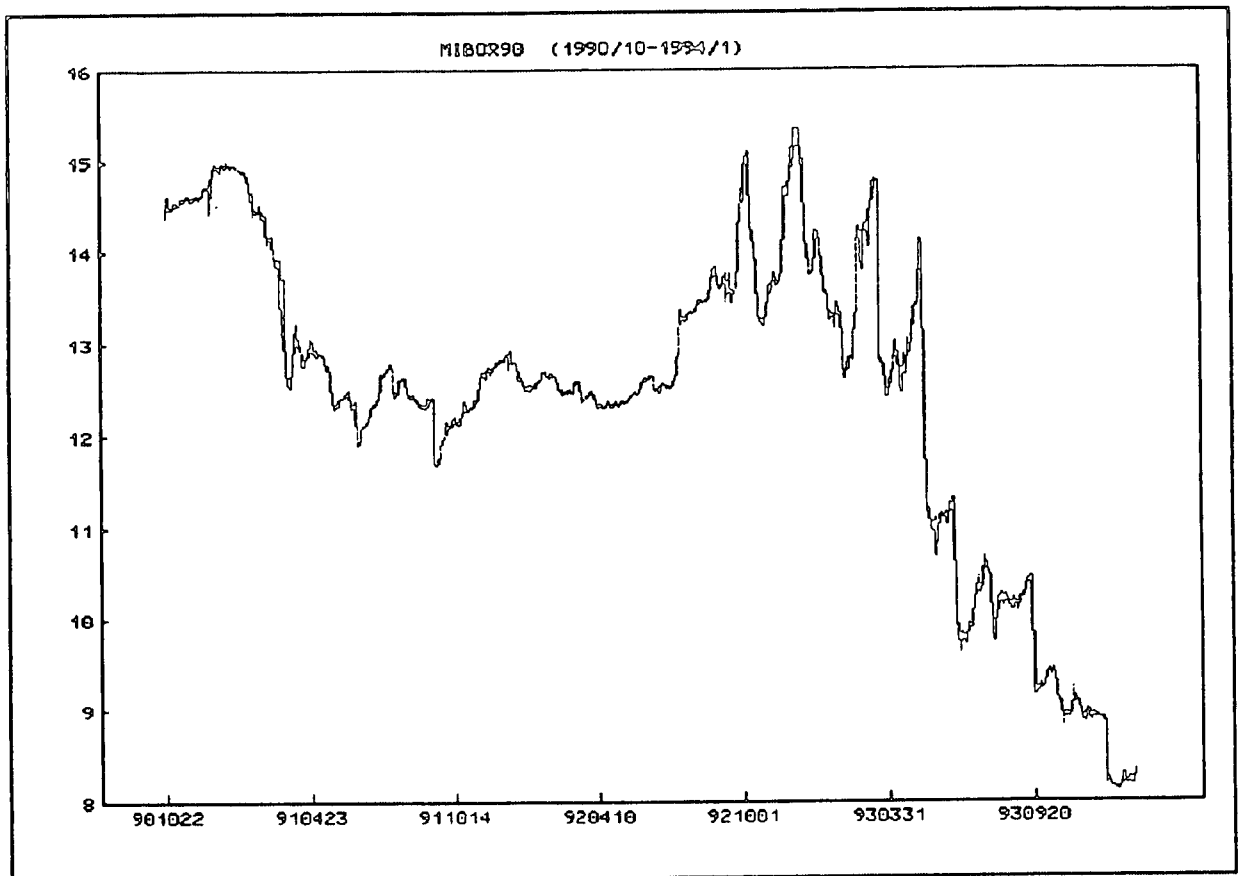


FIGURE 1 (cont). IBEX-35, MIBOR90 AND HISTORICAL VOLATILITY OF THE STOCK RETURNS

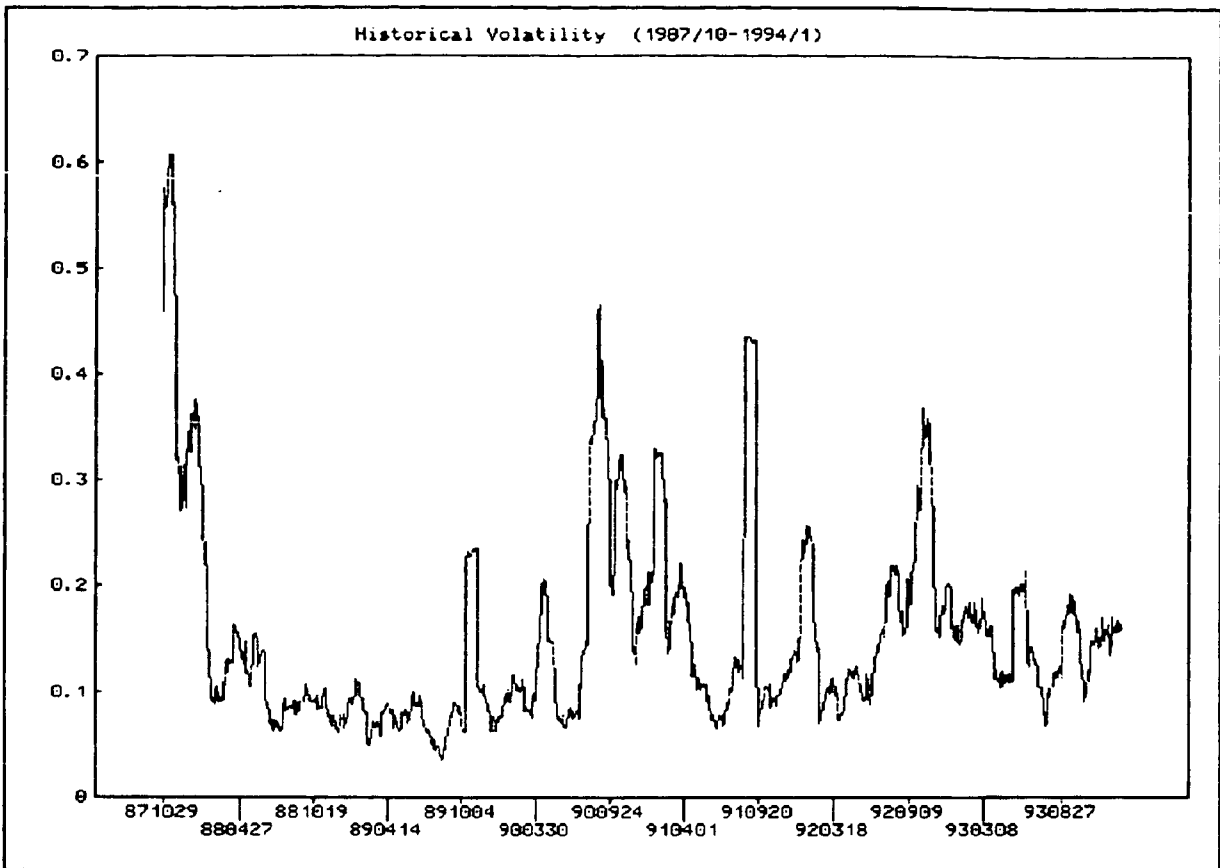


FIGURE 2. SQUARED STOCK RETURNS AND DIFFERENCED INTEREST RATES

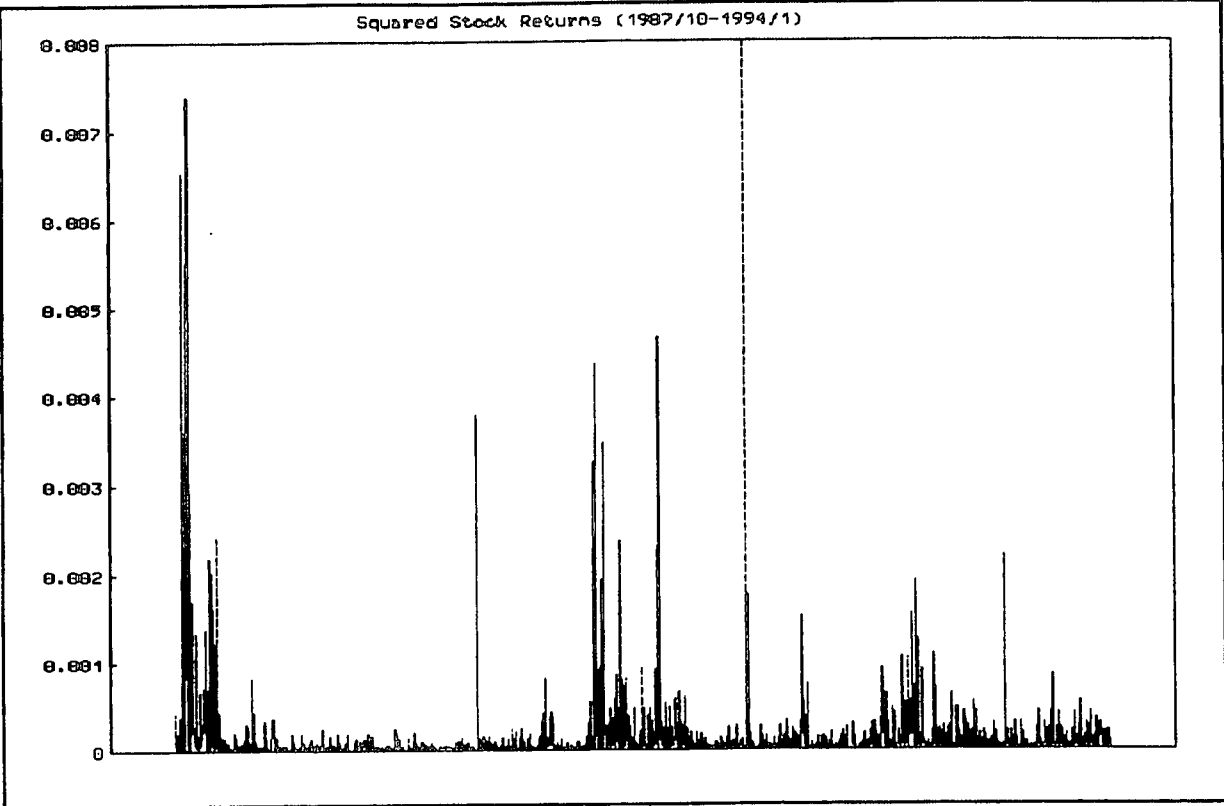


FIGURE 2 (cont). SQUARED STOCK RETURNS AND DIFFERENCED INTEREST RATES

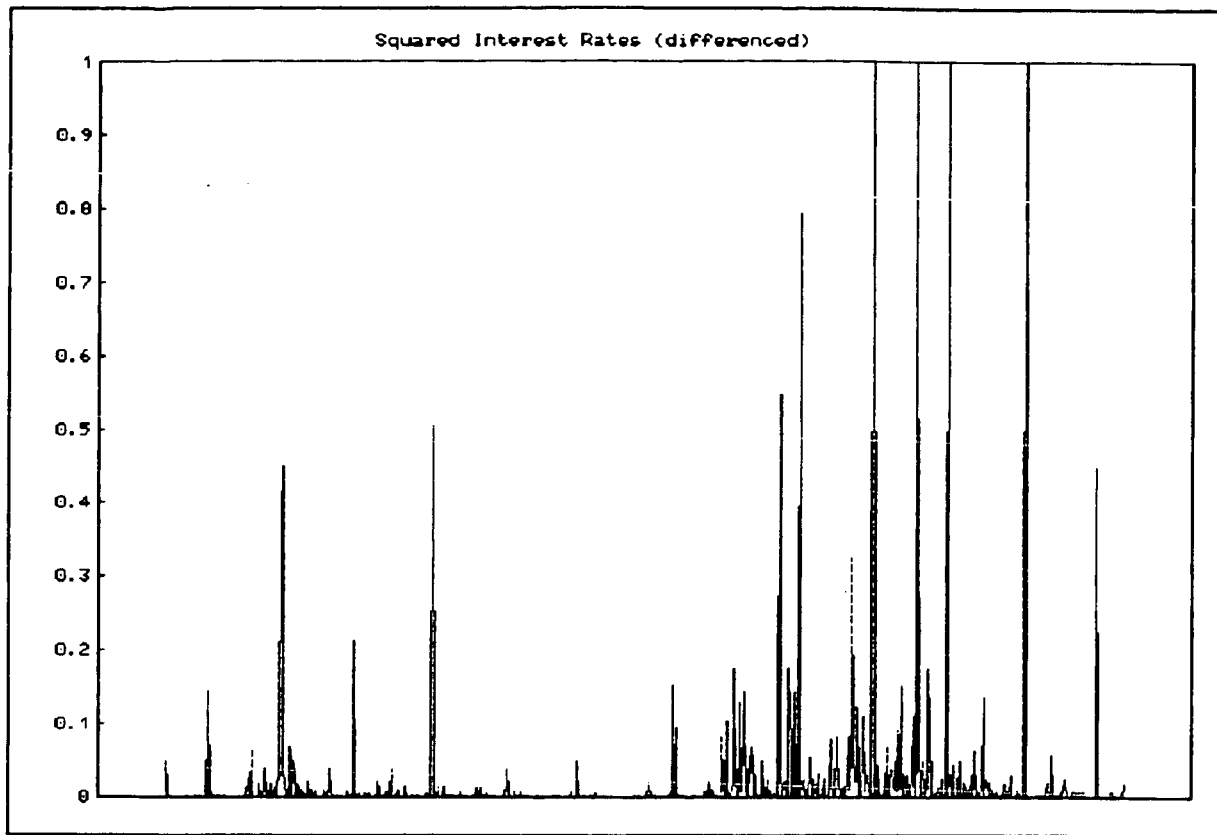


FIGURE 3. BLACK-SCHOLES AND AMIN-NG BIASES

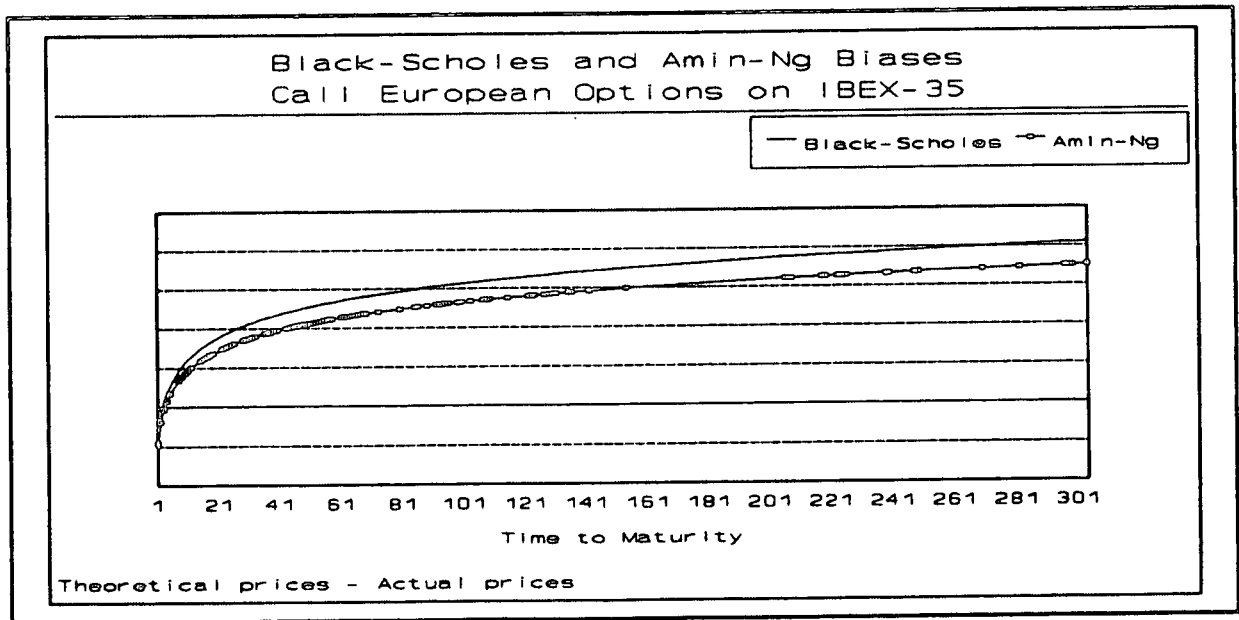
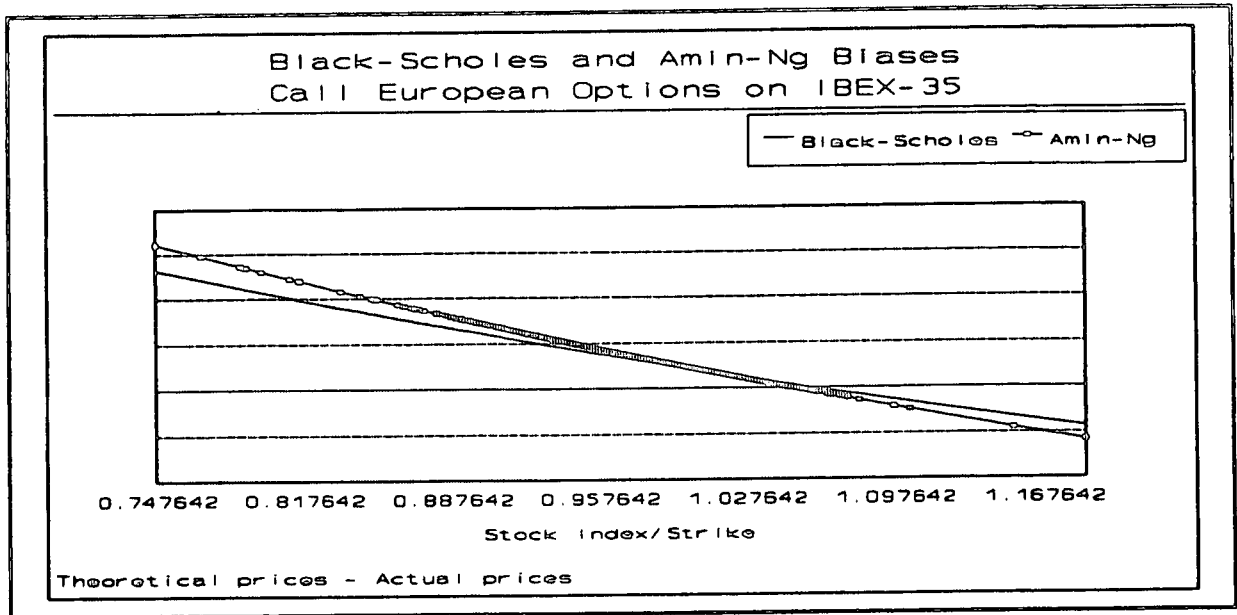
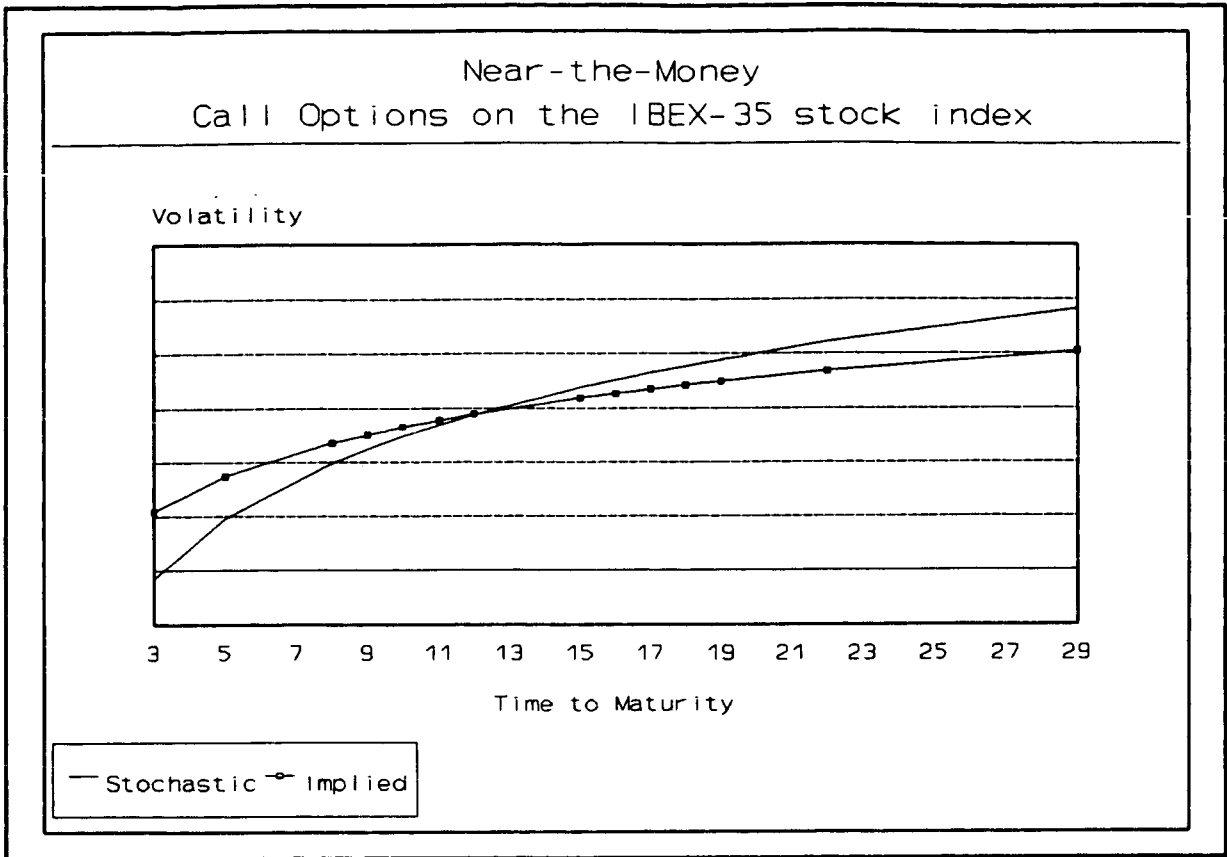


FIGURE 4. STOCHASTIC AND IMPLIED VOLATILITY vs TIME TO MATURITY. NEAR-THE-MONEY



1. See Bollerslev, Chou and Kroner (1992) for a comprehensive survey of these type of models.
2. The probability distribution in continuous-time corresponding to the normal distribution in a static context.
3. Corresponding to the expected instantaneous relative price change of the options contract.
4. When the volatility is correlated with the stock price they also simulate the stock price.
5. The Black-Scholes formula, Hull-White stochastic variance valuation formula, Merton's (1973), Turnbull and Milne's (1991) and Amin and Jarrow (1992) stochastic interest rate option valuation formulas.
6. That is $(e_t \circ \hat{h}^{-1/2})^2$, where e_t were the residuals of the 'mean' adjustment regressions, the AR(p',q') models, and $\hat{h}^{-1/2}$, was obtained from the 'best' GARCH(p,q) model. Both 'mean' regression and the GARCH(p,q) were estimated jointly.
7. All the results can be obtained on request.
8. This problem arises with respect to different closing times of the market of the options and the underlying asset.
9. The intermediate estimated models as well as the results of the likelihood ratio tests can be obtained on request.
10. A thousand replications for random variables drawn from the standard normal distribution with mean 0 and standard deviation of 1 were computed.
11. Results not shown here can be obtained on request.
12. Theoretical biases are reported in Wiggins (1987), Hull and White (1987) and Amin and Ng (1993). Empirical biases can be found in Whaley (1986), Scott (1987), Engle, Kane and Noh (1993) and Noh, Engle and Kane (1993), among many others.
13. Thanks to Antonio Cabrales for pointing this out.
14. For an extensive discussion on this point see Harvey, Ruiz and Shepard (1992) and specially the work of Andersen (1992).

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