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Intertemporal Cournot and Walras Equilibrium: an Illustration*

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Abstract

In an intertemporal general equilibrium model, we compare Cournot and Walras equilibrium outcomes. We show that if the number of agents increases without limit, the intertemporal Cournot equilibrium *does not* converge to the intertemporal Walras equilibrium. The agents realize their market power under Cournot equilibrium and exploit it by consuming more and supplying (and investing) less. This generates and inefficiency in every market they participate which does not vanish as the number of agents tends to infinity.

"S'il y avait 3, 4,...n producteurs en concurrence [...] la valeur de p, qui en résulte, diminuerait indéfiniment par l'accroissement indéfini du nombre n", A. A. Cournot (1838), page 63.

1 Introduction

It is well-known that in the standard Cournot framework, strategic agents become more competitive as their number becomes larger and, in the limit, the Cournot equilibrium converges to the competitive outcome. Much work has been done to generalize this result. It is generally expected that every Cournot equilibrium is approximately competitive in large economies [see, Mas-Colell (1980)]. This paper presents a counter-example. In a dynamic setting, we show that a Cournot equilibrium does not converge to the Walras equilibrium as the number of infinitely-lived agents increases without limit.

The example we present is a dynamic version of the Gabszewicz-Michel (1992) general equilibrium oligopoly model[1]. There are two types of agents and two goods. Each agent produces one good but consumes both. The type is identified with the good produced by the agent. There is a spot market in every time period at which the agents trade. No trade occurs across time-periods. Production has a time lag: inputs in the previous period generate outputs at the current period. A part of the current output is used for current consumption and the rest is used as input to produce future output. Consumption generates utility and each infinitely-lived agent maximizes its intertemporal welfare which is the discounted sum of utilities. A Walrasian agent maximizes its intertemporal welfare for a given set of prices. A Cournot agent maximizes welfare by taking account of the effect its quantity decisions have on the market-clearing prices. We restrict our attention to the space of Markovian strategies and show that, in each period, a Cournot agent overconsumes the good it is endowed with, in order to have a better terms of trade both in the current and future periods. Moreover, the difference between the Walras and Cournot equilibrium does not vanish as the number of strategic agents goes to infinity. A strategic agent generates an unbounded distortion by participating in the market infinite number of times no matter how large the economy is (or, how small the agent is compared to the economy).

Some instances of similar nonconvergence property are Green (1980), Guesnerie-Hart (1985) and Chari-Kehoe (1990). Our model differs from Green (1980) because we have a general equilibrium model in which actions in one period affect the feasible set in the next period and we do not consider trigger strategies. In contrast to Guesnerie-Hart (1985) and Chari-Kehoe (1990), we have a dynamic model and the strategic agent is involved in both consumption and production.

The paper is arranged in the following way. The notation used is listed in section 1.1. We present our model in section 2 and define intertemporal Cournot and Walras equilibrium. In section 3, we compute and compare these equilibria.

1.1 Notation

The two goods are denoted by subscripts 1 and 2, and time by the subscript t where $t = 0, 1, \cdots$. There are two types of agents (1, 2). Each agent is identified with the good it produces. The typical agent of type 1 (or, 2) is denoted by i (or, j). There are n agents of each type.

z^i_{1t} and z^j_{2t} are the stocks of good held by agents i and j, respectively, in period t. Similarly, y^i_{1t} and y^j_{2t} are the amounts utilized for consumption by agents i and j, respectively, in period t.

[1] In this specific case, the model is equivalent to a strategic market game in the Shapley-Shubik tradition. See, for instance, Dubey-Shubik (1978).

Consequently, $(z_{1t}^i - y_{1t}^i)$ and $(z_{2t}^j - y_{2t}^j)$ are the amounts used for investment. (c_{1t}^i, c_{2t}^i) is the bundle of goods consumed by the agent i in period t. Similarly, (c_{1t}^j, c_{2t}^j) is the j-th agent's consumption bundle. The price of goods 1 and 2, in period t, are (p_{1t}, p_{2t}). Let $p_t = \frac{p_{2t}}{p_{1t}}$ be the relative price.

Let $\sigma_{1t}^{-i} = [\sum_{i=1}^n (y_{1t}^i - c_{1t}^i)] - (y_{1t}^i - c_{1t}^i)$ and $\sigma_{2t} = \sum_{j=1}^n (y_{2t}^j - c_{2t}^j)$. Finally, $\{a_t\}$ stands for the time sequence of the variable a.

2 The Model

We consider an economy with two goods (1 and 2) and $2n$ agents. There are two types of agents (1 and 2), n of each type. Each agent produces only one good which identifies the agent. The superscript i stands for an agent of type 1 and the superscript j stands for an agent of type 2 $(i, j = 1, \cdots, n)$. z_{1t}^i (z_{2t}^j) denotes the stocks of good 1 (2) hold by an agent of type 1 (2) in period t. In period $t = 0$, this stock is the initial endowment, in the subsequent periods $(t = 1, 2, \cdots, \infty)$, it is the output produced from previous period's input. A part of the stock, y_{1t}^i and y_{2t}^j, is utilized for consumption in period t by agents of type 1 and 2, respectively, and the rest, $(z_{1t}^i - y_{1t}^i)$, and $(z_{2t}^j - y_{2t}^j)$, are used as inputs, in period t. These inputs generate output in the period $(t+1)$, from the technology,

$$z_{1(t+1)}^i = (z_{1t}^i - y_{1t}^i)^{\alpha_1}, \tag{1}$$

$$z_{2(t+1)}^j = (z_{2t}^j - y_{2t}^j)^{\alpha_2}. \tag{2}$$

The i-th agent consumes a fraction of y_{1t}^i and trades the rest for the consumption of good 2. Let (c_{1t}^i, c_{2t}^i) denote the agent i's consumption in period t. This yields utility,

$$lnc_{1t}^i + lnc_{2t}^i. \tag{3}$$

The intertemporal utility is given by the discounted sum of one-period utilities,

$$U(\{c_{1t}^i, c_{2t}^i\}) = \sum_{t=0}^{\infty} (\delta_1)^t [lnc_{1t}^i + lnc_{2t}^i]. \tag{4}$$

Here, δ_1 is the common discount factor for agents of type 1. Similarly, the j-th agent's consumption in period t is (c_{1t}^j, c_{2t}^j) and its intertemporal utility is $\sum_{t=0}^{\infty} (\delta_2)^t [lnc_{1t}^j + lnc_{2t}^j]$. The agents balance their budget every time period and there is no borrowing or lending: neither across agents nor across time. Let p_{1t} and p_{2t} be the prices of goods 1 and 2, respectively, in period t. The i-th agent's budget balancing requires

$$p_{1t}c_{1t}^i + p_{2t}c_{2t}^i = p_{1t}y_{1t}^i. \tag{5}$$

Replacing the righthandside of equation (5) by $p_{2t}y_{2t}^j$ and the consumption bundle (c_{1t}^i, c_{2t}^i) by (c_{1t}^j, c_{2t}^j), we get the j-th agent's budget constraint.

The agent i maximizes the discounted sum of utilities subject to the conditions of budget balancing and technical feasibility. The optimization problem is[2]

[2]The problem of the j-th agent is similar with appropriate changes in the indices.

$$\max_{\{c^i_{1t},c^i_{2t},y^i_{1t}\}} \sum_{t=0}^{\infty}(\delta_1)^t(lnc^i_{1t} + lnc^i_{2t}),$$
subject to

$$p_{1t}c^i_{1t} + p_{2t}c^i_{2t} = p_{1t}y^i_{1t}, \qquad \text{(P)}$$
$$0 \leq y^i_{1t} \leq z^i_{1t} = (z^i_{1(t-1)} - y^i_{1(t-1)})^{\alpha_1}$$
and $c^i_{1t} \geq 0, c^i_{2t} \geq 0$, given $z^i_{10} > 0$.

Note that, market clearance in period t requires that aggregate supply of good 1 equals its aggregate demand. Type 1 agents supply good 1 and type 2 agents demand good 1. Each agent of type 1 supplies the amount $(y^i_{1t} - c^i_{1t})$ and each agent of type 2 demands $\frac{p_{2t}(y^j_{2t}-c^j_{2t})}{p_{1t}}$. Therefore, the market-clearing price ratio is

$$p_t \equiv \frac{p_{2t}}{p_{1t}} = \frac{\sum_{i=1}^n (y^i_{1t} - c^i_{1t})}{\sum_{j=1}^n (y^j_{2t} - c^j_{2t})}. \qquad (6)$$

Next, we define an intertemporal Walras equilibrium and an intertemporal Cournot equilibrium. The crucial difference between the Walras and the Cournot framework is in the agents' perception of the prices. A Walrasian agent behaves as a price-taker, while in the Cournot-Nash framework, an agent takes account of its influence on the market-clearing prices.

Definition 1: *An intertemporal Walras equilibrium* is a $(3 \times 2n)$ -tuple of sequences of consumption decisions for $2n$-agents and a sequence of relative prices such that:

(i) $\{y^i_{1t}, c^i_{1t}, c^i_{2t}\}$ maximizes $U^i(.)$, subject to its technology, (1), its budget constraint, (5), at the given price ratio, p_t, for all $t = 0, 1, \cdots$ and for $i = 1, \cdots, n$;

(ii) Similarly, $\{y^j_{2t}, c^j_{1t}, c^j_{2t}\}$ maximizes $U^j(.)$, subject to its technology and budget constraint at the given price ratio, p_t, for $j = 1, \cdots, n$;

(iii) the price ratio, p_t, clears the market at period, t.

Definition 2: *An intertemporal Cournot equilibrium* is a $(3 \times 2n)$ -tuple of sequences of consumption decisions for $2n$-agents such that:

(i) $\{y^i_{1t}, c^i_{1t}, c^i_{2t}\}$ maximizes $U^i(.)$, subject to its technology, (1), its budget constraint, (5), and the market-clearing condition, (6), given the consumption decisions of $(2n - 1)$ agents other than itself, for $i = 1, \cdots, n$;

(ii) Similarly, $\{y^j_{2t}, c^j_{1t}, c^j_{2t}\}$ maximizes $U^j(.)$, subject to its technology, budget constraint and the market-clearing condition, given the consumption decisions of other $(2n-1)$ agents, for $j = 1, \cdots, n$.

For simplicity, we restrict our attention to the space of stationary or Markovian strategies. That is, consumption decisions depend only on the current state which are the output stocks.

3 Nonconvergence

In this section, we show that an intertemporal Cournot equilibrium in Markovian strategies does not converge to the unique intertemporal Walras equilibrium as the number of agents tends to infinity. The techniques of stationary dynamic programming are employed for computation of equilibrium.

Note that, in equilibrium, the choice of consumption allocation in any period is not independent of the aggregate amount that each agent devotes for consumption in that period. Suppose the sequence of consumption decisions, $\{y^i_{1t}, c^i_{1t}, c^i_{2t}\}$, maximizes the intertemporal utility, $U^i(.)$. Then it must be true that, in every period t, the choice of consumption allocation, (c^i_{1t}, c^i_{2t}), maximize that period's utility given y^i_{1t}[3]. In equilibrium, it is necessary that the consumption decisions, $\{y^i_{1t}, c^i_{1t}, c^i_{2t}\}$, must be such that, in every period t, the following maximization is solved:

$$\max_{c^i_{1t}, c^i_{2t}} lnc^i_{1t} + lnc^i_{2t},$$
subject to
$$p_{1t}c^i_{1t} + p_{2t}c^i_{2t} = p_{1t}y^i_{1t},$$
and $c^i_{1t} > 0, c^i_{2t} > 0.$

Or, in other words,

$$\max_{c^i_{1t}} lnc^i_{1t} + ln[\frac{p_{1t}}{p_{2t}}(y^i_{1t} - c^i_{2t})]. \tag{7}$$

The difference between the Walras and Cournot agents is in their perception of market-clearing prices. When we look for an intertemporal Walrasian equilibrium, the problem (7) is solved for a given set of prices. In order to derive an intertemporal Cournot equilibrium, this t-period utility maximization is solved by incorporating the market-clearing price from equation (6). In subsections 3.1 and 3.2, we study the implications of these equilibrium outcomes on resource allocation.

3.1 The Intertemporal Walras Equilibrium

When the agents behave as price-takers, the solution to the t-th period utility maximization problem (7) is

$$c^i_{1t} = \frac{y^i_{1t}}{2}. \tag{8}$$

From the budget constraint (5), we get

$$c^i_{2t} = \frac{y^i_{1t}}{2}\frac{p_{1t}}{p_{2t}}. \tag{9}$$

Thus, given the choice of y^i_{1t}, the maximum utility that the agent i can have in period t, is its indirect utility,

$$ln(\frac{y^i_{1t}}{2}) + ln(\frac{y^i_{1t}p_{1t}}{2p_{2t}}) = 2ln(\frac{y^i_{1t}}{2}) + ln(\frac{p_{1t}}{p_{2t}}). \tag{10}$$

Using equation (10), we may rewrite the intertemporal optimization problem, (P), as follows:

[3]If (c^i_{1t}, c^i_{2t}) does not maximize the t-th period utility then given the aggregate consumption, y^i_{1t}, there is another feasibile consumption bundle that does. This alternative bundle will also correspond to a higher intertemporal utility. Therefore, intertemporal utility cannot be maximized at the original consumption choice.

$$
\begin{aligned}
&\max_{\{y^i_{1t}\}} \sum_{t=0}^{\infty} (\delta_1)^t [2ln(\frac{y^i_{1t}}{2}) + ln(\frac{p_{1t}}{p_{2t}})], \\
&\text{subject to} \\
&0 \leq y^i_{1t} \leq z^i_{1t} = (z^i_{1(t-1)} - y^i_{1(t-1)})^{\alpha_1} \text{ and given } z^i_{10} > 0.
\end{aligned} \tag{11}
$$

The objective of the dynamic programming problem (11) is to maximize the discounted sum of indirect utilities and the choice variable is the sequence of aggregate consumption, $\{y^i_{1t}\}$. This problem is not stationary if the sequence of relative price is not a constant sequence. However, this does not pose a problem in terms of finding a solution. The agents are price-takers and, therefore, we may solve the optimization ignoring the discounted sum of the logarithm of relative prices, $\sum_{t=0}^{\infty}(\delta_1)^t ln(\frac{p_{1t}}{p_{2t}})$, which is a constant number[4] and solve

$$
\begin{aligned}
&\max_{\{y^i_{1t}\}} \sum_{t=0}^{\infty} (\delta_1)^t [2ln(\frac{y^i_{1t}}{2})] \\
&\text{subject to} \\
&0 \leq y^i_{1t} \leq z^i_{1t} = (z^i_{1(t-1)} - y^i_{1(t-1)})^{\alpha_1} \text{ and given } z^i_{10} > 0.
\end{aligned} \tag{12}
$$

This is a one-sector optimal growth model with the unique Markovian solution,

$$
y^i_{1t} = (1 - \delta_1\alpha_1) z^i_{1t} \qquad \text{for all } t. \tag{13}
$$

From equations (8) and (9), the optimal consumption choices for the agent i are

$$
c^i_{1t} = \frac{(1 - \delta_1\alpha_1) z^i_{1t}}{2}, \qquad c^i_{2t} = \frac{(1 - \delta_1\alpha_1) z^i_{1t} p_{1t}}{2p_{2t}}. \tag{14}
$$

Similarly, for the optimal choices of agent j, replace the superscript i by the superscript j, production coefficient α_1 by the coefficient α_2, discount factor δ_1 by the discount factor δ_2 and switch the subindices 1 for 2 and vice-versa in equations (13) and (14). Since the agents of each type are identical, we may omit the superscripts i and j. From equations (6), (13) and (14) we derive the equilibrium relative price,

$$
p_t = \frac{(1 - \delta_1\alpha_1) z_{1t}}{(1 - \delta_2\alpha_2) z_{2t}}. \tag{15}
$$

Equations (13), (14) and (15) summarize the intertemporal Walras equilibrium.

3.2 The Intertemporal Cournot Equilibrium

Should the agents behave strategically they do not act as price takers but perceive the influence of their individual supply on the equilibrium terms of trade. In this case, each agent will try to improve its terms of trade by reducing the supply. See, for example, Codognato-Gabszewicz (1991) and Gabszewicz-Michel (1992).

By substituting equation (6) in the maximization problem (7), the t-th period utility maximization reduces to,

[4] We assume this sum is bounded so that any sequence of aggregate consumption is not trivially optimal.

$$\max_{c^i_{1t}\in[0,y^i_{1t}]} lnc^i_{1t} - ln[\sum_{i=1}^{n}(y^i_{1t}-c^i_{1t})] + ln[\sum_{j=1}^{n}(y^j_{2t}-c^j_{2t})] + ln(y^i_{1t}-c^i_{1t}).$$

Or,

$$\max_{c^i_{1t}\in[0,y^i_{1t}]} lnc^i_{1t} - ln(\sigma^{-i}_{1t} + y^i_{1t} - c^i_{1t}) + ln\sigma_{2t} + ln(y^i_{1t}-c^i_{1t}), \tag{16}$$

where, $\sigma^{-i}_{1t} = [\sum_{i=1}^{n}(y^i_{1t}-c^i_{1t})] - (y^i_{1t}-c^i_{1t})$, and $\sigma_{2t} = \sum_{j=1}^{n}(y^j_{2t}-c^j_{2t})$. Solving the maximization problem (16), it can be shown that the i-th agent consumes

$$c^i_{1t} = y^i_{1t} + \sigma^{-i}_{1t} - \sqrt{\sigma^{-i}_{1t}(\sigma^{-i}_{1t}+y^i_{1t})}. \tag{17}$$

By substituting equation (17) into the objective function of the problem (16) and simplifying, we express agent i's indirect utility,

$$w^i(y^i_{1t},\sigma^{-i}_{1t},\sigma_{2t}) = ln\sigma_{2t} + 2ln[\sqrt{\sigma^{-i}_{1t}+y^i_{1t}} - \sqrt{\sigma^{-i}_{1t}}]. \tag{18}$$

This allows us to reduce the optimization problem (P) to the following:

$$\begin{aligned} &\max_{\{y^i_{1t}\}} \sum_{t=0}^{\infty}(\delta_1)^t w^i(y^i_{1t},\sigma^{-i}_{1t},\sigma_{2t}) \\ &\text{subject to} \\ &0 \le y^i_{1t} \le z^i_{1t} = (z^i_{1(t-1)} - y^i_{1(t-1)})^{\alpha_1} \text{ and given } z^i_{10} > 0. \end{aligned} \tag{19}$$

In order to compute an intertemporal Cournot equilibrium, we adapt the technique of Fischer-Mirman (1992). The value function, or, the maximum intertemporal utility an agent derives depends on the current stocks of output in the Markovian framework. We assume that the value function for the agent i is linear-logarithmic[5],

$$W^i(z^1_1,\cdots,z^n_1;z^1_2,\cdots,z^n_2) = \sum_{i=1}^{n}(A^i_1 lnz^i_1) + \sum_{j=1}^{n}(B^j_2 lnz^j_2) + D^i. \tag{20}$$

where A^i_1, B^j_2, and D^i are constants for all $i,j = 1,\cdots,n$ and we omit the time subscripts because all the variables correspond to the same time period. By applying Bellman's principle, we have the functional equation,

$$\begin{aligned} W^i(z^1_1,\cdots,z^n_1;z^1_2,\cdots,z^n_2) = &\max_{0\le y^i_1\le z^i_1}\{w^i(y^i_1,\sigma^{-i}_{1t},\sigma_{2t}) + \\ &\delta_1 W^i[(z^1_1-y^1_1)^{\alpha_1},\cdots,(z^n_1-y^n_1)^{\alpha_1};(z^1_2-y^1_2)^{\alpha_2},\cdots,(z^n_2-y^n_2)^{\alpha_2}]\}. \end{aligned} \tag{21}$$

Using equation (20), the maximand of the functional equation (21) can be written as

$$\max_{0\le y^i_1\le z^i_1}\{w^i(y^i_1,\sigma^{-i}_1,\sigma_2) + \delta_1\alpha_1\sum_{i=1}^{n}[A^i_1 ln(z^i_1-y^i_1)] + \delta_1\alpha_2\sum_{j=1}^{n}[B^j_2 ln(z^j_2-y^j_2)] + \delta_1 D^i\}, \tag{22}$$

[5]By extrapolating from the value functions of finite period problems.

which has the following first-order condition:

$$\frac{1}{\sigma_1^{-i} + y_1^i - \sqrt{\sigma_1^{-i}(\sigma_1^{-i} + y_1^i)}} = \frac{\delta_1 \alpha_1 A_1^i}{z_1^i - y_1^i}. \tag{23}$$

Since all agents of type 1 are identical, we have, $\sigma_{1t}^{-i} = (n-1)(y_{1t}^i - c_{1t}^i)$ which along with equation (17) implies

$$\sigma_1^{-i} = \frac{(n-1)^2 y_1^i}{2n-1}. \tag{24}$$

The condition (23) and the equation (24) suggest that we have an intertemporal Cournot equilibrium in linear strategies. That is, a strategy of using a constant proportion of the current stock every period for consumption is an equilibrium. In fact, it is easy to check that the strategies of the form $y_1^i = \gamma_1^i z_1^i$ and $y_2^j = \gamma_2^j z_2^j$ where $0 \leq \gamma_1^i, \gamma_2^j \leq 1$ and the value function (20) satisfy the Bellman's equation (21), if

$$A_1^i = \frac{1}{1 - \delta_1 \alpha_1}. \tag{25}$$

Equations (23), (24) and (25) imply that an optimal aggregate consumption for the i-th agent is

$$y_{1t}^i = \frac{(2n-1)(1-\delta_1\alpha_1) z_{1t}^i}{(2n-1)(1-\delta_1\alpha_1) + n\delta_1\alpha_1}. \tag{26}$$

Similarly, an optimal aggregate consumption for the j-th agent is

$$y_{2t}^j = \frac{(2n-1)(1-\delta_2\alpha_2) z_{2t}^j}{(2n-1)(1-\delta_2\alpha_2) + n\delta_2\alpha_2}. \tag{27}$$

The following consumption allocations constitute an intertemporal Cournot equilibrium with the aggregate choices in equations (26) and (27):

$$c_{1t}^i = \frac{n y_{1t}^i}{(2n-1)}, \qquad c_{2t}^i = \frac{(n-1) p_{1t} y_{1t}^i}{(2n-1) p_{2t}}; \tag{28}$$

$$c_{1t}^j = \frac{(n-1) p_{2t} y_{2t}^j}{(2n-1) p_{1t}}, \qquad c_{2t}^j = \frac{n y_{2t}^j}{(2n-1)}. \tag{29}$$

We omit superscripts i and j since all agents of each type are identical. The market-clearing price ratio is

$$p_t = \frac{(1-\delta_1\alpha_1)\left[(2n-1)(1-\delta_2\alpha_2) + n\delta_2\alpha_2\right] z_{1t}}{(1-\delta_2\alpha_2)\left[(2n-1)(1-\delta_1\alpha_1) + n\delta_1\alpha_1\right] z_{2t}}. \tag{30}$$

The equations (26)-(30) summarize an intertemporal Cournot equilibrium in Markovian strategies.

3.3 Comparisons

First, in proposition 1, we note that the agents consume more and invests less per unit of stock every period, under the Cournot equilibrium compared to the Walrasian equilibrium.

Proposition 1: *The proportion of stock used for aggregate consumption, in each period, is higher in the intertemporal Cournot equilibrium than the corresponding ratio in the intertemporal Walras equilibrium.*

Proof: Compare equations (13) and (26) and note that $n > 1$.□

The intuition for the result is the following: when agents behave strategically they have interest in restricting their supply in the market in the current and all the future periods and they do so by consuming relatively more and investing relatively less compared to the Walrasian agents.

We conclude by observing, in proposition 2, that even when the number of agents becomes very large, the differences in the aggregate consumption and investment decisions in the intertemporal Cournot equilibrium and the intertemporal Walras equilibrium does not become negligible.

Proposition 2: *If the time horizon is infinite and the number of agents, n, increases without limit, the consumption, investment and the market-clearing prices in the intertemporal Cournot equilibrium do not converge to the corresponding quantities in the intertemporal Walras equilibrium.*

Proof: In equation (26), take limit as n tends to infinity, and compare the aggregate consumption per unit of stock in the Cournot equilibrium to the same ratio in the Walrasian equilibrium, given by the equation (13).□

An agent in the Cournot framework creates inefficiency in the model by manipulating prices every period. From equations (26) and (28), agent i's consumption of own good per unit of stock is

$$\frac{(1-\delta_1\alpha_1)}{(2-\frac{1}{n})((1-\delta_1\alpha_1)+\delta_1\alpha_1}.$$

This is larger than the corresponding ratio in the Walrasian equilibrium [equation (14)] even when the number of strategic agents, n, becomes very large. In fact, in the limit, the difference is

$$\frac{\delta_1\alpha_1}{2},$$

which can be interpreted as the magnitude of strategic market manipulation by an infinitesimal Cournot agent, every time he participates in the market process. This is a constant fraction independent of t, which summed over an infinite time horizon is unboundedly large.

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