Does money matter in shaping domestic business cycles? An international investigation.

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Abstract

We study the contribution of money to business cycle fluctuations in the US, the UK, Japan, and the Euro area using a small scale structural monetary business cycle model. Constrained likelihood-based estimates of the parameters are provided and time instabilities analyzed. Real balances are statistically important for output and inflation fluctuations. Their contribution changes over time. Models giving money no role provide a distorted representation of the sources of cyclical fluctuations, of the transmission of shocks and of the events of the last 40 years.

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1 Introduction

There has been considerable interest in recent years in analyzing the monetary aspects of the business cycle; stylized facts about the transmission of monetary policy shocks have been collected (see Christiano et al., 1999 for an early summary of the evidence) and researchers have constructed dynamic stochastic general equilibrium (DSGE) models which replicate these facts and help guiding monetary policy decisions.

In the majority of the monetary models nowadays employed in academics and in policy institutions the stock of money has disappeared. Two reasons appear to justify this approach: the relationship between monetary aggregates and real activity is weakening over time; the money demand function is considerably unstable. The near universal adoption of the interest rate as the policy instrument by central banks of the developed world, coupled with the recent emphasis in modelling central bank's behavior with a policy rule, has lead researchers to focus on models where the supply of money is infinitely elastic. In these models a coherent determination of the equilibrium level of output, inflation and the nominal rate is possible without any reference to monetary aggregates (see Woodford, 2008, for a recent restatement of this result; McCallum, 2008 and Nelson, 2008, for qualifications and counterarguments).

The dichotomy that these models display is hard to accept as working paradigm for theoretical discussion by those who believe in the quantity theory orthodoxy (see e.g. Alvarez, et. al., 2001), by those who think that balance sheet effects matter (see e.g. Meltzer, 2001) or liquidity constraint are important, and by those working with models where the banking sector and the bond market play a role (see e.g. Canzoneri, et al. 2008). It also appears to be grossly inconsistent with a large body of VAR evidence (see e.g. Gordon and Leeper, 1994, Canova and De Nicolo', 2002, Leeper and Roush, 2003, Sims and Zha, 2006, Favara and Giordani, 2009) highlighting the importance of liquidity effects and of credit markets for the propagation of monetary policy disturbances and stressing that shocks to the money demand equation may have important output and inflation effects. Finally, the existing specifications provide little guidance to evaluate the effects of unconventional quantitative and credit easing measures that Japan, the US, and Europe have recently undertaken.

Are these models providing an accurate description of the role of money in propagating and/or amplifying cyclical fluctuations in output and inflation? Can we safely neglect (the stock of) money when studying domestic cyclical fluctuations and evaluating the desirability of monetary policy actions? Despite the relevance of these issues, the structural literature on the topic is surprisingly scant and the conclusions it has reached quite opaque. McCallum, 2001, and Woodford, 2003, have calibrated New-Keynesian models where money has a transaction role and found that neglecting liquidity premia is, to a first approximation, appropriate. Rudebusch and Svensson, 2002, fitted a semi-structural backward looking model to US data and found that nominal money does not affect either output or inflation but Nelson, 2002, using the same model and different a monetary aggregate concluded that the opposite is true, even controlling for movements in the nominal interest rate. Ireland, 2004, on the other hand, has estimated the parameters of simple specification within a general class of forward looking New-Keynesian models, and found little statistical role for money (a similar result is obtained, in a more complicated model, by Andres, et al., 2009). This evidence however has been recently questioned by Favara and Giordani 2009, who claim that the (potentially false) cross equation restrictions that Ireland's model imposes force "estimates of the impact of money on other variables to zero"(p.420). Unfortunately, the design of their simulation experiments is heavily biased against Ireland estimation technique making results difficult to interpret. In general, no one has investigated whether and how structural economic analyses could be distorted when models without money are used nor whether institutional features (and their differences across time and across country) matter for the conclusions one reaches.

This paper brings fresh evidence to these issues by examining four interrelated questions. First, what is the role of money in amplifying cyclical fluctuations to output and inflation in four large industrialized countries (the US, Japan, the Euro area and the UK)? Second, does this role change over time? Third, can variations in the role of money explain certain changes in output and inflation dynamics observed over the last 40 years? Fourth, is economic inference distorted when models where money plays no role are used? To answer these questions, we take a standard small scale New Keynesian model and give money a role via two somewhat reduced form devices. The specification we employ is sufficiently general to capture several neglected channels though which money could affect output and inflation. For example, it can capture transaction frictions, asset market segmentation, working capital requirements or indirect balance sheets effects. In our model, money matters for three reasons. Since it affects the marginal rate of substitution between consumption and leisure, the stock of money influences the real wage, and thus marginal costs and the Phillips curve. Moreover, since it alters the intertemporal rate of substitution of output at different points in time, it creates a wedge in the Euler equation. Finally, since the stock of money enters in the policy rule, it can indirectly affect output and inflation dynamics. Each of these three channels may contribute to alter the nature of cyclical fluctuations the model can account for and stretch their persistence over time.

We estimate the parameters with a constrained maximum likelihood (ML) technique and test, both statistically and economically, the relevance of real balances for output and inflation fluctuations. We employ a structural econometric approach, rather than a more common SVAR approach, to build the tightest possible link between existing theories and the data. For our purpose, a ML technique are preferable to a Generalize Method of Moment or similar limited information methods because the full implications of the model are taken into account and because interesting parametric restrictions are easily tested within the general specification we estimate. We refrain from employing (informative) a-priori restrictions on the parameters to make the information content of the data and its ability to distinguish interesting theoretical specifications as transparent as possible. While it is common nowadays to estimate DSGE models with a prior, the constraints standard priors imply are often so tight and so much data-based that formal testing becomes difficult, if not impossible (see Canova, 2007).

Our investigation reaches four main conclusions. First, money is statistically important for domestic fluctuations in output and inflation. Depending on the country and the time period, money may matters directly, by affecting Euler equation and the Phillips curve, indirectly, by influencing the determination of nominal interest rate, or both. Second, the role of money is changing over time, both in the sense that estimates change magnitude and significance and that different channels become important. Since money does not stand-in for standard omitted suspects and the results we obtain are robust to the use of alternative monetary aggregates, specification and measurement problems are unlikely to drive the conclusions. Third, our estimates highlight the presence of an important time varying wedge between consumption and output. This wedge could be influenced by the real money stock, for example, because of asset market segmentation or participation constraints. Our results are also consistent with the idea that balance sheet effects affect the determination of marginal costs and the link between marginal costs and the output gap, via working capital requirements or direct credit constraints. Finally, we show that the interpretation of the evidence is altered when money is not allowed to play a role in the model. Researchers could mistakenly interpret the pattern of impulse responses in the US in the pre 1980 period; erroneously measure the causes of inflation volatility in Japan up to 1990 and the reasons for the sustained output recovery after the early 1990s recession in the UK. Furthermore, they would have hard time to account for the fall in inflation variability and persistence experienced in the US over the last 40 years.

The rest of the paper is organized as follows. The next section presents the theoretical model used to organize the data. Section 3 describes the data and its sources. Section 4 presents full sample estimates, tests restrictions, studies subsample evidence and examines whether money may proxy for omitted factors. Section 5 examines the role of money in interpreting the events of the last 40 years. Section 6 concludes.

2 The model

The theoretical framework used builds on the small scale New-Keynesian model without capital accumulation described in Ireland, 2004. The structure is extended in two ways: we allow for external habits in consumption; and posit a monetary policy rule where the growth rate of nominal balances matters for the determination of the nominal rate. A third extension, allowing for price indexation, was considered but discarded as the price indexation parameters is hard to identify in a model where money has a role [1]

Contrary to the recent literature examining source of business cycle fluctuations (see e.g. Smets and Wouters, 2007, or Justiniano, et al. 2010), we prefer to work with a small scale model for three basic reasons. First, in a small scale model, the channels though which money may matter for output and inflation fluctuations are easier to identify and the dynamics simpler to interpret. Second, since many frictions present in medium scale models capture omitted factors, it becomes hard to evaluate the role of money if any of these frictions proxy (in a reduced form sense) for the effects of money. Third, population identification problems, of the type emphasized by Canova and Sala, 2009, are likely to be eased, making estimation more reliable and inference more transparent. The potential drawbacks of our choice are obvious: the likelihood constructed using a small scale model may be misspecified, making parameter estimate inconsistent. Since estimation turns out to be reasonably successful and the resulting shocks sufficiently well behaved, misspecification is probably less of an issue in our

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[1] It is worth mentioning also the differences with Andres et al. 2009: they also use consumption habit although of internal type (we use external habits), but do not allow money growth in the policy rule and, as in Ireland, force money to be complement with consumption in the utility.

exercises.

Since the economy is quite standard, we only briefly describe its features. There is a representative household, a representative final good producing firm, a continuum of intermediate goods firms each producing a differentiated good $i \in [0,1]$ and a monetary authority. At each t the representative household maximizes

$$E_t \sum_t \beta^t a_t \left[U\left(x_t, \frac{M_t}{p_t e_t} \right) - \eta n_t \right] \tag{1}$$

where $x_t = c_t - hc_{t-1}$, $0 < \beta < 1$, $h, \eta > 0$ [2] subject to the sequence of constraints

$$M_{t-1} + T_t + B_{t-1} + W_t n_t + D_t = p_t c_t + \frac{B_t}{R_t} + M_t \tag{2}$$

where c_t is consumption, n_t are hours worked, p_t is the price level, M_t are nominal balances, W_t is the nominal wage, B_t are one period nominal bonds with gross nominal rate R_t, T_t are lump sum nominal transfers at the beginning of each t and D_t are dividends distributed by the intermediate firms. a_t and e_t are disturbances to preferences and to the money demand, whose properties will be described below. Let $m_t \equiv \frac{M_t}{p_t}$ denote real balances and $\pi_t \equiv \frac{p_t}{p_{t-1}}$ the gross inflation rate during period t.

The representative final good producing firm uses y_t^i units of intermediate good i, purchased at the price p_t^i to manufacture y_t units of the final good according to the constant returns to scale technology $y_t = [\int_0^1 (y_t^i)^{(\theta-1)/\theta} di]^{\theta/(1-\theta)}$ with $\theta > 1$. Profit maximization yields demand functions for each i of the form

$$y_t^i = \left(\frac{p_t^i}{p_t} \right)^{-\theta} y_t \tag{3}$$

so that θ measures the constant price elasticity of demand for each intermediate good. Competition within the sector implies that $p_t = (\int_0^1 (p_t^i)^{1-\theta} di)^{1/(1-\theta)}$.

An intermediate goods producing firm i hires n_t^i units of labor to produce y_t^i units of intermediate good using the production function $y_t^i = z_t n_t^i$, where z_t is an aggregate productivity shock. Since intermediate goods substitute imperfectly for one another in finished goods production, the intermediate firms act as monopolistic competitors in their pricing decisions. We assume that, when firms change prices, they face cost of adjustment, measured in terms of finished goods, of the form $\frac{\phi}{2}\left(\frac{p_t^i}{\pi^s p_{t-1}^i} - 1 \right)^2 y_t$

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[2] We have also experimented with a specification where external habits enter multiplicatively in utility, i.e. $x_t = \frac{c_t}{c_{t-1}^h}$ but discarded it since the likelihood displays severe numerical problems.

where $\phi > 0$ and π^s measures steady state inflation. The pricing problem faced by the representative firm is therefore to maximize

$$E_t \sum_t [\beta^t a_t U_1 \left(x_t, \frac{M_t}{p_t e_t} \right)] \left(\frac{D_t^i}{p_t} \right) \tag{4}$$

subject to (3), where $\beta^t a_t U_1(x_t, \frac{M_t}{p_t e_t})$ measures the marginal utility value to the household of an additional unit of profits received at t and real dividends are

$$\frac{D_t^i}{p_t} = \left(\frac{p_t^i}{p_t} \right)^{1-\theta} y_t - \left(\frac{p_t^i}{p_t} \right)^{-\theta} \left(\frac{w_t y_t}{z_t} \right) - \frac{\phi}{2} \left(\frac{p_t^i}{\pi^s p_{t-1}^i} - 1 \right)^2 y_t \tag{5}$$

The monetary authority is characterized by a set of rules, where the current nominal interest rate depends on past values of the nominal interest rate and on either current, past or future values of output, of inflation and of the growth rate of nominal balances:

$$R_t = R_{t-1}^{\rho_r} E_t y_{t-p}^{(1-\rho_r)\rho_y} E_t \pi_{t-p}^{(1-\rho_r)\rho_\pi} E_t \Delta M_{t-p}^{(1-\rho_r)\rho_m} \epsilon_t \tag{6}$$

where ϵ_t is a monetary policy shock, $-4 \leq p \leq 4$. The rule in (6) is similar in spirit to the one employed by Christiano et al. 2006 and does not allow, for example, for a time varying inflation objective or a learning mechanism. Adding these features requires controversial assumptions: the law of motion of the inflation target or the constant gain in the learning function need to be arbitrarily specified. Furthermore, without additional arbitrary restrictions, the likelihood function is unable to separate disturbances driving the time varying inflation target from the monetary policy shocks ϵ_t or to assess the credibility of the learning mechanism.

The flexibility we built in (6) is important. Since the samples span up to 50 years of data, allowing for the possibility that monetary policy was, on average, backward or forward looking avoids important specification errors. p is treated as a free parameter, jointly estimated with the others structural parameters of the model [3]

The four disturbances $v_t = (a_t, e_t, z_t, \epsilon_t)$ are assumed to follow an AR(1) process $\log v_t = \bar{v} + N \log v_{t-1} + u_t$, where N is diagonal with entries $(\rho_a, \rho_e, \rho_z, 0)$, respectively. The covariance matrix of the u_t (denoted by Σ) is also diagonal, with entries $\sigma_a^2, \sigma_e^2, \sigma_z^2, \sigma_\epsilon^2$. In a symmetric equilibrium $y_t^i = y_t$, $n_t^i = n_t$, $p_t^i = p_t$, and $D_t^i = D_t$.

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[3] An earlier version of the paper has permitted the growth rate of the dollar exchange rate to enter (6) for the UK and Japan and the model was closed with an interest parity relationship, driven by a risk premium shock. Since the exchange rate was estimated to be insignificant and the UIP relationship dichotomized from the rest of the equations, a closed economy specification was adopted for all countries.

Log-linearizing the equilibrium around the steady states leads to:

$$\begin{aligned}
\hat{y}_t &= \frac{1}{1+h}E_t\hat{y}_{t+1} + \frac{h}{1+h}\hat{y}_{t-1} - \frac{\omega_1}{1+h}\left[\hat{R}_t - E_t\hat{\pi}_{t+1} - (\hat{a}_t - E_t\hat{a}_{t+1})\right] \\
&+ \frac{\omega_2}{1+h}\left[(\hat{m}_t - \hat{e}_t) - (E_t\hat{m}_{t+1} - E_t\hat{e}_{t+1})\right] && (7) \\
\hat{m}_t &= \gamma_1(\hat{y}_t - h\hat{y}_{t-1}) - \gamma_2\hat{R}_t + (1 - (R^s - 1)\gamma_2)\hat{e}_t && (8) \\
\hat{\pi}_t &= \beta E_t\hat{\pi}_{t+1} + \psi\left(\frac{1}{\omega_1}(\hat{y}_t - h\hat{y}_{t-1}) - \frac{\omega_2}{\omega_1}(\hat{m}_t - \hat{e}_t) - \hat{z}_t\right) && (9) \\
\hat{R}_t &= \rho_r\hat{R}_{t-1} + (1-\rho_r)\rho_y\hat{y}_{t-p} + (1-\rho_r)\rho_\pi\hat{\pi}_{t-p} \\
&+ (1-\rho_r)\rho_m\Delta(\hat{m}_{t-p} + \hat{\pi}_{t-p}) + \hat{\epsilon}_t && (10)
\end{aligned}$$

where

$$\omega_1 = -\frac{U_1(x^s, \frac{m^s}{e^s})}{y^s U_{11}(x^s, \frac{m^s}{e^s})} \qquad (11)$$

$$\begin{aligned}
\omega_2 &= -\frac{m^s}{e^s}\frac{U_{12}(x^s, \frac{m^s}{e^s})}{y^s U_{11}(x^s, \frac{m^s}{e^s})} && (12) \\
\gamma_2 &= \frac{R^s}{(R^s-1)(m^s/e^s)}\left(\frac{U_2(x^s, \frac{m^s}{e^s})}{(R^s-1)e^s U_{12}(x^s, \frac{m^s}{e^s}) - R^s U_{22}(x^s, \frac{m^s}{e^s})}\right) && (13) \\
\gamma_1 &= (R^s - 1 + R^s\omega_2\frac{y^s}{m^s})(\frac{\gamma_2}{\omega_1}) && (14) \\
\psi &= \frac{\theta - 1}{\phi} && (15)
\end{aligned}$$

The superscript s denotes steady state values of the variables, U_j is the first derivative of U with respect to argument $j = 1, 2$ and U_{ij} is the second order derivative of the utility function, $i, j = 1, 2$.

The Euler condition (equation (7)) includes, in addition to standard arguments, terms involving real money balances and the money demand shock. These terms are irrelevant for output determination if and only if the utility function is separable in consumption and real balances, i.e. $U_{12} = 0$ (see equation (12)). Equation (8) is a money demand equation: real balances depend positively on current output and negatively on lagged output and the nominal rate. There are important cross coefficient restrictions in this equation and, for example, the output elasticity of money demand depends on the interest semi-elasticity. Real balances enter the Phillips curve (equation (9) as they affect the marginal rate of substitution between consumption and leisure which, in equilibrium, is equal to the real wage and the real wage is a crucial determinant of marginal costs. $U_{12} \neq 0$ is again necessary for real balances to play a role in the determination of the inflation rate [4].

Finally, the policy rule allows, but does not require, the growth rate of real balances to be an important determinant of interest rate decisions. The typical (log-linear) interest rate rule is unlikely to be a good approximation to the way monetary policy is run when the nominal interest rate is close to the zero bound. This may not be a big problem for the US, UK and the Euro area since our samples end at the beginning of 2008. However, it may be in Japan, since in the late 1990s and the early 2000s the nominal rate was very low. The specification we use neglects the potentially important non-linear effects that the zero bound creates, but can be easily transformed into a money growth rule when the interest rate reaches this bound. Thus, it provides a flexible tool to account for the historical experience of the four countries. We further discuss this issue when commenting on the estimation results.

This paper focuses attention on estimates of ω_2 and ρ_m. The first measures the direct role of money in determining the magnitude and the persistence of output and inflation fluctuations; the second, the long run indirect effects that money has on these two variables through the nominal rate. When ω_2 is zero, real balances have no direct role in propagating cyclical fluctuations in output and inflation. When both ω_2 and ρ_m are zero, money could be omitted from business cycle and monetary policy analyses without statistical or interpretation losses.

2.1 The role of money for inflation and output dynamics: an overview

Prior to estimation, it is useful to highlight what difference money may make in the model and along which dimensions we should expect giving money a role may help us to understand better certain real world phenomena. It is well known that the advanced world witnessed a significant decline in the volatility of output and inflation and a considerable fall in the persistence of inflation over the last 30 years (see e.g. Stock and Watson, 2005, Canova at el. 2007). Since the explanations put forward to account for these changes (see e.g. Clarida, et al., 2000, Sims and Zha, 2006, Campbell and Herkowitz, 2006), give money no role, we investigate i) whether allowing ω_2 and ρ_m to be different from zero may make a quantitative difference for the variability and the

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[4] The implications summarized by equation (9) hold also under an alternative pricing scheme, such as Calvo. A flexible price model is not nested in our specification (we require that $\phi > 0$). Nevertheless, since a small ϕ makes the Phillips curve steeper, $\phi \to 0$, approximates, in a reduced form sense, a flexible price specification

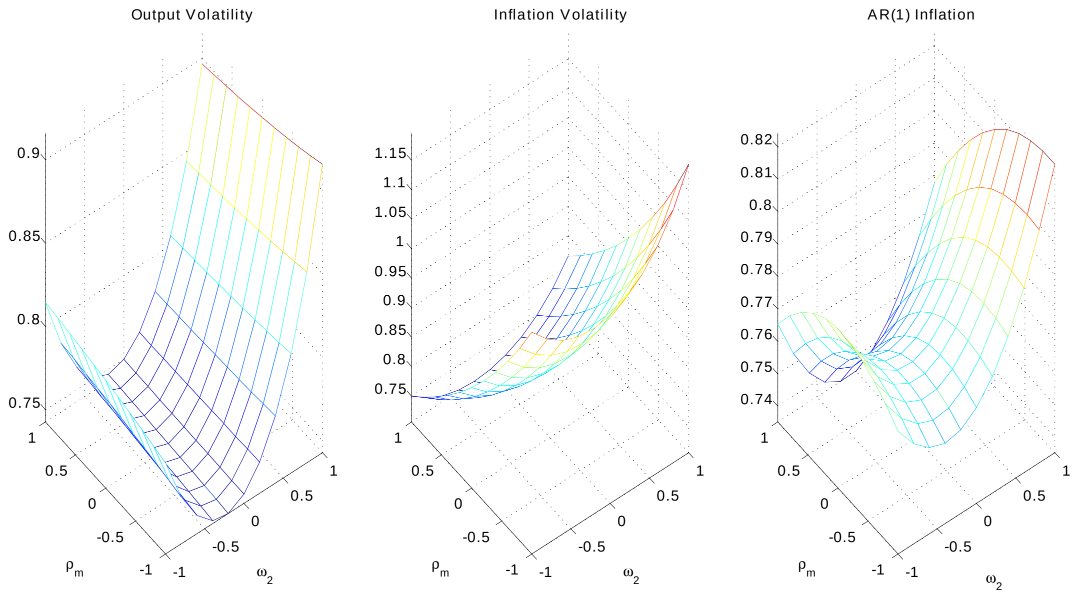

Figure 1: Surface plots

persistence of output and inflation; and ii) whether changes in these two parameters could account for some of the observed changes in these moments.

We parameterize the model as follows: the rule is specified to be backward looking, and $p = 1$. We set $\beta = .99, \psi = 0.74, \omega_1 = 2.0, \gamma_2 = 0.07, \sigma_a = 0.16, \sigma_z = 0.0059, \sigma_e = 0.009, \sigma_\epsilon = 0.009\ h = 0.71, \rho_r = 0.68, \rho_y = 0.17, \rho_\pi = 4.37$, and $\rho_a = 0.90, \rho_z = 0.70, \rho_e = 0.97$. These values are close to those we obtain estimating the model with US data and comparable to those of Ireland, 2004. We then let ω_2 and ρ_m vary within a range and trace out how the volatility of output and inflation and the persistence of inflation change - volatility is measured here by the unconditional population standard deviation. For ω_2 the interval is [-1.0,1.0], which allows for complementarity and substitutability of consumption and real balances in utility; for ρ_m the interval is also [-1.0,1.0], where negative values capture the possibility that reacting to nominal balances is a way to temper a possibly too strong policy reaction to inflation.

Figure 1 shows that the unconditional moments of output and inflation depend on the magnitude of ω_2 and ρ_m and that variations in both parameters may account for some of the variations in the volatility of output and inflation and in the persistence of inflation observed in the real world. For example, the variability of output is larger when $\omega_2 \neq 0$, and the portion of inflation fluctuations accounted by the model significantly increases when $\omega_2 \neq 0$ and/or $\rho_m \neq 0$. In addition, variations in both ω_2 and ρ_m induce significant variations in inflation variability. Inflation persistence also depends on the magnitude of ω_2 and ρ_m and two forces determine the shape of the surface. When $\omega_2 \neq 0$, the persistence of marginal cost rises, making inflation persistence increase. On the other hand, when $\rho_m \neq 0$, inflation persistence is reduced since variations in the money stock, produced by changes in the money demand function, make interest rate move and inflation react. These two channels provide conflicting forces which, depending on the parameterization used, produce a complicated pattern for inflation persistence. With the one we have employed the nonlinear effect is relatively mild and the indirect interest rates effect is small.

In sum, in addition to the reasons cited in the introduction, a-priori forcing $\rho_m = 0$ and $\omega_2 = 0$ may be restrictive as far as inflation and output volatility and persistence are concerned and variations over time in these two parameters represent unexplored channels which may help us to better understand the changes advanced economies have experienced over time.

3 The data and the estimation approach

We assume that the investigator observes data for output, the inflation rate, the nominal rate and real balances. The sample differs across countries: it goes from 1959:1 to 2008:2 for the US, from 1970:1 to 2007:4 for the Euro area, from 1965:1 to 2008:2 for the UK and from 1980:1 to 2007:4 for Japan. US data is from the FRED database at the Federal Reserve Bank of St. Louis; Euro data is from the Eurowide model dataset (update 7) available at the ECB web page; UK data comes from the Bank of England and the Office of National Statistics; data for Japan is collected from IMF and OECD data bases. The inflation rate in the four countries is measured by the growth rate of the GDP deflator; the nominal rate is the three month T-bill rate in the US and the UK, the call rate in Japan, and three month rate in the Euro area. The M2 stock in the US and the Euro area, the M2 plus certificate of deposits in Japan, and the M4 retail stock in the UK are our nominal monetary aggregates. GDP and nominal monetary aggregates are scaled by the GDP deflator and by civilian population in the 16-65 group to transform them in real per-capita terms. We use a relatively large monetary aggregate in our exercises to be consistent with the literature and to avoid, as best as possible, well known instabilities. The sensitivity of the results to changes in the measure of monetary aggregate used is examined below.

Per-capita real GDP and real money balances display an almost deterministic upward trend in all countries. Since the drift is idiosyncratic, both across variables and countries, we separately eliminate it from the log of the series using a linear specification. The inverted U-shaped patterns that interest rates and inflation display are more difficult to deal with. Consistent with the literature, we demean both series.

One feasible alternative to the strategy we use to match the data to the model's counterparts is to allow one of the shocks (typically, the technology shock z_t) to be non-stationary and remove the upward trend in per-capita real output and real balances using a model consistent methodology. We do not follow this approach for two reasons. When technology shocks have a unit root, per-capita output and real balances share the same trend, which is not the case in any of the four countries. On the other, it is unclear whether all non-cyclical fluctuations can be safely attributed to non-stationary technology shocks. Chang et al., 2006, for example, have recently fit a model with non-stationary preference shocks to US data with good results.

The model (7)-(10) contains 20 parameters; 5 structural ones $\eta_1 = (h, \rho_r, \rho_y, \rho_\pi, \rho_m)$, 3 semi-structural ones $\eta_2 = (\psi, \omega_2, \gamma_2)$, 8 auxiliary ones, $\eta_3 = (\rho_a, \rho_z, \rho_e, \sigma_a, \sigma_z, \sigma_e, \sigma_\epsilon, p)$ plus the elasticity parameter ω_1, the discount factor β, the steady state nominal interest rate, R^s and the output to real balance ratio, $\frac{y^s}{m^s}$ which we group into $\eta_4 = (\omega_1, \beta, R^s, \frac{y^s}{m^s})$. Our exercise is geared to obtain restricted likelihood-based estimates of $\eta = (\eta_1, \eta_2, \eta_3)$, conditional on selected values for η_4 - these are calibrated because the likelihood function (of the log-linear model) has little information for them [5]

The model is solved using standard methods. Its solution has the format:

$$x_{1t+1} = A_1(\eta)x_{1t} + A_2(\eta)u_t \tag{16}$$

$$x_{2t} = A_3(\eta)x_{1t} + A_4(\eta)u_t \tag{17}$$

where $x_{2t} = [\hat{y}_t, \hat{\pi}_t, \hat{R}_t, \hat{m}_t]$, $x_{1t} = [\hat{y}_{t-1}, \hat{\pi}_{t-1}, \hat{R}_{t-1}, \hat{m}_{t-1}, \Delta\hat{M}_{t-p-1}, \hat{v}_{1t}, \hat{v}_{2t}, \hat{v}_{3t}, \hat{v}_{4t}]$ and the matrices $A_i(\eta), i = 1, 2$ are complicated functions of the structural parameters η.

Likelihood-based estimation of the free parameters entering (16) and (17) is simple: given some η, and a sample $[t_1, \ldots, t_2]$, the likelihood, denoted by $f(y_{[t_1,t_2]}|\eta)$, is computed by means of the Kalman filter and the prediction error decomposition, and the original η values updated using gradient methods. The Kalman filter is easy to use

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[5] We read R^s for each country and each sample from the average level of inflation, once we set β. For all countries and samples, $\frac{m}{y}$ is fixed to 1.5 (following Chari et. al., 2002).

since (16) and (17) form a linear state space system, where (16) is a transition equation and (17) an observation equation.

Unfortunately, the likelihood function is ill-behaved with the data of any of the countries and displays multiple peaks, sharp cliffs and large flat areas. This is due, in part, to uninformative samples and, in part, to the highly non-linear mapping from the structural parameters to the objective function. To find the maximum of the function, we employ the following multi-step approach:

1. For each data set, each country and each sample, we randomly draw 500 initial conditions. If the model does not have a solution, the draw is discarded. For the remaining draws, the likelihood function is maximized using the csminwel.m routine written by Chris Sims [6]. We take as an estimate of η the vector producing the largest likelihood value across draws, excluding runs where convergence failed.

2. We repeat step 1 for each specification of the monetary policy rule. We compare specifications using the Schwarz approximation to the log Bayes factor (see Canova, 2007). The preferred specification satisfies two conditions: optimizes this criteria and results in estimated residuals which deviate less from the mean zero, iid assumption.

3. The output produced in step 2 is screened to eliminate specifications that violate basic economic principles (e.g. specifications where adjustment costs are negative) or produce unreasonably large standard errors for the structural shocks.

4. For each data set, each country and each sample, steps 1-3 are repeated changing the values of η_4 within a reasonable range and results compared.

To measure the statistical relevance of money in the model, we repeat the four estimation steps conditioning on $\omega_2 = 0$ and $\rho_m = 0$ and compare restricted and unrestricted specifications using a likelihood ratio statistic (computed as $2 * (\log L_u - \log L_r)$ and compared to a $\chi^2(2)$) and a Schwarz approximation to the log Bayes factor (computed as $(\log L_u - \log L_r) - 0.5 * (k_u - k_r) * log(T)$, where k_u and k_r are the number of unrestricted and restricted parameters estimated). It is important to stress, that contrary to the existing literature, we do not restrict ω_2 (and ρ_m) to be positive and search for the policy rule that best fits the data for every country and every sample.

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[6] Standard Matlab routines (such as fminunc.m or fminserach.m) fail to move from the initial conditions in more than 90 percent of the draws.

Bayesian methods have an edge over classical likelihood methods in situations like ours where the likelihood function is poorly behaved. However, when the likelihood has problematic features, inference crucially depends on the shape, the location and the spread of the multivariate prior density. Since prior elicitation is often subjective, questions emerge about the meaning of the inferential conclusions one draws from such a framework. Our choice of letting the data speak and of employing an ex-post criteria to eliminate economically non-interpretable estimates is equivalent to assume that the prior for the parameter vector is multivariate uniform with truncation in the area of the parameter space producing indeterminate solutions or economically unreasonable results (as e.g. in Fernandez Villaverde, 2009).

4 The results

4.1 Specification issues

Before examining estimates of the parameters of interest, it is useful to document the properties of the model, to make sure it fits the data reasonably well and that no predictable pattern is left in the residuals, and discuss estimates of certain parameters which may give important information about the quality of the fit. Tables 1 to 4 report point estimates and standard errors obtained in the basic model and in the restricted model (where ω_2 and ρ_m are forced to be zero) for the four countries. Standard errors are computed from the Hessian of the function at the maximum. Figure 2 plots the residuals computed using point estimates of the parameters, the Kalman filter and the solution of the basic model.

Point estimates of all the parameters in all countries have the right sign and reasonable magnitude. The major exception is γ_2, the interest semi-elasticity of money demand, which at times is counterintuitively negative. This appear to occur more frequently when ω_2 and ρ_2 are forced to be zero, but in all the cases when a negative sign is recorded, estimates are statistically insignificant. Standard errors of the estimates are generally tight indicating that our search has located a sharp likelihood peak. Comparing basic and restricted specifications, one can notice sharp changes in the point estimates of habit parameter, of the mongrel parameter ψ, and in the persistence of the interest rate rule. Thus, in a model where money has no role, these three parameters partly adjust to account for the potentially omitted money variable.

In almost all the specifications we have estimated (across countries and time periods)

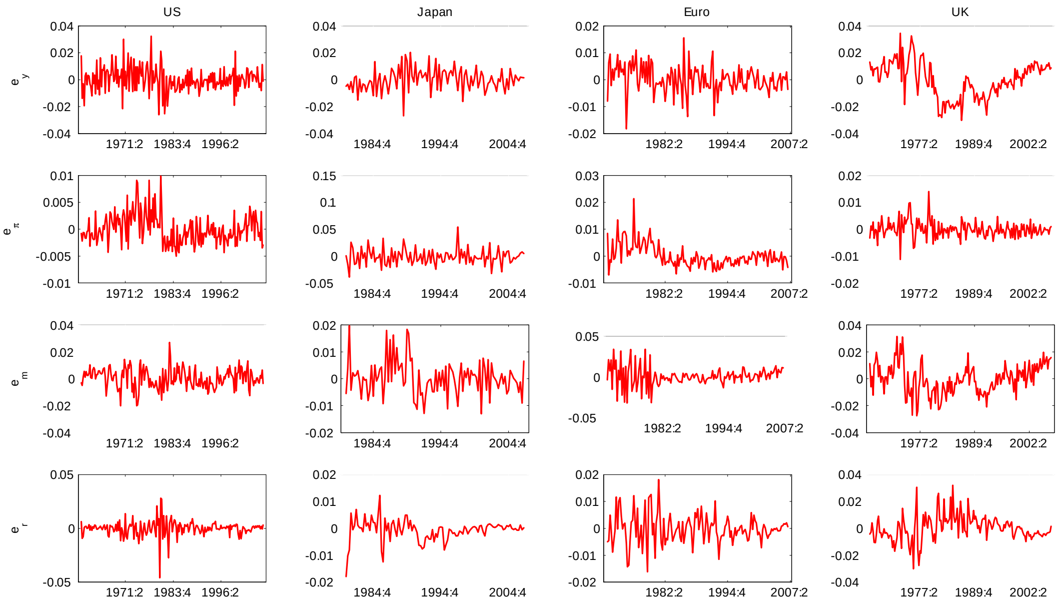

Figure 2: Model residuals.

estimates of ψ are significant. If we treat θ, the elasticity of substitution between varieties as fixed in the estimation and calibrate it to a standard value, our results imply that the costs of adjusting prices are important. Therefore, some nominal rigidities are necessary to understand the mechanics of transmission of shocks and a flexible price model where money has a role appears to be insufficient to account for the richness of the cross country and cross time evidence.

In section 2 we mentioned that the monetary policy rule is flexibly specified to avoid specification errors.For the US a backward rule is preferred and $p = 1$ is selected. For the other countries, a contemporaneous rule is instead selected. Interestingly, in no countries a forward looking specification produces the highest likelihood value.

The residuals of all the equations appear to be reasonably behaved: the mean is not statistically different from zero and serial correlation appears to be generally absent, except perhaps with the residuals of the UK output equation and the Euro inflation equations. However, the volatility bursts (see, for example, Euro money demand equation and the US inflation equation) and outliers (see the policy rule residuals in the US in the middle of the sample) are present at some dates.

Volatility changes and outliers could, in principle, be due to breaks in the long run properties of the per-capita output and money series. To check for this possibility, we have re-estimated the model allowing for a segmentation in the trend of these two variables at 1980:1 for the US; at 1992:4 for the UK and the Euro area, and at 1990:4 for Japan. It turns out that this alteration adds noise to the estimate but does not change the basic features of the estimated residuals. Furthermore, since the exact location of the break does not matter - we can move the break dates within a year (backward or forward) of the selected ones without much changes - volatility changes are unlikely to be due specification errors.

There is also little evidence that the residuals of the model are predictable using lags of the endogenous variables. For example, in the US, a regression of the residuals over lags of the endogenous variables yield coefficients which are very small - the largest point estimate (-0.16) is the one obtained regressing the residuals of the inflation equation on the second lag of inflation. For other countries, the regression coefficients are even smaller. Thus, apart from volatility changes, the model we use appears to be sufficiently well specified to make inference about the role of money credible.

4.2 Is money statistically important? Full sample estimates

In the full sample, the coefficient measuring the direct impact of real balances, ω_2, is negative in three of the four countries, but all estimates are statistically insignificant. The indirect effect of real balances, ρ_m, is estimated to be positive and moderate in all countries and is everywhere significant, except in the UK. The point estimates of this parameter imply that a one percent deviation of real balances from their steady state value has short run effects on interest rates ranging from 0.25 to 0.50 percentage points, depending on the country, in line with the estimates of Barthelemy et al., 2008.

A joint test for the importance of ω_2 and ρ_m gives a stronger role to the stock of real balances in the model. In all countries, money is statistically important in explaining data fluctuations. The likelihood ratio test strongly rejects the hypothesis that both coefficients are zero, while the Bayes factor is conclusively against this hypothesis except in the US (here the evidence only mildly against the hypothesis). Since the LR test and Schwarz criteria give similar conclusions, small sample biases are unlikely to be important for full sample estimates. Note that since both statistics accounts the fit of the whole system, the results they provide are more powerful than simple t-statistics in evaluating the role of money in the model.

Do these estimates depend on the calibrated values of η_4? As the appendix shows, the selected values of η_4 have little influence on the conclusions reached. For example, calibrating ω_1 to one, does change the magnitude of ω_2 but both the sign and the significance of the estimates are unaffected. Thus, a negative estimate for ω_2 is not obtained because only $\omega_1 + \omega_2$ is identifiable and ω_1 is calibrated to be too high.

4.3 Is money statistically important? Subsample evidence

There is considerable evidence that the time series features of output, inflation, nominal rate and money have changed over time and figure 2 confirms, to a large extent, that this is the case in all the countries. Therefore, we have re-estimated the specification allowing both the trend and the cyclical relationships to be varying over time. This exercise permits us to examine whether time variations in the role of money have any possibility to explain the changes in output and inflation dynamics experienced since the mid 1980s. However, since both the likelihood ratio and the t-tests are considerably biased in small samples, care should be exercised in interpreting the results one obtains.

The parameters of interest are estimated using the samples 1959:1-1979:4 and 1980:1-2008:2 for the US, the samples 1980:1-1990:4 and 1991:1-2007:4 for Japan, the samples 1970:1-1992:4 and 1993:1-2007:4 for the Euro area, and the samples 1965:1-1992:4 and 1993:1-2008:2 for the UK. The location of the breaks reflect the history of the individual countries: for the US, 1980 was the year when inflation and the nominal rate started declining from their peak levels; for the Euro area, the break roughly corresponds to the time when the Maastricht Treaty was implemented and to the beginning of the decline of inflation and the nominal rate from the 1980s levels. For the UK, the break is selected to separate the inflation targeting period from previous monetary policy experiences. Finally, the second subsample in Japan starts in correspondence with the bursting of the land bubble which, by many, was considered the trigger of the so-called "Lost Decade". We a-priori select a break date (and analyze the sensitivity of the estimates moving the break date backward and forward) rather than formally test for breaks for two reasons. Standard break tests are of univariate nature and when applied to the four variables used here do not select a single break date. Therefore, considerable judgment needs to be used also after formal testing. Moreover, since the data does not necessarily fit the assumptions underlying these tests, classical type II errors could make structural estimates problematic. These errors can be in part avoided with the selected strategy.

The structural relationships appear to have changed over time in all countries; variations are present in the parameters governing the private sector behavior; the monetary policy rule; and the variance of certain shocks. For the parameters regulating the role of money, conclusions depend on the country.

For the US, the coefficient measuring the direct role of money in transmitting fluctuation to output and inflation is significant in both subsamples. Since this coefficient was insignificant in the full sample, time heterogeneities appear to be important. Notice also that the sign and the magnitude of the estimated coefficient depend on the sample: consumption and real balances are complement in the utility function in the pre-1980 sample, but they are substitute in the post-1980 sample. The coefficient measuring the indirect effect of money also displays important variations: it is negative and insignificant in the pre-1980 sample but becomes positive and significant in the post 1980 sample. Since both the LR test and the log Bayes factor suggest that money matters and rejection of the null hypothesis is stronger in the two subsamples than for the full sample, it is reasonable to conjecture that i) the channels through which the stock of real balances affects output and inflation fluctuations may have changed over time, and ii) full sample estimates provide a misleading picture of the role of money in the US economy.

Money has a marginal role in explaining output and inflation fluctuations in Japan in the first sample, with the Bayes factor being inconclusive about its relevance in the model. Here, the direct and the indirect effects of money are individually significant and the point estimate of ρ_m is negative. As we have mentioned, a negative value is economically meaningful once it is taken into account that by reacting to nominal balances the central bank may temper a possibly too strong policy reaction to inflation. In the second subsample, estimates of both parameters are smaller and insignificant and both the LR and the Bayes factor give money a rather limited role in the propagating cyclical fluctuations. For our purposes, it is important to emphasize that the role of money has dramatically changed over time also in Japan, making full sample estimates somewhat dubious.

	Full sample		Sample 1		Sample 2	
	With money	No money	With money	No money	With money	No money
ψ	0.7430	0.1620	0.0009	0.1065	2.0342	0.1071
	(0.1643)	(0.0356)	(0.0013)	(0.0521)	(0.4474)	(0.0140)
	0.9900	0.9900	0.9900	0.9900	0.9900	0.9900
ω_2	-0.5125	0.0000	24.1444	0.0000	-0.9781	0.0000
	(0.5226)		(8.1261)		(0.1989)	
γ_2	0.0744	0.2474	-0.0278	0.1032	0.3625	0.3300
	(0.0817)	(0.0760)	(0.0283)	(0.1787)	(0.0708)	(0.0713)
h	0.7162	0.9410	0.8846	0.9410	0.1469	0.9574
	(0.0848)	(0.0646)	(0.0262)	(0.0394)	(0.1106)	(0.0167)
ρ_r	0.6824	0.8229	0.8080	0.5441	0.6284	0.7268
	(0.0383)	(0.0320)	(0.0707)	(0.1150)	(0.0366)	(0.0369)
ρ_y	0.1779	0.7095	0.3571	0.2289	0.4796	0.9310
	(0.0760)	(0.1512)	(0.0938)	(0.0655)	(0.1136)	(0.1428)
ρ_p	4.3719	3.5155	2.1510	2.5050	6.0769	5.4809
	(0.3911)	(0.1848)	(0.4814)	(0.2664)	(0.5471)	(0.5524)
ρ_m	1.5814	0.0000	-0.0363	0.0000	1.8107	0.0000
	(0.2964)		(0.2187)		(0.2816)	
ρ_a	0.9629	0.9245	0.9204	0.9859	0.9863	0.9851
	(0.0183)	(0.0077)	(0.0291)	(0.0230)	(0.0034)	(0.0053)
ρ_e	0.9775	0.9765	0.9689	0.9813	0.9807	0.9788
	(0.0083)	(0.0083)	(0.0002)	(0.0223)	(0.0089)	(0.0082)
ρ_z	0.7009	0.7969	0.8700	0.6034	0.9509	0.7055
	(0.0739)	(0.0287)	(0.0803)	(0.2736)	(0.0300)	(0.0671)
σ_r	0.0099	0.0085	0.0067	0.0077	0.0092	0.0094
	(0.0684)	(0.0593)	(0.1172)	(0.1131)	(0.0763)	(0.0767)
σ_a	0.1618	0.1030	0.1316	0.3094	0.4889	0.5348
	(0.5429)	(0.1736)	(0.1683)	(2.0135)	(0.3885)	(0.5664)
σ_e	0.0093	0.0094	0.0110	0.0103	0.0088	0.0080
	(0.0508)	(0.0517)	(0.0676)	(0.0717)	(0.0673)	(0.0691)
σ_z	0.0054	0.0045	0.4911	0.0125	0.0029	0.0072
	(0.0798)	(0.2286)	(1.5415)	(0.4582)	(0.0984)	(0.1557)
p	1	1	1	1	1	1
Log L	2674.93	2664.85	1180.82	1146.16	1589.29	1566.04
Sample size	197	197	84	84	113	113
LR-test p-value	0.0000		0.0000		0.0000	
Schwarz	4.79		30.24		18.50	

Table 1: Parameter estimates for US. In the estimation β is calibrated at 0.99 and ω_1 at 2.

	Full sample		Sample 1		Sample 2	
	with money	no money	with money	no money	with money	no money
ψ	0.6361	0.1590	0.0460	0.3088	0.1085	0.1122
	(0.1477)	(0.0099)	(0.0043)	(0.6299)	(0.2057)	(0.0138)
	0.9900	0.9900	0.9900	0.9900	0.9900	0.9900
ω_2	-0.2254	0.0000	1.2148	0.0000	0.6241	0.0000
	(0.1354)		(0.2093)		(1.1793)	
γ_2	0.0002	-0.0647	-0.2875	0.1315	0.0744	-0.9839
	(0.1402)	(0.1707)	(0.1772)	(3.0041)	(1.6573)	(3.6494)
h	0.9881	0.7263	0.3922	0.8709	0.3633	0.3549
	(0.0187)	(0.0402)	(0.2232)	(1.0910)	(1.1472)	(0.2146)
ρ_r	0.8419	0.9064	0.5787	0.7480	0.8262	0.8129
	(0.082)	(0.0138)	(0.0711)	(1.0807)	(0.1039)	(0.0993)
ρ_y	0.1836	1.1968	0.9794	1.3213	1.9470	1.4477
	(0.0363)	(0.0656)	(0.1370)	(6.5565)	(2.0552)	(0.5383)
ρ_p	0.0671	0.4748	1.2652	0.4912	-0.4152	0.6430
	(0.4427)	(0.1626)	(0.3589)	(2.6899)	(2.6917)	(3.9775)
ρ_m	1.0471	0.0000	-1.3553	0.0000	1.0130	0.0000
	(0.4473)		(0.5408)		(2.5827)	
ρ_a	0.9950	0.9862	0.9911	0.9842	0.9842	0.9679
	(0.0023)	(0.0018)	(0.0013)	(0.1433)	(0.0336)	(0.0118)
ρ_e	0.9990	0.9557	0.9825	0.9370	0.9884	0.9674
	(0.0037)	(0.0126)	(0.0087)	(0.2245)	(0.0117)	(0.0209)
ρ_z	0.0467	0.0375	0.0276	0.0007	0.0455	0.0230
	(0.0817)	(0.0344)	(0.0603)	(1.6813)	(0.1708)	(0.2967)
σ_r	0.0060	0.0067	0.0080	0.0084	0.0099	0.0065
	(0.1553)	(0.1148)	(0.1368)	(2.6828)	(1.2895)	(1.6335)
σ_a	0.4457	0.1947	0.3893	0.3061	0.1020	0.0751
	(0.3466)	(0.0996)	(0.3446)	(4.8069)	(3.3294)	(1.0439)
σ_e	0.0095	0.0082	0.0100	0.0101	0.0065	0.0064
	(0.1206)	(0.0771)	(0.1226)	(1.9665)	(0.4139)	(0.4479)
σ_z	0.0259	0.1072	0.4021	0.0603	0.1288	0.1101
	(0.2806)	(0.0864)	(0.2026)	(3.4141)	(1.7468)	(0.2132)
p	0	0	0	0	0	0
Log L	1423.66	1396.00	545.16	540.50	926.79	936.43
Sample size	112	112	44	44	68	68
LR-test p-value	0.0000		0.0090		1.0000	
Schwarz	22.94		0.87		-13.85	

Table 2: Parameter estimates for Japan. In the estimation β is calibrated at 0.99 and ω_1 at 2.

	Full sample		Sample 1		Sample 2	
	with money	no money	with money	no money	with money	no money
ψ	6.3863	6.5444	0.0373	0.6887	0.1450	0.1009
	(1.8553)	(0.9615)	(0.0078)	(0.1024)	(0.0284)	(0.0364)
	0.9900	0.9900	0.9900	0.9900	0.9900	0.9900
ω_2	-1.9986	0.0000	-0.8089	0.0000	-0.0272	0.0000
	(1.3404)		(0.2178)		(0.0671)	
γ_2	0.1238	-0.0370	0.2723	-0.1271	0.1782	0.1190
	(0.0991)	(0.1812)	(0.2000)	(0.1845)	(0.1610)	(0.5721)
h	0.1375	0.2152	0.5723	0.6495	0.9110	0.9634
	(0.0850)	(0.0814)	(0.1101)	(0.0692)	(0.0567)	(0.0342)
ρ_r	0.5640	0.5454	0.7394	0.8534	0.8877	0.9199
	(0.0696)	(0.0554)	(0.0402)	(0.0195)	(0.0259)	(0.0759)
ρ_y	0.5427	0.2596	1.3873	1.8212	2.2281	3.0527
	(0.1186)	(0.1371)	(0.1953)	(0.2643)	(0.3486)	(2.9861)
ρ_p	4.3265	4.9937	-0.5509	3.2928	4.5742	1.3242
	(0.3497)	(0.4082)	(0.8814)	(0.6420)	(1.0750)	(4.5498)
ρ_m	0.6695	0.0000	0.5959	0.0000	0.8109	0.0000
	(0.2347)		(0.3828)		(0.6799)	
ρ_a	0.9782	0.9934	0.9346	0.8937	0.9790	0.9956
	(0.0069)	(0.0019)	(0.0135)	(0.0222)	(0.0049)	(0.0310)
ρ_e	0.9876	0.9938	0.9328	0.9475	0.9767	0.9935
	(0.0020)	(0.0063)	(0.0098)	(0.0131)	(0.0107)	(0.0058)
ρ_z	0.9692	0.9908	0.7524	0.5821	0.2354	0.2325
	(0.0051)	(0.0064)	(0.0472)	(0.0848)	(0.1277)	(0.1201)
σ_r	0.0130	0.0138	0.0094	0.0084	0.0039	0.0036
	(0.1529)	(0.1747)	(0.1482)	(0.1175)	(0.1331)	(0.1259)
σ_a	0.2358	0.7668	0.1119	0.0745	0.1695	0.8241
	(0.3974)	(0.2108)	(0.1920)	(0.1781)	(0.2637)	(11.6015)
σ_e	0.0131	0.0129	0.0115	0.0140	0.0077	0.0076
	(0.0544)	(0.0575)	(0.0670)	(0.0707)	(0.0957)	(0.1259)
σ_z	0.0022	0.0027	0.0317	0.0043	0.0103	0.0148
	(0.1348)	(0.0621)	(0.2384)	(0.1621)	(0.2933)	(0.3284)
p	0	0	0	0	0	0
Log L	2132.74	2119.91	1212.76	1214.75	953.22	957.24
Sample size	153	153	91	91	62	62
LR-test p-value	0.0000		1.0000		1.0000	
Schwarz	7.79		-6.50		-8.14	

Table 3: Parameter estimates for the Euro area. In the estimation β is calibrated at 0.99 and ω_1 at 2.

	Full sample		Sample 1		Sample 2	
	with money	no money	with money	no money	with money	no money
ψ	-0.0007	-0.0008	0.0813	0.0298	13.4829	8.2998
	(0.0014)	(0.0032)	(0.0519)	(0.0095)	(1.9491)	(1.6284)
ω_2	1.1985	0.0000	-0.6567	0.0000	-2.5079	0.0000
	(2.5277)	(0.0000)	(0.2108)	(0.0000)	(0.1885)	(0.0000)
γ_2	1.1783	0.8326	0.2110	0.2893	0.0886	0.4771
	(2.5605)	(3.1799)	(0.2184)	(0.2274)	(0.0313)	(0.2433)
h	0.0337	0.3826	0.5963	0.7327	0.5217	0.5859
	(0.2139)	(0.1525)	(0.1504)	(0.1420)	(0.0791)	(0.0749)
ρ_r	0.9521	0.9963	0.4880	0.5701	0.7517	0.3743
	(0.1538)	(0.0043)	(0.1233)	(0.1451)	(0.0572)	(0.0631)
ρ_y	0.0166	0.4663	0.7018	1.0157	0.4869	0.5233
	(0.0693)	(0.0566)	(0.1741)	(0.2958)	(0.7857)	(0.3722)
ρ_p	1.4905	-0.0763	0.6751	-1.5171	44.3673	41.8592
	(3.4930)	(3.2169)	(1.3835)	(1.8921)	(3.7231)	(8.7637)
ρ_m	0.2876	0.0000	-0.3505	0.0000	1.8257	0.0000
	(0.4422)	(0.0000)	(0.2325)	(0.0000)	(1.5568)	(0.0000)
ρ_a	0.8891	0.7798	0.9047	0.9591	0.9307	0.8811
	(0.2943)	(0.0568)	(0.0366)	(0.0067)	(0.0211)	(0.0322)
ρ_e	1.0000	0.8930	0.9280	0.9279	0.9490	0.9282
	(0.0560)	(0.0411)	(0.0198)	(0.0381)	(0.0097)	(0.0131)
ρ_z	0.9153	0.9990	0.5609	0.5635	0.9280	0.8830
	(0.1484)	(0.0459)	(0.1499)	(0.1260)	(0.0598)	(0.0271)
σ_r	0.0061	0.0090	0.0101	0.0116	0.0236	0.0542
	(0.4083)	(0.4336)	(0.1467)	(0.1500)	(0.3235)	(0.3429)
σ_a	0.2202	0.5092	0.0785	0.1956	0.0496	0.0294
	(1.4855)	(0.3229)	(0.3995)	(0.2764)	(0.2984)	(0.3024)
σ_e	0.0410	0.3481	0.0131	0.0129	0.0073	0.0072
	(1.8409)	(0.4025)	(0.0958)	(0.1236)	(0.0866)	(0.0839)
σ_z	0.2444	0.0262	0.0130	0.0363	0.0013	0.0016
	(3.1862)	(0.3786)	(0.4511)	(0.3609)	(0.0990)	(0.0939)
p	0	0	0	0	0	0
log L	1716.40	1209.66	600.71	599.89	1005.80	1002.15
Sample size	104	104	44	44	60	60
LR-test p-value	0.0000		0.4400		0.0212	
Log Bayes factor	502.09		-2.56		-0.44	

Table 4: Parameter estimates for the UK. In the estimation β is calibrated at 0.99 and ω_1 at 2.

When the interest rate is close to zero, the monetary policy rule we consider can be interpreted as a money growth rule. For the second sample in Japan, the estimated parameters imply, for example, that money growth was mildly linked to inflation movements and strongly and negatively linked to output gap fluctuations. In particular, a one percent fall in output below trend would have induced an almost two percent increase in money growth. Thus, our specification captures well shift in policy instruments and can reasonably represent the experience of quantitative easing which occupies part of this sample.

Money does not seem to matter much in the Euro area in the first sample. Here estimates of ω_2 are significant and negative but the overall role of money in the model is rather marginal. In the second subsample, the estimate of the direct effect of money becomes smaller and insignificant, while the estimate of the indirect effect becomes larger but remains insignificant. Overall, the role of money in transmitting fluctuations to output and inflation in the Euro is quite small in both subsamples. While the heterogeneity in the structure of the Euro economies could be responsible, in part, for the result, it is also true that repeating estimation over the two subsamples using Germany data only marginally affects the conclusions. Hence, while estimates do change over time in a direction consistent with expectations, the overall conclusions seems to be that money has little role in propagating cyclical fluctuations in the Euro area [7]

The UK is the only country where the role of money appears to have increased over time. Estimates of ρ_m are insignificant in both subsamples, while ω_2 becomes more significant in the inflation targeting sample. Note also, that while still insignificant, the point estimate of ρ_m changes sign and becomes larger in the second subsample. As in the US, subsample estimates are considerably different from those obtained in the full sample, suggesting the existence of a common pattern of the time variations in the two countries.

We have examined the sensitivity of the conclusions to changes in the break date in two ways. First, we have moved the break date forward or backward by up to two years and found very little changes in the properties of the estimates. Second, for the Euro area and Japan we have considered an additional break point in correspondence of the creation of the ECB (1998) and the separation of the Bank of Japan from the Ministry of Finance (1997). While the main features of our conclusions remain, point

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[7] This result should be contrasted with Barthelemy et al, 2008, who instead find a significant role for money in the Euro area. Differences could be due to three factors: the model specification is different; the sample they use cut across our two samples and the estimation details are different.

estimates turn out to be very imprecise due to the small size of the last two samples.

In sum, while not uniform across countries and time periods, the evidence suggests that money matters in shaping business cycles fluctuations or, more mildly, that matters more than the literature is willing to assume. Since statistical significance does not necessarily imply distortions when interpreting the evidence, designing optimal policies, or conducting conditional forecasting exercises, section 5 examines the economic relevance of our findings.

4.4 Discussion

A few important points can be made from the estimates we have presented. First, money matters for output and inflation fluctuations, because it affects the marginal rate of substitution between consumption and leisure and creates a time varying wedge between consumption and output. The complementarity between consumption and real balances in the utility found, for example, in the US and in Japan in the pre-1980 sample, it is easily reconcilable with models where money has a transaction role. The substitutability we have found, for example the US and the UK in the second subsample, indicates that money has instead a store-of-value role and is consistent with the segmented market approach of Alvarez et al., 2001. In that setup, only a fraction of consumers have access to asset markets and the limited participation constrain implies that real balances become a proxy for future consumption, making real balances substitutable with current consumption.

It is worth also mentioning that a negative ω_2 could be generated with a utility function of the form $\frac{c_t^{1-\sigma_c}}{1-\sigma_c}\frac{(m_t/e_t)^{1-\sigma_m}}{1-\sigma_m}$, and a strong concavity of the utility function with respect to real balances ($\sigma_m >> 1$). When the economy is hit by a shock that increases inflation, the nominal rate increases, leading to a fall in the money demand. When consumption and real balances are substitute, consumption rises following such a shock, leisure falls and work effort and output increase.

Recall that our model restricts real balances to enter with the same coefficient in the Euler equation and the Phillips curve. Therefore, an alternative explanation for the substitutability between real balances and consumption comes from looking at the role that money may have in the determination of marginal costs. When firms face a cash-in-advance or a working capital constraint, the production function can be written as a function of labor and real balances (as e.g. in Benhabib and Farmer, 2000). Hence, if real balances become an input in the production process, they can contribute to the marginal costs. Future work in the area should try to disentangle alternative explanations for the substitutability between consumption and real balances in utility since they provide alternative views about the mechanics of transmission in monetary economies. The switch in the sign of ω_2 we have obtained across subsamples in some countries also deserves careful scrutiny as it may indicate interesting changes in the way the economy works.

Second, money also matters because variations of the growth rate of the nominal quantity of money alter the nominal interest rate, which is a crucial determinant of output and inflation fluctuations. As far as we know, the typical monetary policy rules nowadays estimated in the structural literature do not include the growth rate of nominal balance among the regressors. Since reacting to nominal balances is also, indirectly, a way to react to inflation, it is likely that, in typical rules neglecting money, the inflation coefficient is biasedly estimated and the richness of second round dynamics due to money muffled. Given the estimated effects are typically important in all countries, policy analyses conducted in models where the monetary policy rule abstract from money may have important repercussions for the interpretation of the evidence. This point is well known in the SVAR literature (see e.g. Leeper and Roush, 2003 and Sims and Zha, 2006), but it is hardly mentioned in the more structural literature (see Smets and Wouters, 2007 or Justiniano, et al., 2010). We show below that interpretation mistakes are indeed important.

Taken together these two observations indicate that the IS channel is not necessarily the main mechanism through which monetary policy affects the economy. The two additional channels the model possesses constitute alternative and unexplored mechanisms through which a larger portion of the cyclical fluctuations present in the data could be accounted for.

Third, in contrast with some of the existing SVAR literature, money is important not only in the 1970s and 1980s, when central banks were explicitly or implicitly employing the growth rate of nominal aggregates as the main indicator for monetary policy decisions, but also in the 2000s, when the dynamics of monetary aggregates are hardly mentioned by Federal Reserve officials and the inflation targeting rhetoric has moved monetary aggregates to the back burner of the policy discussion. Below we investigate whether this somewhat surprising result could be spuriously due to measurement errors and/or omitted variables.

Fourth, although the estimates obtained indicate that real balances play an impor-

	Basic	$\omega_2 = 0$	$\rho_m = 0$	$\omega_2 = h = 0$	$\rho_m = h = 0$
ω_2	-0.9781 (0.198)	0.0000	-0.1441 (0.1940)	0.000	-0.198 (0.181)
ρ_m	1.810 (0.284)	1.875 (0.278)	0.000	1.852 (0.283)	0.000
$\log L$	1589.29	1578.41	1562.75	1573.87	1569.21

Table 5: Alternative specifications: US, 1980:1-2008:2.

tant coincident indicator role for output and inflation fluctuations in three countries, one should be careful in translating such evidence into a statement concerning the use of real (or nominal) balances in unconditional forecasting regressions of output and inflation. The model used is stylized and, for forecasting purposes, may not be as successful as time series specifications which leave real or nominal money out. Thus, our results should be interpreted as giving money a conditional role. That is, conditional on the model, giving or not giving money a role may lead to different policy conclusions. Section 5 investigates to what extent this is the case.

Fifth, our results for the US stand in contrast with the findings of Ireland, 2004, who estimated ω_2 to be positive and insignificant using post 1980 data (and set ρ_m identically to zero). There are a few reasons which may account for the differences: the model specification is slightly different - we use a model with consumption habit, while Ireland does not - and the sample is slightly longer. Table 5 shows that in the longer sample, setting $h = \rho_m = 0$ recovers Ireland's result (see the last column) [8]. However, the likelihood of this specification is considerably smaller than the likelihood of our basic setup and conditioning on not having nominal balances growth in the interest rate rule makes it more likely that ω_2 is small and insignificant. Thus, to properly measure the role of money for output and inflation fluctuations one needs to jointly consider the direct and the indirect effects of money. Without the latter, the likelihood function is twisted in the direction of giving money no direct role [9].

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[8] Since Ireland forces ω_2 to be positive, and the ML estimate is negative, is not surprising that he finds that it is around zero. The estimates presented in the last column mimic Ireland's result in the sense that a confidence interval of standard length would include the zero value.

[9] During the search with US data, we have encountered one or more local peaks where estimates of the effect of real balances in the Euler equation and the Phillips curve is indeed positive and small and the growth rate of nominal balances is marginally insignificant in the interest rule. Hence, the procedure employed to maximize the function is also important to deliver proper conclusions and the strategy outlined in section 3 helps in locating where its global maximum is.

4.5 Does money stand-in for something else?

Despite the evidence we have presented in section 4.1, one could still argue that estimates of ω_2 and ρ_m are suspect. Is it possible that money truly does not matter, but turns out to be significant because it captures influences omitted from our specification? Could it be possible that the importance of money changes over time because nominal balances proxy for exchange rates, credit, oil or asset prices in certain periods but not in others? To properly answer these questions a much more complicated model ought to be developed and one needs to understand under what conditions real balances may capture effects which are due the potential omission of these factors. Such an investigation is beyond the scope of this paper, but one can guess that a very complex set of circumstances is needed to make nominal balances proxy for certain variables in one sample but not in another one.

Here we are concerned with the much modest task of providing additional information about the role played by real balances in the model when some of these factors are taken into account. Because of data constraints, the analysis is limited to the US for the second sample, but we do not have reasons to believe that the data of other countries or other periods will behave differently.

	M2	M1	Currency	Credit	SP500	House Prices	Oil
ω_2	-0.9781	-0.9454	-1.0499	-1.1056	0.3817	0.1307	0.1986
	(0.1989)	(0.2646)	(0.2976)	(0.4825)	(0.3546)	(0.0895)	(1.5340)
ρ_m	1.8107	0.8487	1.7138	1.1908	0.9229	2.0508	1.8687
	(0.2816)	(0.2186)	(0.2946)	(0.1874)	(0.1986)	(0.4496)	(3.1508)
$\log L$	1589.93	1516.48	1456.78	1514.66	1704.94	1869.09	1438.36

Table 6: Robustness checks, US sample 2. Currency measures the currency component of M2, Credit measures total Consumer Credit Outstanding

We conduct a number of exercises. First, we use M1 or currency in place of M2. If the effects of money are due to demand or time deposits, they should be eliminated when smaller monetary aggregates are used in the estimation. Second, we use total consumer credit outstanding in place of M2. If the outside component of monetary aggregates matters more than the inside component, one should observe significant changes in the estimates. Third, we allow oil to enter the production function, following Nakov and Pescatori, 2009, and assume an exogenous process for the price of oil. The resulting specification implies that, in addition to the standard arguments appearing in (9), the Phillips curve also depends on the oil price variable (since it is a component of marginal costs). If money stands-in for oil, the significant effects we have found should disappear in this specification. Finally, we add stock price growth or house price growth to the monetary policy rule and close the model assuming an exogenous process for these prices. If nominal money growth proxies for these omitted factors, ρ_m should become insignificant when these variables are allowed in the specification.

Overall, it appears that money is important for output and inflation fluctuations on its own and not because it proxies for standard omitted suspects. For example, changing the monetary aggregate does not change our conclusions: estimates of ω_2 and ρ_m remain significant and the sign of the estimates are unaltered. Similarly, the inside and the outside component of money play a similar role. Results appear to change somewhat when asset prices are added to the policy rule. In this case, point estimates of ω_2 become positive but are statistically insignificant, while estimates of ρ_m remain statistically significant and economically important. Thus, it appears that the direct role of money could be alternatively captured by inserting asset prices in the interest rate rule, a result which is again consistent with the idea that asset markets could be segmented. Finally, when we add oil to the model, estimates of both parameters become insignificant. Since other structural parameters, such as the habit coefficient, the output elasticity and the interest semi-elasticity of money demand, also become insignificant, we conjecture that the large swing in oil prices in 2007-2008 combined with a small sample is responsible for this outcome. In fact, when the full sample is used, having or not having oil in the model does not change either the magnitude or the significance of both ω_2 and ρ_m.

5 The economic relevance of money

The evidence so far collected indicates that money statistically matters. But, is money economically important? In other words, can we safely neglect money when studying domestic business cycles and designing optimal monetary policies?

To study whether the omission of real balances is crucial for understanding certain economic phenomena, we first present responses to the four shocks for the US in the sample 1959-1979, when money is allowed to play a role and when it does not (i.e. ω_2 and ρ_m are identically set to zero). This sample/country combination is chosen since

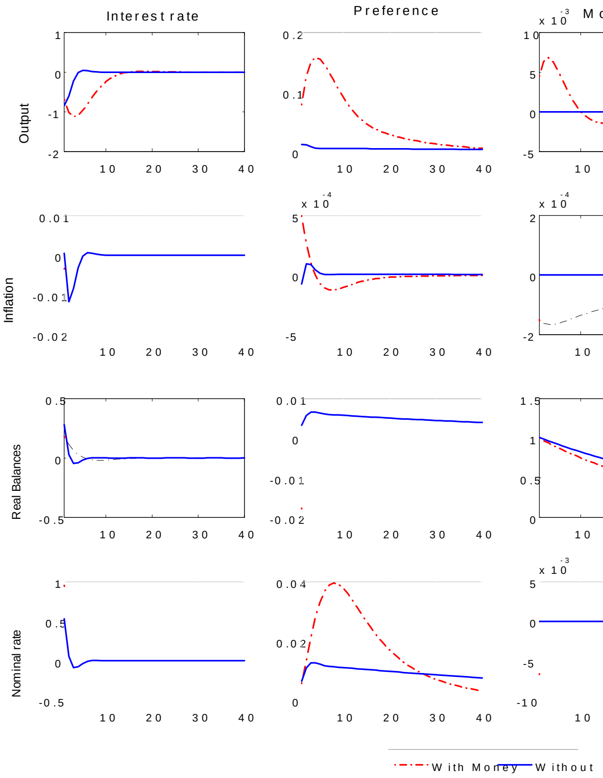

Figure 3: Responses to shocks US, 1959-1979

it could tell us something about the causes of the Great Inflation of the 1970s.

There are important differences in the responses to the shocks in the two specifications. For example, technology shocks have different effects on output and inflation and an interest rate disturbance induces more persistent output and inflation fluctuations in a model with money. Notice that in a model without money, the instantaneous response of output and inflation to technology shocks is perverse (positive technology shocks induce negative output responses and positive inflation responses) because the interest rate instantaneously increases. This suboptimal interest rate response does not seems to be due to monetary policy incorrectly leaning against technological improvements. Rather, it seems to be an omitted variable problem: technology shocks capture, in part, the effects of negative money demand shocks.

In addition, while in response to contractionary interest shocks output falls in both specifications, inflation instantaneously fall when money matters and it is instantaneously unchanged when money does not matter. Hence, a VAR specification where interest rate is instantaneously causally prior to inflation is consistent with the evidence obtained in a model without but not with money. Finally, while interest rate shocks do not induce liquidity effects in any specifications (real balances positively comove with the nominal rate), money demand shocks have this feature in a model with money.

Interestingly, one aspect of figure 2 is not affected by the role given to money in the model: inflation in the 1970s is not a monetary phenomena. Technology shocks, in fact, dominate the forecast error variance of inflation at the 5 year horizon in both specifications (they explain 94 percent of the forecast error variance of inflation when money matters and 92 percent when money does not matter). The contribution of both money demand and interest rate shocks is instead negligible.

Distortions in interpreting the evidence are obtained in other instances. For illustration, we consider two. First, we study sources of inflation variability in Japan in the pre-1990 sample. In a model without money, the five year ahead forecast error variance of inflation is due primarily to technology shocks (92 percent); in a model with money, money demand shocks have the dominant role (69 percent). Two reasons can account for this difference: the size of the shocks is different in the two specifications; the transmission mechanism of various shocks is altered. The relative variance of technology shocks is indeed different in the two specifications. However, it is the transmission of money demand shocks which is mainly altered: when money is not allowed to matter, money demand shocks have no effect on inflation at any horizon and their effect is spuriously captured by technology and preference shocks.

Second, we have computed a historical decomposition of UK output in the second subsample and examined which factors have contributed most to the recovery following the deep recession of the early 1990s. To do this, the model is estimated with data up to 1992:4 and the sequence of one-step ahead forecast errors is computed for the next 10 years using the Kalman filter. These errors are then decomposed into the component attributable to each of the four structural shocks. Figure 4 plots the time path of these forecast errors due to each structural shock, when money has a role (left panel) and when it does not have one (right panel).

Clearly, the two panels interpret the experience differently. The contribution of policy shocks is roughly unchanged in the two specifications, but in a model without money, the contribution of technology and money demand shocks is negligible over the entire forecasting horizon. Hence, the model without money makes the predictable component of output absorb the contribution of these shocks; that is, researchers would

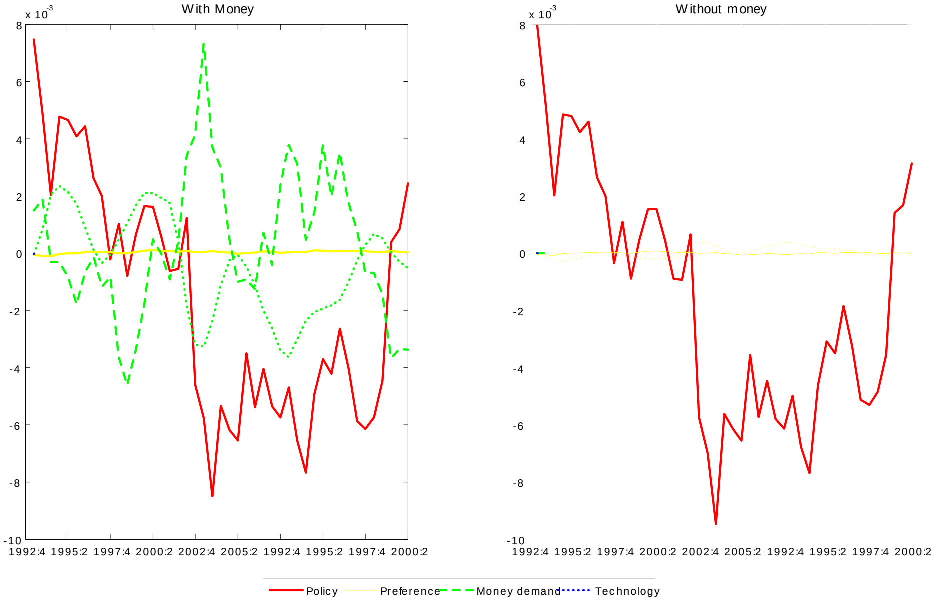

Figure 4: One step ahead historical decomposition of UK output

infer that most of the post 1992 output recovery in the UK was predictable. Including money in the model instead tell us that unexpected variations play a larger role and that technology and interest rate shocks almost equally contribute to the output recovery in the first 3-4 years of the sample.

Finally, we examine to what extent a specification with money is better endowed than the alternative one without money in interpreting the time profile of output and inflation volatility and persistence observed in the US over the last 40 years. As mentioned in section 2, variations over time in ω_2 and ρ_m do affect these moments. Here we study whether the interpretation of the episode is also different.

When analyzing how certain features of the model affect important functions of the data, it is typical to conduct counterfactuals where, for example, part of the parameters are changed and others are kept fixed at some values. Such a procedure is clearly inappropriate when parameters are jointly estimated with system-wide methods. There is in fact a correlation structure among estimates and if some parameters are changed, the remaining parameters must be adjusted using the estimated covariance matrix. Furthermore, the alternative values that are selected may be "unreasonable" or highly unlikely from the point of view of the sample under consideration, making the results difficult to interpret. Both of these problems are absent if one compares, as we do here,

	With Money		Without Money	
	Sample 1	Sample 2	Sample 1	Sample2
Output volatility	21.84	8.14	3.73	3.21
Inflation volatility	1.21	0.68	0.37	0.56
Inflation persistence	0.99	0.72	0.71	0.78

Table 7: Volatility is measured by the standard deviation of the variables computed using the solution of the model at the parameter estimates. Sample 1 goes from 1959:1 to 1979:4 and sample 2 from 1980:1-2008:2.

a specification where all parameters are unrestrictedly estimated with another where ω_2 and ρ_m are both set to zero and the remaining parameters free to adjust.

Table 7 indicates that the model with money captures both the decline in the volatility of output and inflation and in the persistence of inflation across the two subsamples. Apart from changes in ω_2 and ρ_m, the parameters that change most across subsamples are ψ, which increases from 0.0009 to 2.03; h which falls from 0.88 to 0.14; ρ_π which increases from 2.15 to 6.07; and the standard deviation of the technology shock which falls from 0.49 to 0.02. Hence, the so-called Great Moderation episode is due to a number of reasons: the parameters regulating private sector decisions change, the parameters of the interest rate rule are altered and the variability of at least one shock falls. The model without money produces a fall in output volatility across subsamples, but fails to capture the reduction in the volatility and the persistence of inflation. Here the only parameters significantly changing in the two samples are the interest rate smoothness parameter ρ_r and the inflation coefficient ρ_π, both of which increase. Thus, giving money a role allows the model to fit the time variations in the moments of the inflation process better.

6 Conclusions

The role that money has in shaping business cycles is a crucial but insufficiently investigated issue within the class of models nowadays used in academic discussions and policy analyses. To fill the gap, this paper investigates a number of questions using a small scale model where money plays a somewhat reduced form role and its parameters estimated by maximum likelihood.

Our results bring new light in several dimensions. We show that money is statistically important for domestic fluctuations in output and inflation. Depending on the country and the time period, money may matter directly, by affecting Euler equation and the Phillips curve, indirectly, by influencing the determination of nominal interest rate, or both. We document that the role of money is changing over time, both in the sense that estimates change magnitude and significance and that different channels become important. Since money does not stand-in for standard omitted suspects and since alternative monetary aggregate give similar conclusions, specification and measurement problems are unlikely to be crucial in delivering results. Overall, our estimates suggest that liquidity premia have limited importance and highlight the presence of time varying wedge between consumption and output. This wedge could be influenced by the real money stock, for example, because of asset market segmentation or participation constraints. Our estimates are also consistent with the idea that balance sheet effects, in the form of working capital requirements or direct credit constraints, may affect the determination of marginal costs and the link between marginal costs and the output gap. Finally, we show that the interpretation of the evidence may be distorted when money is not allowed to play a role in the model. Researchers could mistakenly interpret the pattern of impulse responses; erroneously measure the causes of inflation volatility and the reasons for output fluctuations; and they would have hard time to account for the fall in inflation variability and persistence that the US has experienced over the last 40 years if money is excluded from the model.

While our investigation delivers sharp conclusions, more works needs to be done to understand better the role of money in various economies. As we have mentioned, the specification used is consistent with several "structural" interpretations and disentangling them is crucial for policy purposes. Does real balances matter because balance sheet effects are important or because working capital requirements bind? Is asset segmentation a relevant feature of industrialized economies? Furthermore, our evidence is consistent with different channels being important at different point in time. Why this is the case? What kind of institutional changes are necessary to alter the channels through which money matter? Does financial market liberalizations, both within and across countries, have anything to do with what we observe? We plan to study these issues in future work.

References

[1] Alvarez, F., Lucas, R., Weber, W., 2001. Interest rates and inflation. American Economic Review, Papers and Proceedings, 91, 219-225.

[2] Andres, J., Lopez Salido, D. and Nelson, E., 2009. Money and the natural rate of interest: Structural estimates for the United States and the Euro area. Journal of Economic Dynamics and Control, 33, 758-776.

[3] Barthelemy, J., Clerc, L. and Marx, M., 2008. A two pillar DSGE monetary policy Model for the Euro Area. Bank of France working paper NER-R 219.

[4] Benhabib, J., Farmer, R., 2000. The monetary transmission mechanism. Review of Economic Dynamics, 3, 523-550.

[5] Campbell, J., Herkovitz, Z., 2006. The Role of Collateralized Household Debt In Macroeconomic Stabilization. Federal Reserve Bank of Chicago, manuscript.

[6] Canova, F., 2007. Methods for Applied Macroeconomic Research, Princeton University Press, Princeton, N.J.

[7] Canova, F., De Nicolo, G., 2002. Money Matters for Business Cycle Fluctuations in the G7. Journal of Monetary Economics, 49, 1131-1159.

[8] Canova, F., Gambetti, L., Pappa, E., 2007. The structural dynamics of output growth and inflation: some international evidence. Economic Journal, 117, C167-C191.

[9] Canova, F., Sala, L., 2009. Back to square one: identification issues in DSGE models. Journal of Monetary Economics, 56, 431-449.

[10] Canzoneri, M., Cumby, R., Diba, B., Lopez Salido, D. , 2008, Monetary aggregates and liquidity in a NeoWicksellian framework. Journal of Money, Credit and Banking, 40, 1667-1698.

[11] Chang, Y., Doh, T. Schorfheide, F., 2006, Non-stationary Hours in a DSGE model. Journal of Money, Credit and Banking, 39, 1357-1373.

[12] Chari, V.V., Kehoe, P. and McGrattan, E., 2002. Can sticky prices generate volatile and persistent real exchange rates. Review of Economic Studies, 69, 533-563.

[13] Christiano, L., Eichenbaum, M. and Evans, C., 1999. Monetary policy Shocks: what have we learned and to what end? in J. Taylor and M.Woodford (eds.) , Handbook of Macroeconomics, Elsevier, Amsterdam, 65-148.

[14] Christiano, L., Motto, M., Rostagno, M., 2006. Shocks, structures or policies? A comparison of the EU and the US, ECB manuscript.

[15] Clarida, R., Gali, J., Gertler, M. 2000. Monetary Policy Rule and Macroeconomic Stability: Evidence and Some Theory. Quarterly Journal of Economics, CXV, 147-180.

[16] Favara, and Giordani, P., 2009. Reconsidering the role of money for output, prices and interest rates. Journal of Monetary Economics, 56, 419-430.

[17] Fernandez Villaverde, J., 2009, The econometrics of DSGE models, CEPR working paper 7157.

[18] Gordon, D., Leeper E., 1994. The dynamic Impact of monetary policy: an exercise in tentative identification. Journal of Political Economy, 102, 1228-1247.

[19] Ireland, P., 2004. Money's role in the Monetary Business Cycle. Journal of Money, Credit and Banking, 36, 969-983.

[20] Justiniano, A., Primiceri, G., and Tambalotti, A., 2010. Investment shocks and the Business cycle. Journal of Monetary Economics, 57, 132-145.

[21] Leeper, E. and Roush, J., 2003. Putting M back in Monetary Policy. Journal of Money, Credit and Banking, 35, 1217-1256.

[22] Meltzer, A., 2001. The transmission process in Deutsche Bundesbank: The Monetary Transmission Process: Recent Development and Lessons for Europe, Palgrave, New York, 112-130.

[23] McCallum, B., 2001. Monetary policy analysis in models without money. Federal Reserve Bank of St. Louis, July/ August, 145-160.

[24] McCallum, B., 2008. How important is money in the conduct of Monetary Policy? A Comment, Journal of Money, Credit and Banking, 40, 1783-1790.

[25] Nakov, A., Pescatori, A., 2007. Oil and the Great Moderation. Journal of Money Credit and Banking, 42, 1-32.

[26] Nelson, E., 2002. Direct effects of money base on aggregate demand: theory and evidence. Journal of Monetary Economics, 49, 687-708.

[27] Nelson, E., 2008. Why money growth determines inflation in the long run. Answering Woodford Critique. Journal of Money Credit and Banking, 40, 1791-1814.

[28] Rudebusch, G. and Svensson, L., 2002, Eurosystem monetary targeting: lessons from US data. European Economic Review,46, 417-446.

[29] Sims, C.A., Zha, T., 2006. Were there Regime Switches in US Monetary Policy?. American Economic Review, 96, 54-81.

[30] Smets, F. and Wouters, R., 2007. Shock and frictions in US business cycle: a Bayesian DSGE approach. American Economic Review, 97, 586-606.

[31] Stock, J., Watson, M., 2005. Understanding changes in International Business Cycle Dynamics. Journal of the European Economic Association.

[32] Woodford, M., 2003. Interest and Prices. Foundation of a theory of Monetary Policy. Princeton University Press, Princeton.

[33] Woodford, M., 2008. How important in money in the conduct of monetary policy? Journal of Money Credit and Banking, 40, 1561-1598

Additional material to the paper
”Does money matter in shaping domestic business cycles? An international investigation”
Fabio Canova and Tobias Menz

Appendix A: Derivation of the log-linearized conditions

Household’s problem

The Lagrangian for the household’s maximization problem is given by

$$\begin{aligned}
\mathcal{L} &= E_0\Bigg\{\sum_{t=0}^{\infty}\beta^t\Bigg[a_t\left(U\left(x_t,\frac{M_t}{p_te_t}\right)-\eta n_t\right) \\
&+ \Lambda_t\left(M_{t-1}+T_t+B_{t-1}+W_tn_t+D_t-p_tc_t-\frac{B_t}{R_t}-M_t\right)\Bigg]\Bigg\}
\end{aligned}$$

where $x_t \equiv c_t - hc_{t-1}$ is the flow of consumption good and h is an external habit formation parameter. Letting real balances be denoted by $m_t = M_t/p_t$, the first order conditions are

$$c_t: \quad a_tU_1(x_t, m_t/e_t) = \Lambda_t p_t \tag{1}$$

$$n_t: \quad a_t\eta = \Lambda_t W_t \tag{2}$$

$$B_t: \quad \frac{\Lambda_t}{R_t} = \beta E_t\Lambda_{t+1} \tag{3}$$

$$M_t: \quad \Lambda_t - a_tU_2(x_t, m_t/e_t)\frac{1}{p_te_t} = \beta E_t\Lambda_{t+1} \tag{4}$$

Letting $w_t = W_t/p_t$ denote the real wage rate and combining (1) and (2) yields the optimal labor-leisure condition

$$\eta = w_tU_1(x_t, m_t/e_t) \tag{5}$$

Letting $\pi_t = p_t/p_{t-1}$ denote the gross inflation rate in period t and combining (1) and (3) yields the intertemporal Euler equation

$$a_t U_1(x_t, m_t/e_t) = \beta R_t E_t \left[\frac{a_{t+1}U_1(x_{t+1}, m_{t+1}/e_{t+1})}{\pi_{t+1}}\right] \tag{6}$$

Finally, combining (1), (3) and (4) yields the money demand equation

$$(R_t - 1)e_t U_1(x_t, m_t/e_t) = R_t U_2(x_t, m_t/e_t) \tag{7}$$

Firm's problem

The first order condition from the firm's maximization of future discounted streams of real dividends is given by

$$\begin{aligned} 0 \;&=\; (1-\theta)\left(\frac{p_t^i}{p_t}\right)^{-\theta}\left(\frac{y_t}{p_t}\right) + \theta\left(\frac{p_t^i}{p_t}\right)^{-\theta-1}\left(\frac{y_t w_t}{z_t p_t}\right) - \phi\left(\frac{p_t^i}{\pi^s p_{t-1}^i} - 1\right)\left(\frac{y_t}{\pi^s p_{t-1}^i}\right) \\ &\quad + \beta\phi E_t\left[\frac{a_{t+1}U_1(x_{t+1}, m_{t+1}/e_{t+1})}{a_t U_1(x_t, m_t/e_t)}\left(\frac{p_{t+1}^i}{\pi^s p_t^i} - 1\right)\left(\frac{y_{t+1}p_{t+1}^i}{\pi^s (p_t^i)^2}\right)\right] \end{aligned}$$

Since we consider a symmetric equilibrium in which all firms behave identically, $p_t^i = p_t$, the above equation simplifies to

$$\begin{aligned} 0 \;&=\; (1-\theta)\left(\frac{y_t}{p_t}\right) + \theta\left(\frac{y_t w_t}{z_t p_t}\right) - \phi\left(\frac{\pi_t}{\pi^s} - 1\right)\left(\frac{y_t \pi_t}{\pi^s p_t}\right) \\ &\quad + \beta\phi E_t\left[\frac{a_{t+1}U_1(x_{t+1}, m_{t+1}/e_{t+1})}{a_t U_1(x_t, m_t/e_t)}\left(\frac{\pi_{t+1}}{\pi^s} - 1\right)\left(\frac{y_{t+1}\pi_{t+1}}{\pi^s p_t}\right)\right] \end{aligned}$$

Dividing by y_t/p_t and substituting for w_t from (5) yields

$$\begin{aligned} \theta - 1 \;&=\; \frac{\theta\eta}{z_t U_1(x_t, m_t/e_t)} - \phi\left(\frac{\pi_t}{\pi^s} - 1\right)\frac{\pi_t}{\pi^s} \\ &\quad + \beta\phi E_t\left[\frac{a_{t+1}U_1(x_{t+1}, m_{t+1}/e_{t+1})}{a_t U_1(x_t, m_t/e_t)}\left(\frac{\pi_{t+1}}{\pi^s} - 1\right)\left(\frac{\pi_{t+1}y_{t+1}}{\pi^s y_t}\right)\right] \end{aligned} \tag{8}$$

Log-linearization

In what follows, a hat-variable denotes percentage-deviation from the steady state, for example, $\hat{R}_t = \log(R_t/R^s) \simeq \frac{dR_t}{R^s}$. Since all output is consumed in equilibrium, $y_t = c_t$. Furthermore, equation 6 in the steady state collapses to $1 = \beta\frac{R^s}{\pi^s}$. Finally, for notational convenience, we omit the arguments of the derivatives of the utility function , i.e. we will use U_1 for $U_1(x^s, m^s/e^s)$ and so on.

Euler equation

Log-linearizing (6) around a deterministic steady state we obtain

$$U_1(\hat{R}_t + E_t\hat{a}_{t+1} - E_t\hat{\pi}_{t+1}) + U_{11}y^s(E_t\hat{y}_{t+1} - h\hat{y}_t) + U_{12}\frac{m^s}{e^s}(E_t\hat{m}_{t+1} - E_t\hat{e}_{t+1})$$
$$= U_1\hat{a}_t + U_{11}y^s\left(\hat{y}_t - h\hat{y}_{t-1}\right) + U_{12}\frac{m^s}{e^s}\left(\hat{m}_t - \hat{e}_t\right)$$

Collecting terms gives

$$\begin{aligned} U_{11}y^s(1+h)\hat{y}_t &= U_{11}y^s E_t\hat{y}_{t+1} + U_{11}y^s h\hat{y}_{t-1} \\ &+ U_1[\hat{R}_t - E_t\hat{\pi}_{t+1} - (\hat{a}_t - E_t\hat{a}_{t+1})] \\ &- U_{12}\frac{m^s}{e^s}[(\hat{m}_t - \hat{e}_t) - (E_t\hat{m}_{t+1} - E_t\hat{e}_{t+1})] \end{aligned}$$

Defining

$$\omega_1 \equiv -\frac{U_1(x^s, m^s/e^s)}{y^s U_{11}(x^s, m^s/e^s)} \quad \text{and} \quad \omega_2 \equiv -\frac{(m^s/e^s)U_{12}(x^s, m^s/e^s)}{y^s U_{11}(x^s, m^s/e^s)}$$

the linearized Euler equation is

$$\begin{aligned} \hat{y}_t &= \frac{1}{1+h}E_t\hat{y}_{t+1} + \frac{h}{1+h}\hat{y}_{t-1} \\ &- \frac{\omega_1}{1+h}\left[\hat{R}_t - E_t\hat{\pi}_{t+1} - (\hat{a}_t - E_t\hat{a}_{t+1})\right] \\ &+ \frac{\omega_2}{1+h}\left[(\hat{m}_t - \hat{e}_t) - (E_t\hat{m}_{t+1} - E_t\hat{e}_{t+1})\right] \end{aligned} \tag{9}$$

Money demand

Linearizing the equation (7) yields

$$\begin{aligned}R^s e^s U_1 \hat{R}_t + (R^s-1)e^s U_1 \hat{e}_t + (R^s-1)e^s \left[y^s U_{11}(\hat{y}_t - h\hat{y}_{t-1}) + \frac{m^s}{e^s} U_{12}(\hat{m}_t - \hat{e}_t) \right] \\ = R^s U_2 \hat{R}_t + R^s y^s U_{21}(\hat{y}_t - h\hat{y}_{t-1}) + R^s m^s/e^s U_{22}(\hat{m}_t - \hat{e}_t)\end{aligned}$$

In the steady state $(R^s-1)e^s U_1 = R^s U_2$. Hence, collecting terms we obtain

$$\begin{aligned}&[(R^s-1)e^s U_{12} - R^s U_{22}] \frac{m^s}{e^s} \hat{m}_t \\ &\qquad = [R^s U_{21} - (R^s-1)e^s U_{11}] y^s (\hat{y}_t - h\hat{y}_{t-1}) - \frac{R^s}{R^s-1} U_2 \hat{R}_t \\ &\qquad + \left[((R^s-1)e^s U_{12} - R^s U_{22}) \frac{m^s}{e^s} - (R^s-1)e^s U_1 \right] \hat{e}_t\end{aligned}$$

Defining

$$\begin{aligned}\gamma_2 &= \frac{R^s}{(R^s-1)(m^s/e^s)} \left(\frac{U_2(x^s, m^s/e^s)}{(R^s-1)e^s U_{12}(x^s, m^s/e^s) - R^s U_{22}(x^s, m^s/e^s)} \right) \\ \gamma_1 &= \left(R^s - 1 + R^s \omega_2 \frac{y^s}{m^s} \right) \left(\frac{\gamma_2}{\omega_1} \right)\end{aligned}$$

we can write the linearized money demand equation as

$$\hat{m}_t = \gamma_1(\hat{y}_t - h\hat{y}_{t-1}) - \gamma_2 \hat{R}_t + (1 - (R^s-1)\gamma_2)\hat{e}_t \tag{10}$$

Phillips curve

When linearizing the optimal pricing rule (8), the equation simplifies considerably because the price adjustment cost term is zero in the steady state and we are left with $\theta - 1 = \frac{\theta\eta}{z^s U_1(x^s, m^s/e^s)}$. Hence

$$\begin{aligned}0 &= -(\theta-1)\hat{z}_t - (\theta-1)\frac{U_{11}y^s}{U_1}(\hat{y}_t - h\hat{y}_{t-1}) - (\theta-1)\frac{U_{12}\frac{m^s}{e^s}}{U_1}(\hat{m}_t - \hat{e}_t) \\ &\quad -\phi\hat{\pi}_t + \beta\phi E_t \hat{\pi}_{t+1}\end{aligned}$$

Solving for $\hat{\pi}_t$, defining $\psi = (\theta - 1)/\phi$ and using the definitions of ω_1 and ω_2:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \psi \left(\frac{1}{\omega_1}(\hat{y}_t - h\hat{y}_{t-1}) - \frac{\omega_2}{\omega_1}(\hat{m}_t - \hat{e}_t) - \hat{z}_t \right) \tag{11}$$

Policy rule

Log-linearizing the assumed policy rule implies

$$\begin{aligned} \hat{R}_t &= \rho_r \hat{R}_{t-1} + (1-\rho_r)\rho_y \hat{y}_{t-p} + (1-\rho_r)\rho_\pi \hat{\pi}_{t-p} \\ &+ (1-\rho_r)\rho_m \Delta(\hat{m}_{t-p} + \hat{\pi}_{t-p}) + \hat{\epsilon}_t \end{aligned} \tag{12}$$

Summarizing, the log-linearized optimality conditions are

$$\begin{aligned} \hat{y}_t &= \frac{1}{1+h} E_t \hat{y}_{t+1} + \frac{h}{1+h}\hat{y}_{t-1} - \frac{\omega_1}{1+h}\left[\hat{R}_t - E_t\hat{\pi}_{t+1} - (\hat{a}_t - E_t\hat{a}_{t+1})\right] \\ &+ \frac{\omega_2}{1+h}\left[(\hat{m}_t - \hat{e}_t) - (E_t\hat{m}_{t+1} - E_t\hat{e}_{t+1})\right] \qquad (13) \\ \hat{m}_t &= \gamma_1(\hat{y}_t - h\hat{y}_{t-1}) - \gamma_2\hat{R}_t + (1 - (R^s - 1)\gamma_2)\hat{e}_t \qquad (14) \\ \hat{\pi}_t &= \beta E_t\hat{\pi}_{t+1} + \psi\left(\frac{1}{\omega_1}(\hat{y}_t - h\hat{y}_{t-1}) - \frac{\omega_2}{\omega_1}(\hat{m}_t - \hat{e}_t) - \hat{z}_t\right) \qquad (15) \\ \hat{R}_t &= \rho_r\hat{R}_{t-1} + (1-\rho_r)\rho_y\hat{y}_{t-p} + (1-\rho_r)\rho_\pi\hat{\pi}_{t-p} \\ &+ (1-\rho_r)\rho_m\Delta(\hat{m}_{t-p} + \hat{\pi}_{t-p}) + \hat{\epsilon}_t \qquad (16) \end{aligned}$$

where

$$\begin{aligned} \omega_1 &= -\frac{U_1(x^s, \frac{m^s}{e^s})}{y^s U_{11}(x^s, \frac{m^s}{e^s})} \qquad (17) \\ \omega_2 &= -\frac{m^s}{e^s}\frac{U_{12}(x^s, \frac{m^s}{e^s})}{y^s U_{11}(x^s, \frac{m^s}{e^s})} \qquad (18) \\ \gamma_2 &= \frac{R^s}{(R^s - 1)(m^s/e^s)}\left(\frac{U_2(x^s, \frac{m^s}{e^s})}{(R^s-1)e^s U_{12}(x^s, \frac{m^s}{e^s}) - R^s U_{22}(x^s, \frac{m^s}{e^s})}\right) \qquad (19) \\ \gamma_1 &= (R^s - 1 + R^s\omega_2\frac{y^s}{m^s})(\frac{\gamma_2}{\omega_1}) \qquad (20) \\ \psi &= \frac{\theta - 1}{\phi} \qquad (21) \end{aligned}$$

Appendix B: Estimates in alternative specifications

	Basic	$\omega_2 = 0$	$\rho_m = 0$	$\omega_2 = h = 0$	$\rho_m = h = 0$
ψ	2.0427	1.0055	0.0136	1.3679	1.9232
	(0.4450)	(0.2765)	(0.0102)	(0.3586)	(0.4926)
	0.9900	0.9900	0.9900	0.9900	0.9900
ω_2	-0.9794	0.0000	-0.1441	0.0000	-0.1987
	(0.2861)		(0.1940)		(0.1816)
γ_2	0.3647	0.4039	0.2854	0.3428	0.3520
	(0.0959)	(0.1044)	(0.0769)	(0.1020)	(0.0609)
h	0.1447	0.7324	0.8330	0.0000	0.0000
	(0.1006)	(0.0975)	(0.1431)		
ρ_r	0.6281	0.6408	0.7645	0.6375	0.6101
	(0.0378)	(0.0404)	(0.0979)	(0.0386)	(0.0412)
ρ_y	0.4769	0.5066	0.6811	0.4144	0.4584
	(0.1181)	(0.1049)	(0.0995)	(0.1024)	(0.1144)
ρ_p	6.0743	6.2789	0.4611	6.4268	6.8726
	(0.5099)	(0.5616)	(1.3736)	(0.5534)	(0.5774)
ρ_m	1.7993	1.8759	0.0000	1.8526	0.0000
	(0.2543)	(0.2789)		(0.2837)	
ρ_a	0.9862	0.9563	0.9327	0.9854	0.9878
	(0.0301)	(0.0156)	(0.0112)	(0.0007)	(0.0099)
ρ_e	0.9807	0.9732	0.9714	0.9800	0.9812
	(0.0332)	(0.0118)	(0.0111)	(0.0096)	(0.0126)
ρ_z	0.9517	0.6153	0.7805	0.9444	0.9560
	(0.0279)	(0.1048)	(0.1110)	(0.0336)	(0.0267)
σ_r	0.0092	0.0091	0.0098	0.0093	0.0103
	(0.0835)	(0.0902)	(0.0964)	(0.0761)	(0.0876)
σ_a	0.4849	0.1677	0.1164	0.4688	0.6066
	(3.5114)	(0.4051)	(0.2162)	(0.0926)	(1.4836)
σ_e	0.0088	0.0085	0.0091	0.0085	0.0083
	(0.0714)	(0.0742)	(0.0784)	(0.0657)	(0.0600)
σ_z	0.0029	0.0042	0.0417	0.0041	0.0030
	(0.1013)	(0.0989)	(0.3479)	(0.0888)	(0.0960)
Log L	1589.29	1578.42	1532.75	1573.88	1569.21

Table 1: Comparison with Ireland, US, 1980:1-2008:2. In the estimation β is calibrated at 0.99 and ω_1 at 2.

	M1	M0	Credit	SP500	House Prices	Oil	Goverment
ψ	1.9317	1.8982	1.4166	0.0533	0.0513	0.0761	0.3916
	(0.5031)	(0.2721)	(0.3635)	(0.0100)	(0.0222)	(0.0768)	(0.0505)
	0.9900	0.9900	0.9900	0.9900	0.9900	0.9900	0.9900
ω_2	-0.9454	-1.0499	-1.1056	0.3817	0.1307	0.1986	-0.4426
	(0.2646)	(0.2976)	(0.4825)	(0.3546)	(0.0895)	(1.5340)	(0.1223)
γ_2	0.3979	0.3373	0.0215	0.3431	1.0765	1.0870	0.7937
	(0.1428)	(0.0823)	(0.0389)	(0.4429)	(0.3796)	(2.9938)	(0.2274)
h	0.1446	0.1704	0.8380	0.8935	0.9610	0.9790	0.6957
	(0.1096)	(0.1193)	(0.0955)	(2.2193)	(0.2081)	(3.1747)	(0.0781)
ρ_r	0.6524	0.6475	0.6002	0.7819	0.5332	0.0033	0.6504
	(0.0406)	(0.0387)	(0.0417)	(0.1714)	(0.0811)	(0.1841)	(0.0409)
ρ_y	0.5959	0.5468	0.2831	1.1451	0.7566	0.2181	0.2347
	(0.1304)	(0.1188)	(0.1122)	(0.9329)	(0.1223)	(2.8471)	(0.0697)
ρ_p	6.6062	7.0230	7.1866	3.9549	4.3659	1.0174	4.0976
	(0.6091)	(0.6586)	(0.5458)	(4.3341)	(0.6476)	(2.3430)	(0.3428)
ρ_m	0.8487	1.7138	1.1908	0.9229	2.0508	1.8687	-0.1065
	(0.2186)	(0.2946)	(0.1874)	(1.9861)	(0.4496)	(3.1508)	(0.0485)
ρ_a	0.9897	0.9637	0.9673	0.1877	-0.1515	2.4511	0.9709
	(0.0007)	(0.0066)	(0.0235)	(0.1968)	(0.1847)	(3.0195)	(0.0140)
ρ_e	0.9893	0.9510	0.9706	0.9082	0.9310	0.9144	0.9529
	(0.0067)	(0.0097)	(0.0072)	(0.0435)	(0.0045)	(0.1397)	(0.0162)
ρ_z	0.9481	0.9172	0.4928	0.9856	0.9393	0.9546	0.7599
	(0.0297)	(0.0200)	(0.1377)	(0.0274)	(0.0332)	(0.2492)	(0.0554)
σ_r	0.0096	0.0093	0.0092	0.7903	0.8037	0.4226	0.0098
	(0.0825)	(0.0831)	(0.0791)	(0.0823)	(0.0778)	(1.3992)	(0.0688)
σ_a	0.6772	0.2034	0.2453	0.0075	0.1475	0.0054	0.1922
	(0.1855)	(0.2040)	(0.8852)	(0.7180)	(0.1206)	(2.6459)	(0.5538)
σ_e	0.0149	0.0107	0.0150	0.0086	0.0135	0.0261	0.0310
	(0.0680)	(0.0644)	(0.0722)	(0.1867)	(0.1691)	(2.4934)	(0.0531)
σ_z	0.0029	0.0028	0.0042	0.0975	0.1216	0.0996	0.0057
	(0.0925)	(0.0978)	(0.1042)	(0.5791)	(0.1024)	(2.2782)	(0.1022)
Log L	1516.48	1456.78	1514.66	1704.94	1869.09	1438.36	2324.89

Table 2: Robustness checks, US, sample 1980:1-2008:2. In the estimation β is calibrated at 0.99 and ω_1 at 2.

	$\omega_1 = 1$			$\omega_1 = 2$		
	Full sample	Sample 1	Sample 2	Full sample	Sample 1	Sample 2
ψ	0.1494	0.0175	0.5829	0.7433	0.0009	2.0427
	(0.0485)	(0.0110)	(0.1574)	(0.1595)	(0.0013)	(0.4450)
ω_2	-0.1155	-0.2313	-0.6372	-0.5112	24.1444	-0.9794
	(0.3202)	(0.3495)	(0.1257)	(0.4827)	(8.1261)	(0.2861)
γ_2	0.0881	0.0970	0.3259	0.0746	- 0.0278	0.3647
	(0.0871)	(0.1779)	(0.0813)	(0.0683)	(0.0283)	(0.0959)
h	0.6972	0.9554	0.2078	0.7169	0.8846	0.1447
	(0.0777)	(0.1010)	(0.1163)	(0.0696)	(0.0262)	(0.1006)
ρ_r	0.7190	0.6711	0.6408	0.6822	0.8080	0.6281
	(0.0376)	(0.0824)	(0.0329)	(0.0352)	(0.0707)	(0.0378)
ρ_y	0.2460	0.2769	0.4669	0.1774	0.3571	0.4769
	(0.0909)	(0.0882)	(0.1135)	(0.0690)	(0.0938)	(0.1181)
ρ_p	4.0873	2.3676	6.0247	4.3702	2.1510	6.0743
	(0.3095)	(0.3252)	(0.4626)	(0.3835)	(0.4814)	(0.5099)
ρ_m	1.4961	0.2139	1.7117	1.5788	-0.0363	1.7993
	(0.2490)	(0.3201)	(0.2578)	(0.1957)	(0.2187)	(0.2543)
ρ_a	0.9532	0.9866	0.9611	0.9630	0.9204	0.9862
	(0.0171)	(0.0042)	(0.0122)	(0.0060)	(0.0291)	(0.0301)
ρ_e	0.9773	0.9841	0.9746	0.9774	0.9689	0.9807
	(0.0087)	(0.0081)	(0.0117)	(0.0085)	(0.0002)	(0.0332)
ρ_z	0.7451	0.8395	0.9232	0.7005	0.8700	0.9517
	(0.0472)	(0.0806)	(0.0196)	(0.0623)	(0.0803)	(0.0279)
σ_r	0.0092	0.0071	0.0091	0.0099	0.0067	0.0092
	(0.0660)	(0.0865)	(0.0756)	(0.0672)	(0.1172)	(0.0835)
σ_a	0.1367	0.3990	0.1852	0.1625	0.1316	0.4849
	(0.4022)	(0.6678)	80.3140)	(0.1825)	(0.1683)	(3.5114)
σ_e	0.0092	0.0104	0.0089	0.0093	0.0110	0.0088
	(0.0526)	(0.0745)	(0.0687)	(0.0442)	(0.0676)	(0.0714)
σ_z	0.0128	0.0317	0.0066	0.0054	0.4911	0.0029
	(0.1092)	(1.0556)	(0.1232)	(0.0811)	(1.5415)	(0.1013)
Log L	2722.96	1165.78	1603.98	2674.93	1180.82	1589.29
Sample size	198	198	83	83	115	115

Table 3: Parameter estimates for US. In the estimation β is calibrated at 0.99.