Credit Constraints, Firms' Precautionary Investment, and the Business Cycle*

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Abstract

This paper studies the macroeconomic implications of firms' investment composition choices in the presence of credit constraints. Following a negative and persistent aggregate productivity shock, firms shift into short-term investments because they produce more pledgeable output and because they help alleviate future borrowing constraints. This produces a short-run dampening of the effects of the shock, at the expense of lower long-term investment and future output, relative to an economy with no credit market imperfections. The effects are exacerbated by a steepening of the term structure of interest rates that further encourages a shift towards short-term investments in the short-run. Small temporary shocks to the severity of financing frictions generate large and long-lasting effects on output through their impact on the composition of investment. A positive financial shock produces much stronger effects than an identical negative shock, while the responses to positive and negative shocks to aggregate productivity are roughly symmetric. Finally, the paper introduces a novel explanation for the countercyclicality of financing constraints of firms.

Keywords: Investment composition; Financial frictions; Business cycles; Idiosyncratic production risk; Firm heterogeneity

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1 Introduction

How do financial constraints affect the dynamics of the aggregate composition of investment? Does the behavior of the composition of investment in the presence of financing constraints have important consequences for how such frictions influence the response of an economy to shocks? Firm managers typically face a choice between multiple projects that may differ in terms of their duration, liquidity risk and pledgeability of returns, amongst other characteristics, and financing constraints may have important implications for firms' investment composition choices along these dimensions. Indeed firm-level empirical work has found that credit constrained firms tend to invest in safer and shorter-term projects[1], display a more procyclical pattern of their share of R&D investment[2], and purchase used, rather than new, capital[3]. At the aggregate level, Dew-Becker (2011) shows that the average duration of aggregate investment falls in downturns, and Aghion et al. (2010) provide evidence that long-term investment as a share of total investment decreases following shocks that can be expected to make firms more credit constrained, and also document that this effect is stronger for less financially developed economies.

In this paper we perform a quantitative theoretical exploration of the implications for aggregate investment and output dynamics of the interaction between investment composition choice, idiosyncratic risk and financing constraints. More specifically, the dimension of project choice that will be studied is the duration or maturity of investment projects, which in turn generates endogenous and time-varying implications for other project characteristics such as liquidity risk and pledgeability of returns. We first develop a model which produces results consistent with the evidence described above and use it to address several questions. Does the interaction between investment choice, idiosyncratic risk and financing constraints dampen or amplify the short-run effects of aggregate productivity shocks? Does it influence how the effects of productivity shocks propagate through time? How does it influence the impact of financial shocks?

These questions are dealt with by analyzing a dynamic general equilibrium model of an economy in which heterogeneous firms produce a consumption good using a short-term and a long-term technology and labor provided by a representative household. Firms suffer idiosyncratic liquidity shocks proportional to their total capital stock and face an endogenous constraint that limits the amount of debt finance they can receive to a fraction of the present value of their lowest possible realization of future earnings. The nature of the liquidity shock means that long-term investment increases future liquidity risk more than short-term investment, and the borrowing constraint implies that long-term investment typically produces less pledgeable output on the margin than short-term

[1] See Chevalier and Scharfstein (1996), Andrade and Kaplan (1998), Peyer and Shivdasani (2001), and Ahn et al. (2006).

[2] See Aghion et al. (2012).

[3] See Eisfeldt and Rampini (2007).

investment.

Firms have an incentive to shift investment towards the long-term project in recessions because its returns are relatively acyclical whereas short term activities offer poor returns in downturns. In the presence of financing constraints, this decreases the amount of pledgeable output for a given level of total investment and renders the severity of these constraints countercyclical. This explanation for the countercyclicality of financing constraints is the first result of the paper and provides an alternative to the two main explanations offered in the literature, based on countercyclical agency costs on one hand[4], and collateral constraints and lack of indexation of debt contracts on the other[5].

The second contribution of the paper is the description and quantitative evaluation of a novel mechanism that delivers short-run dampening of productivity shocks and long-run propagation. If a negative unexpected aggregate productivity shock hits the economy, credit constraints tighten for firms and this causes constrained firms to shift into short-term investment because it produces relatively more pledgeable output.[6] Credit constraints are expected to remain tight for some time and this anticipation of future constraints further encourages constrained firms to shift into short-term investment because long-term investment carries more liquidity risk and increases the probability of suffering future credit constraints.[7] The interaction of investment duration choice and financing constraints acts as a financial decelerator that dampens the contemporaneous response to the negative productivity shock.

The behavior of the interest rate exacerbates these effects. In the analysis we compare the transitional dynamics following the negative temporary shock of an economy where the interest rate is assumed fixed with one in which it is endogenous. Interest rates initially drop moderately following the shock for very few periods, and subsequently rise sharply, so the term structure of interest rates steepens considerably following the shock. This makes long-term investment relatively less attractive than short-term investment following the shock, and is an additional force that encourages a further shift out of long-term investment in the short run. While the behavior of the interest rate is similar in economies with and without credit frictions, their effect on the composition of investment

[4] As in Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), or Christiano et al. (2010).

[5] As in Kiyotaki and Moore (1997) or Iacoviello (2005).

[6] Credit constraints tighten following the negative productivity shock due to the novel mechanism outlined above based on the procyclicality of pledgeable output as a result of the incentive to shift to long-term activities in downturns, but also because of two other more standard reasons. One is the large decrease in firms' wealth following the unexpected shock. Another is the increase in interest rates which decreases the present value of pledgeable output and tightens borrowing constraints.

[7] The importance for firm behavior of the anticipation of future financing constraints has been the focus of recent work. Surveys by Graham and Harvey (2001) and Bancel and Mittoo (2004) find that CFOs consider financial flexibility (having enough internal funds to avoid having to fore-go positive Net Present Value projects in the future) to be the primary determinant of their policy decisions. Almeida, Campello and Weisbach (2004) report that the expectation of future financing problems significantly affects firms' investment policies, and Caggese and Cunat (2007) find that it significantly impacts hiring decisions.

is stronger in the presence of credit constraints because the large rise in interest rates in the medium-term tightens future borrowing constraints and the anticipation of such constraints induces a precautionary reaction of firms in the short-run to further shift out of long-term investment.[8] In the version of the model with fixed interest rates the maximum output drop following the negative technology shock is 22% smaller in a model featuring financing constraints relative to one in which financial markets are frictionless. When we endogenize the interest rate, the dampening effect rises to 31%.

The decrease in the share of long-term investment propagates the effect of the shock through time. Convex costs of adjustment in investment mean that the large initial fall in the stock of long-term capital takes a long time to recover. The fall is larger with financing constraints, and the recovery is slower because adjustment costs and investment are hindered by the constraints on financing some firms face. As a result, a trade-off arises between contemporaneous amplification and long-term propagation of the effects of shocks; stronger dampening is associated to larger propagation.

A financial shock, in the form of a tightening of financing constraints, generates a large shift out of long-term investment. Even if the financial shock is short-lived, the large drop in the stock of long-term capital takes a long time to recover and propagates the effects of the financial shock for many periods after the severity of financing constraints subsides. Also in this case, general equilibrium effects through the response of the interest rate result in significantly larger effects. Following the shock the slope of the term structure of interest rates increases sharply and further encourages a shift out of long-term investment. A similar contemporaneous dampening effect is at play in the case of a financial shock as well because the shift into short-term investment raises current output at the expense of future output.

The nature of our solution method allows us to study whether the sign of the shock influences the size and shape of the response of the economy to shocks. While the reaction to positive and negative technology shocks is relatively symmetric, the reaction to a tightening and a loosening of credit constraints is highly asymmetric. A loosening of credit constraints generates an increase in output more than three times the size, in absolute value, than the decrease in output following a symmetric tightening of constraints. The intuition for this large asymmetry is that in equilibrium firms converge broadly into two groups, one in which firms hold few assets and find it hard to invest and grow out of their credit constraints because they are severely credit constrained, and a group of older firms that hold a large amount of assets and are relatively unconstrained. A neg-

[8] At the heart of the general equilibrium effects is a pecuniary externality in the investment duration choice. When firms decide how much to invest in the long-term project, they do not internalize the impact of their choices on future interest rates and thereby on current and future tightness of borrowing constraints and investment choices of other firms. In this sense, this externality generates a dynamic macroeconomic complementarity: the anticipation of low output and high interest rates in the future feeds back into low long-term investment in the present and low future output. A similar mechanism in the context of entrepreneurial risk-taking is identified by Angeletos and Calvet (2006).

ative financial shock hurts credit constrained firms' total investment, which was already relatively small in size before the shock, and produces a moderate drop in output, but a positive shock enables these firms to overcome their constraints after a few periods and significantly increase their investment, resulting in a large output increase. We are able to capture this strong asymmetry in the response to financial shocks because our framework allows for credit constraints to bind occasionally, considers firm heterogeneity along two important dimensions, and uses a solution method that can capture non-linearities in the response to shocks. These features, in addition to investment composition choice, thus seem important when constructing models designed to study the macroeconomic implications of credit constraints.

A large body of research has studied the role of firms' financing frictions in amplifying business cycles. Most of this work focuses on how firms' investment capacity is affected by tighter borrowing constraints in recessions or following a tightening of monetary policy, either directly through a balance sheet channel (Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke at al. (1999)) or indirectly through a contraction in the supply of intermediated finance (Holmstrom and Tirole (1997), Gertler and Kiyotaki (2010)). Common to most of these models is the assumption that firms can invest in only one type of project, which precludes analyzing questions regarding the composition of investment. In addition, these models typically assume permanently binding credit constraints, which, in combination with the assumption of lack of investment type choice, means that the effect of the anticipation of future constraints is limited to general equilibrium effects that affect entrepreneurs through asset or product prices. Entrepreneurs who have an incentive to insure against future credit constraints are unable to do so because their possible actions are limited to investing as much as their permanently binding constraints allow in their single investment opportunity.

Matsuyama (2007) and Aghion et al. (2010) are two exceptions in this literature. They both address the behavior of the composition of investment in overlapping generations models with homogeneous entrepreneurs that suffer from financial constraints, although they mostly focus on questions related to economic growth. Matsuyama (2007) considers one-period investment projects that differ exogenously in their investment size, the pledgeability of their returns, and their productivity, and uses the framework to explain a wide range of phenomena related to economic growth and development. The focus in our paper instead is on short-run aggregate dynamics and on duration as a primary source of investment project heterogeneity. Furthermore, in our paper differences in duration deliver endogenous and time-varying differences in pledgeability, which is assumed exogenously in their work, and liquidity risk, which is absent. Aghion et al. (2010) on the other hand assume that entrepreneurs can invest in a short-term project that takes one period to complete and in a long-term project that takes two periods to complete, suffers from liquidity risk and has a positive externality on the productivity of future generations. Financial frictions discourage investment in long-term projects and tend to make them procyclical, and this is the source of a mechanism that can successfully explain the documented negative correlation between mean growth and volatility. Common to our paper is the notion that long-term projects generate more liquidity risk. But our paper differs in several key modeling choices, in that it tries to develop a more realistic framework amenable to quantitative evaluation and comparison to the data, and in its main results. To highlight some of the most relevant differences, first, the short run implications in their model, that more severe credit frictions amplify the response of output to exogenous business cycle shocks, is the opposite to ours, in which more severe credit frictions dampen the response.[9]Second, one key modeling choice distinction is that, in their model, long-term projects (which could be also interpreted as riskier, rather than longer-term, projects) are assumed to be more productive and to deliver this high productivity fast. Third, this modeling choice has another important implication, which is that in their model firms do not accumulate long-term capital. In our model instead many firms hold large stocks of long-term capital, which by its nature is totally or partially irreversible and depreciates slowly, and this has a large impact on the short-run reaction of long-term investment to shocks. Fourth, in their model the amplification and propagation effects operate entirely through a pure externality. While this is surely an important channel, one would imagine that the most important effects work through future output and productivity changes that the firm internalizes. Finally, relative to both Matsuyama (2007) and Aghion et al. (2010), our paper introduces long-lived firms and allows for firm heterogeneity in the dimensions of asset holdings and long-term capital holdings, enabling us to do a realistic quantitative evaluation of the importance of our proposed mechanism.[10]

Long-term investment in our paper is modeled as the production of units of a capital good that is firm-specific. Firm-specificity of capital introduces irreversibility in investment, and relates our results to the literature studying how credit constraints interact with investment irreversibility to exacerbate aggregate fluctuations. Caggese (2007) studies a partial equilibrium setup in which firms invest in variable and fixed capital, and there is total irreversibility in fixed investment. Khan and Thomas (2011) introduce a dynamic stochastic general equilibrium framework to study the interaction of collateral constraints and partial irreversibility of investment, but do not address issues of composition of investment.

[9]Comparing this result is complicated by the fact that they do not perform a quantitative analysis of their model, but the differences are not a matter of timing; in our model, dampening lasts for around 15 quarters, relative to an economy with no credit frictions, which means that the result is not limited to very high frequency dynamics.

[10]Introducing firm-level heterogeneity allows us, amongst other things, to generate a realistic distribution of the severity of firms' credit constraints and to capture the possibility that, in a general equilibrium context, financially unconstrained firms react in such a way that they compensate for constrained firms' behavior and aggregate dynamics are hardly affected by the presence of financing constraints.

Finally, the corporate finance literature has studied how financial constraints may impact aspects of firms' investment other than the quantity of investment. While most of this work is empirical[11], some theoretical papers have also addressed this topic. In Almeida et al. (2011), firms that expect future financing constraints prefer to invest in more liquid, shorter-term and safer assets. In Eisfeldt and Rampini (2007), firms that suffer from credit constraints have a preference for used capital, which requires a lower up-front investment than new capital.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 discusses the calibration of the model and analyzes firms' optimal decisions in the steady state. Section 4 presents the main results of the model concerning the reaction of the economy to productivity shocks, and section 5 extends the analysis to financial shocks and asymmetries in the response to shocks. Section 6 concludes.

2 Model

We introduce an infinite-horizon, discrete time, perfect foresight model of an economy populated by a continuum of identical households and a continuum of financially constrained firms, both of measure one. There are three goods in this economy; short-term capital, long-term capital, and a homogenous final good that can be used for consumption and investment. Firms are heterogeneous along two dimensions; their holdings of the final good, and their holdings of long-term capital. We begin our description of the economy with a detailed study of the optimization problem facing each firm, then follow with a brief discussion of households and equilibrium.

2.1 Firms

Production opportunities

We assume a large number of firms, each producing the final good via two production processes, both of which use capital (short-term or long-term) and labor.

A firm can invest in a *short-term* production opportunity that converts $k_{S,t+1}$ units of short-term capital into units of the final good next period using the technology

$$f_S(k_{S,t+1}) = \theta_{S,t+1} k_{S,t+1}^{\alpha_S},$$

[11]Examples of empirical papers are Chevalier and Scharfstein (1996), Andrade and Kaplan (1998), Peyer and Shivdasani (2001), Ahn et al. (2006), Eisfeldt and Rampini (2007) and Aghion et al. (2012).

where $\theta_{S,t+1}$ is an aggregate productivity term given by

$$\theta_{S,t+1} = \theta_S \theta_{t+1},$$

and where θ_S is a constant productivity parameter of the short-term technology and θ_{t+1} is an aggregate productivity factor common to both short and long-term production technologies. Investment in the short-term technology depreciates fully in one period.

A firm can also invest in a *long-term* production opportunity that converts $k_{L,t+1}$ units of long-term capital into units of the final good next period using the technology

$$f_L(k_{L,t+1}) = \theta_{L,t+1} k_{L,t+1}^{\alpha_L}$$

where $\theta_{L,t+1}$ is an aggregate productivity term given by

$$\theta_{L,t+1} = \theta_L \theta_{t+1}$$

and where θ_L is a constant productivity parameter of the long-term technology. Long-term capital is firm-specific and depreciates at a rate of δ_L, where $0 < \delta_L < 1$. Accumulation of long-term capital follows the rule

$$k_{L,t+1} = (1 - \delta_L) k_{L,t} + i_{L,t}, \tag{1}$$

where $i_{L,t}$ represents period t investment in long-term capital. A firm can choose not to utilize all long-term capital in a given period, in which case the unused part immediately depreciates. The possibility of disinvestment is captured by the condition that

$$i_{L,t} \geq -(1 - \delta_L) k_{L,t}. \tag{2}$$

Positive investment in long-term capital suffers from convex costs of adjustment according to the function $c(i_{L,t})$, which satisfies $c(0) = 0$, $c'(\cdot) > 0$ and $c''(\cdot) > 0$.[12] Not utilizing part of the capital stock in a given period is equivalent to disinvesting, with an endogenous cost equal to the value for the firm of those units of capital not being utilized.[13]

Firms face the possibility of suffering a liquidity shock s_t each period with probability p. The size of the liquidity shock is a function of the stock at the end of a period of each

[12] Even though adjustment costs are not modeled for short-term investment, this is not because of the belief that these costs are not relevant for short-term investment. It is because in our model short-term investment depreciates fully within one period and one can introduce them by adjusting the short-term production function accordingly. In the interest of simplicity we choose then not to model them separately.

[13] Adjustment costs of investment are introduced because the macroeconomic dynamic stochastic general equilibrium (DSGE) literature has found that they are an important feature in bringing the performance of these models closer to the data (Smets and Wouters (2007)).

type of capital, according to the function

$$s_t = \varepsilon \left(k_{S,t+1} + k_{L,t+1} \right), \tag{3}$$

where $\varepsilon > 0$. The occurrence of the liquidity shock is captured by the indicator function 1_s, which takes value 1 if the liquidity shock occurs, and value 0 if it doesn't.

Firms need one unit of labor to operate irrespective of the scale, in exchange for a wage w_t that is paid at the beginning of the period. Given that revenues are not received until the following period, the wage bill needs to be financed with internal or external funds. The wage is determined according to the sharing rule

$$w_t = (1 - \alpha_S)\theta_{S,t+1} k_{S,t+1}^{\alpha_S} + (1 - \alpha_L)\theta_{L,t+1} k_{L,t+1}^{\alpha_L}. \tag{4}$$

Firms' financing

Firms need to finance wages and investment, including adjustment costs. They can do so by using retained earnings or by borrowing. Firms are able to borrow an amount b_t (or save an amount $-b_t$) using one-period debt contracts at the riskless interest rate $1 + r_{t+1}$.

A firm's holdings of the final good, which we will also refer to as "asset holdings" or "asset position", computed at the beginning of a period immediately after dividends for that period are paid, is denoted $a_{f,t}$, and the dynamics of this asset position are given by

$$a_{f,t+1} = f_S \left(k_{S,t+1} \right) + f_L \left(k_{L,t+1} \right) - d_{t+1} - b_{t+1}(1 + r_{t+1}) - 1_s s_t, \tag{5}$$

where d_{t+1} are dividends paid to shareholders, $k_{L,t+1}$ is given by expression (1), and b_{t+1} can be expressed, using the budget constraint for a firm, as

$$b_{t+1} = i_{L,t} + c(i_{L,t}) + k_{S,t+1} + w_{t+1} - a_{f,t} \quad \text{if } i_{L,t} \geq 0, \tag{6}$$

or

$$b_{t+1} = k_{S,t+1} + w_{t+1} - a_{f,t} \quad \text{if } i_{L,t} < 0. \tag{7}$$

We will denote $a_{f,t+1}^{-}$ the asset position that results in the case that the liquidity shock occurred, and conversely $a_{f,t+1}^{+}$ the asset position in the case it did not.

Firms face financing constraints that limit their ability to obtain external finance using both debt and equity. Firms are unable to obtain any equity finance, which means that their dividends cannot be negative, so

$$d_t \geq 0. \tag{8}$$

Not being able to issue equity also means that firm assets can never be below zero, or

$$a_{f,t} \geq 0, \tag{9}$$

because, should the firm be forced to exit (which occurs with positive exogenous probability, as will be discussed next) with a negative asset position, the shareholders would have to contribute additional funds to cover the negative asset position. This would violate constraint (8).

We assume that firm shareholders force firms to pay out dividends every period according to the rule

$$d_{t+1} = \max\left\{\tau\left(f_S\left(k_{S,t+1}\right) + f_L\left(k_{L,t+1}\right) - b_{t+1}(1 + r_{t+1}) - 1_s s_t - a_{f,t}\right), 0\right\}, \tag{10}$$

which establishes that a fixed fraction of earnings, if these are positive, have to be paid out to shareholders.

Firms also face a borrowing limit given by:

$$b_{t+1} \leq \lambda_t \frac{f_S\left(k_{S,t+1}\right) + f_L\left(k_{L,t+1}\right) - s_t}{1 + r_{t+1}} \tag{11}$$

which means that they can borrow up to a fraction λ_t of the present discounted value of the lowest possible earnings next period, which occur if the firm suffers a liquidity shock.[14] Earnings obtained in periods after $t+1$ cannot be pledged by committing to roll over the debt because, as will be explained next, firms cannot commit to be in operation the following period.[15]

Firm exit

Firms face an exogenous exit shock which occurs with probability η, which can be interpreted as a permanent negative idiosyncratic productivity shock that forces firms to exit. Firms distribute all of their asset holdings $a_{f,t}$ as dividends before exiting but their

[14] It is also important to note that b_{t+1} is debt net of asset holdings given how it is defined. Asset holdings and their returns could in principle be pledgeable, and if that is the case then firms could hold a positive balance of assets in their balance sheet and simultaneously be borrowing. If debt is not risky, however, as is the case in this model, firms have no motive to do so. See Acharya et al. (2007) for a detailed discussion of this point.

[15] This specification for borrowing constraints can have several microeconomic justifications. As a first step, if we restrict the possible financial contracts to riskless debt, then it follows that only next period's output can be pledgeable. This is because firms might be forced to exit next period and thus cannot roll over debt and pledge output occurring in $t+2$ or after. One needs then to justify why only a fraction of this output is pledgeable. Following Hart and Moore (1994) and Kiyotaki and Moore (1997), one could assume that each project requires the input of the firm manager and that without her services revenues are a fraction $(1 - \lambda_t)$ lower. The she can threaten to withdraw her services and renegotiate her liability down to a fraction λ_t of revenues. Another justification could be along the lines of Holmstrom and Tirole (1997). One could assume that the borrower has to contribute with an unobservable effort to increase the expected return of the project and that in order for her to have the proper incentives at least a particular fraction of the expected returns need to accrue to her and cannot be pledged to lenders.

long-term capital holdings $k_{L,t}$ lose all of their value as they are firm-specific. A firm that exits is immediately replaced by a new firm with no assets or long-term capital.

Firm Optimization

Firms maximize the present discounted value of dividends d_t distributed to their shareholders, the households, with perfect foresight on the path of future interest rates, aggregate productivity and financing constraints. Given a sequence starting at time t of the interest rate, aggregate productivity and financing constraints $z_t = \{r_s, \theta_s, \lambda_s\}_{s=t}^{s=\infty}$, a firm's value function, calculated at the beginning of period t conditional on continuing to operate (so assuming it has not suffered the exogenous exit shock), and immediately after the dividend payment, is given by

$$\begin{aligned} J(a_{f,t}, k_{L,t}, z_t) \quad = \quad & \max_{d_{t+1}, k_{S,t+1}, k_{L,t+1}, b_{t+1}, w_t} M_{t,t+1} \left[d_{t+1} + \eta \left(p a^{-}_{f,t+1} + (1-p) a^{+}_{f,t+1} \right) \right. \\ & \left. +(1-\eta) \left(pJ(a^{-}_{f,t+1}, k_{L,t+1}, z_{t+1}) + (1-p) J(a^{+}_{f,t+1}, k_{L,t+1}, z_{t+1}) \right) \right], \end{aligned}$$

subject to (5), (4), (8), (10), and (11), where $M_{t,t+1}$ is the discount factor of households that the firm uses to discount future dividends.

Firms' optimal choices for short term investment, long term capital holdings, wages, dividends and borrowing are given by $k_{S,t+1}(a_{f,t}, k_{L,t}, z_t)$, $k_{L,t+1}(a_{f,t}, k_{L,t}, z_t)$, $b_{t+1}(a_{f,t}, k_{L,t}, z_t)$, $w_t(a_{f,t}, k_{L,t}, z_t)$, and $d_{t+1}(a_{f,t}, k_{L,t}, z_t)$.[16]

2.2 Households

There is a continuum of measure one of infinitely lived households. Given that there is perfect insurance between households, we will treat the household sector as a representative agent. Let $\Gamma_t(a_f, k_L)$ denote the joint distribution of asset holdings and long-term capital holdings in the population of firms, of which there is a continuum also of measure one. Households have preferences represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t u(C_t), \tag{12}$$

[16] It is important to point out that in the context of this model and in the absence of any assumptions that could influence dividend payout behavior (such as the one we make in (10)), an optimizing firm would delay dividend payments until it is forced to exit. This is because firms use households' intertemporal marginal rate of substitution, can save at the same interest rate as households, and face borrowing constraints, which means that holding liquidity inside the firm to avoid potential future underinvestment is more valuable than distributing it out to shareholders. Firms in practice pay out dividends regularly and in large amounts, and this is the justification for our assumption (expression (10)) from an empirical viewpoint. From the perspective of theory, one could justify this assumption on the basis of Jensen's (1986) hypothesis that dividends reduce agency costs associated with free cash flow.

where C_t is consumption in period t. They are endowed with one unit of labor services which they provide inelastically to the firms in exchange for wage W_t equal to

$$W_t = \int w_t(a_{f,t}, k_{L,t}, z_t) d\Gamma_t(a_f, k_L). \tag{13}$$

Households can borrow or lend at the riskless interest rate r_{t+1} and they own shares in the firms, which give them the right to any dividends distributed by the firms, which are equal to

$$D_t = \int d_t(a_{f,t}, k_{L,t}, z_t) d\Gamma_t(a_f, k_L).$$

Their budget constraint for period t is given by

$$C_t = N_t + W_t - A_{t+1}, \tag{14}$$

where

$$N_t = (1 + r_t)A_t + D_t$$

is the wealth of a consumer at the beginning of period t, and A_{t+1} are savings made in period t.

2.3 Equilibrium

Given a sequence starting at time t of aggregate productivity and borrowing constraints which we denote by $x_t = \{\theta_s, \lambda_s\}_{s=t}^{s=\infty}$, let firms' choices for short term investment, long term capital holdings, and borrowing be given by $k_{S,t+1}(a_{f,t}, k_{L,t}, x_t)$, $k_{L,t+1}(a_{f,t}, k_{L,t}, x_t)$, and $b_{t+1}(a_{f,t}, k_{L,t}, x_t)$, respectively, and choices by households for consumption and savings be given by $C_t(N_t, x_t)$ and $A_{t+1}(N_t, x_t)$, respectively. Let $\Gamma_t(a_f, k_L)$ denote the joint distribution of asset holdings and long-term capital holdings in the population of firms. We are now ready to define an equilibrium.

Definition 1 *An equilibrium is a sequence of interest rates* $\{r_t\}$, *a sequence of consumption and savings policies* $\{C_t(N_t, x_t)\}$ *and* $\{A_{t+1}(N_t, x_t)\}$, *a sequence of short term investment, long term capital holdings, and borrowing policies* $\{k_{S,t+1}(a_{f,t}, k_{L,t}, x_t)\}$, $\{k_{L,t+1}(a_{f,t}, k_{L,t}, x_t)\}$, *and* $\{b_{t+1}(a_{f,t}, k_{L,t}, x_t)\}$, *and a sequence of distributions for firms' asset and long-term capital holdings* $\{\Gamma_t\}$, *such that, given the initial distribution* Γ_0 *and an initial wealth of households* N_0 :

(i) $C_t(N_t, x_t)$ *and* $A_{t+1}(N_t, x_t)$ *are optimal given* $\{r_t\}$,

(ii) $k_{S,t+1}(a_{f,t}, k_{L,t}, x_t)$, $k_{L,t+1}(a_{f,t}, k_{L,t}, x_t)$, *and* $b_{t+1}(a_{f,t}, k_{L,t}, x_t)$ *are optimal given* $\{r_t\}$,

(iii) Γ_t *is consistent with the investment and borrowing decisions of firms,*

(iv) *the bond market clears:*

$$\int b_{t+1}(a_{f,t}, k_{L,t}, x_t)d\Gamma_t(a_f, k_L) = A_{t+1}(N_t, x_t). \tag{15}$$

The bond market clearing condition (15) requires that net aggregate borrowing of the firm sector (the left-hand side of the equation) is equal to aggregate saving of the household sector (the right-hand side).

3 Calibration and Steady State Analysis

3.1 Calibration

We calibrate the economy at the quarterly frequency, and the chosen values for the parameters are shown in Table 1.

The utility function for households is chosen to be iso-elastic of the form

$$u(C_t) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma},$$

with a constant relative risk aversion coefficient, γ, of 2. The intertemporal preference rate β is set at 0.99 to match a 4 percent annualized average interest rate.

The calibration of parameters concerning the firm sector can be divided into three categories: those that affect their investment opportunities, those that control firms' financing constraints, and those that determine firm entry and exit.

The firm technology parameters are calibrated as follows. The capital share of both technologies, α_S and α_L, is set to 0.36, consistent with most macroeconomic studies. There are four remaining parameters affecting firms' technologies. These are the productivities of the short-term and long-term technologies, θ_S and θ_L respectively, the probability p of suffering a liquidity shock, and the size ε of the liquidity shock per unit of capital. These four parameters are jointly calibrated to match four moments in the data: (i) the aggregate amount of long-term investment as a share of total investment, (ii) the ratio between the median asset holdings of young firms and the median asset holdings of the whole population of firms, (iii) the standard deviation of the ratio of profits to sales, and (iv) the probability of negative profits. The ratio of long-term to total investment is calculated for the U.S. using data from the Bureau of Economic Analysis (BEA). The BEA provides data on gross private domestic investment, which includes fixed residential and nonresidential investment and changes in private inventories, and also on research and development (R&D) spending. Long-term investment is considered to include non-residential fixed investment in structures, residential fixed investment, and R&D spending, and short-term investment includes fixed investment in equipment and

Table 1: Parameter Values

Parameters	Explanation	Value
γ	Coefficient of relative risk aversion	2
β	Households' discount rate	0.99
α_S	Capital share in short-term technology	0.36
α_L	Capital share in long-term technology	0.36
θ	Aggregate productivity factor (steady state)	1
θ_S	Productivity factor of short-term technology	0.7
θ_L	Productivity factor of long-term technology	0.8
p	Probability of a negative liquidity shock	0.25
ε	Size of liquidity shock per unit of total capital	3
δ_L	Depreciation of long-term capital	0.025
ζ	Multiplicative parameter of convex adjustment cost function	$400,000$
χ	Exponential parameter of convex adjustment cost function	4
λ_{SS}	Pledgeability of next period's earnings	0.8
τ	Dividend payout ratio	0.975
η	Exogenous probability of firm exit	0.025

software and changes in private inventories. The share of long-term investment using this measure has been on average 55% during $1990 - 2007$. The ratio between the median asset holdings of young stock-exchange listed firms (those with less than 1 year following their IPO) and the median asset holdings of the whole population of listed firms is 0.47 in the U.S., using data from Capital IQ-Compustat for 2002-2011. Using this same data, we get a standard deviation of the ratio of profits to sales of 0.35 and a probability of negative profits of 25%, both calculated at the annual frequency. A summary of all the empirical moments used to calibrate parameters can be found in Table 2.

The depreciation rate of long-term capital is set at $\delta_L = 0.025$, which is a standard value for quarterly Real Business Cycle (RBC) models. The adjustment cost function for long-term investment, $c(i_{L,t})$, is assumed to adopt the functional form

$$c(i_{L,t}) = \chi i_{L,t}^{\zeta},$$

with values $\zeta = 4$ and $\chi = 400,000$ set to generate aggregate adjustment costs that are roughly equal to 0.2% of aggregate investment, in the lower end of the estimates discussed in studies of adjustment costs of capital such as Gourio and Kashyap (2007), and also to be broadly in line with the firm-level standard deviation of investment rates (investment over capital) observed in the data. Cooper and Haltiwanger (2006) show using Longitudinal Research Database data between 1972 and 1988 that the standard deviation of the investment rate is 0.306.

The steady state value for the parameter regulating borrowing constraints (λ_{SS}) is set to match the average net leverage ratio for Compustat publicly listed firms, which is

Table 2: Empirical Moments Matched in the Calibration

Empirical Moment	**Source**	**Data**	**Model**
Targets to calibrate θ_L, θ_S, p and ε			
- Long-term investment over total investment	(1)	0.55	0.62
- Median assets of young firms over median assets of all firms	(2)	0.47	0.52
- Probability of negative profits (annual)	(2)	0.25	0.33
- Standard deviation of the ratio of profits to sales (annual)	(2)	0.35	0.27
Targets to calibrate ζ and χ			
- Adjustment costs as share of aggregate investment	(3)	0.2% – 6.2%	0.2%
- Standard deviation of investment over capital (annual)	(4)	0.306	0.175
Targets to calibrate λ			
- Average net leverage ratio	(5)	0.079	0.068
Targets to calibrate τ			
- Average payout ratio of U.S. corporations	(6)	105% – 116%	85%
Targets to calibrate η			
- Job destruction rate (annual)	(7)	10%	10%

(1) Own calculations using Bureau of Economic Analysis (BEA) data, (2) Own calculations using Capital IQ-Compustat for U.S. listed firms, (3) Gourio and Kashyap (2007), (4) Cooper and Haltiwanger (2006), (5) Bates et al. (2009), (6) Weston and Siu (2003), (7) Bilbiie et al. (2012)

7.92% according to Bates et al. (2009) using data from 1980 to 2006. They calculate net leverage as the ratio of total debt minus cash holdings to the book value of total assets. Net leverage in our model is calculated as the current value of debt b_{t+1} minus holdings of the asset over the total value of book assets, assuming assets are marked to market and valued according to the current firm value $J(a_{f,t}, k_{L,t}, z_{SS})$, where z_{SS} is the steady state defined by $\{r_{SS}, \theta_{SS}, \lambda_{SS}\}$. [17] The payout ratio for firms with positive earnings (τ) is set to be roughly in line but somewhat below the average payout ratio of U.S. corporations including share repurchases, which Weston and Siu (2003) show was between 105% and 116% in the period 1998-2001. A lower payout ratio compensates for the fact that our firms are assumed not to be able to issue new equity.

Finally, we set the size of the exogenous firm exit shock $\eta = 0.025$ to match the U.S. empirical level of 10 percent job destruction per year, following Bilbiie et al. (2012). Exiting firms are immediately replaced by new firms with no holdings of assets or long-term capital.

[17] Net leverage and leverage are the same in our model because firms do not simultaneously hold the asset and borrow. For this reason, the relevant counterpart in the data is net leverage.

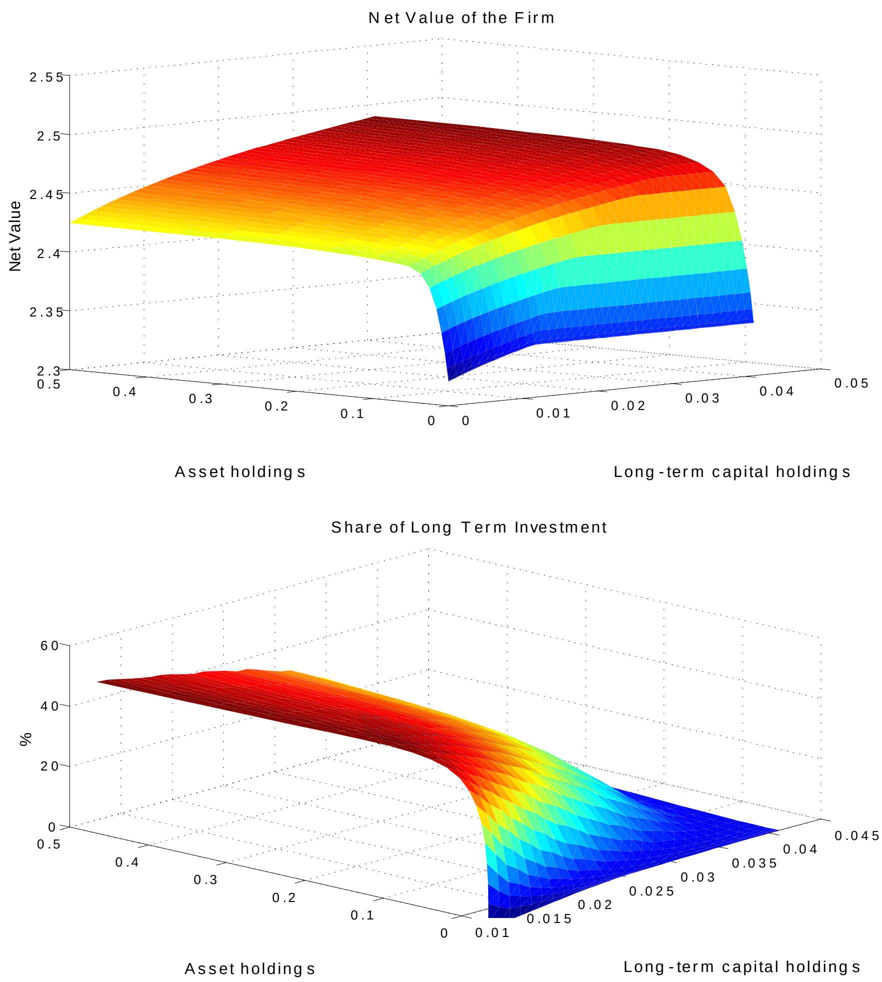
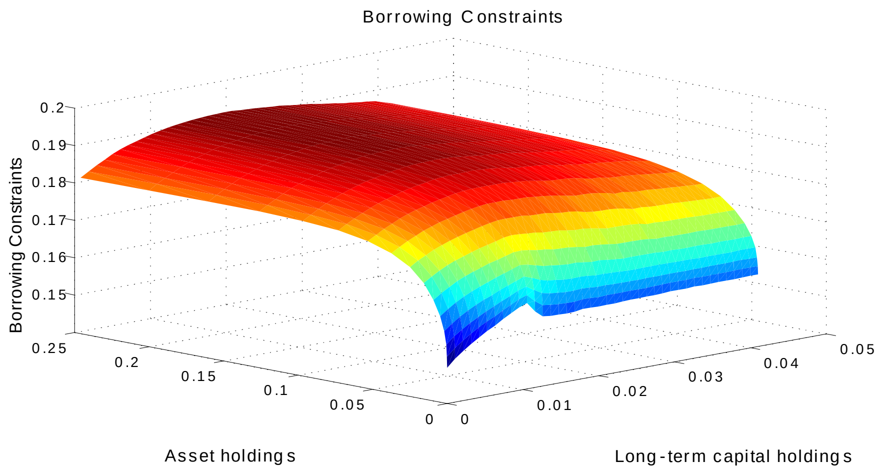


Figure 1: FIRM VALUE AND THE COMPOSITION OF INVESTMENT. The top figure displays the net value of the firm in the space of asset holdings $a_{f,t}$ and long-term capital holdings $k_{L,t}$. The net value of the firm is calculated as firm value $J(a_{f,t}, k_{L,t}, z_t)$ minus asset holdings. The bottom figure displays long-term investment as a share of total investment in the same space defined by asset holdings and long-term capital holdings.

3.2 Firm Policies and Firm Heterogeneity in the Steady State

We now take a closer look at firm policies in the steady state equilibrium that arises from the calibration described above. We solve the model numerically using value function iteration and simulation techniques. A detailed explanation of the solution method for the steady state equilibrium can be found in the Appendix.

The top graph of Figure 1 shows the net value of the firm in the space of asset holdings $a_{f,t}$ and long-term capital holdings $k_{L,t}$. The net value of the firm is calculated as firm value $J(a_{f,t}, k_{L,t}, z_t)$ minus asset holdings. Net value is increasing and concave in both arguments, and drops sharply as asset holdings approach zero. The strong concavity of the value function with respect to asset holdings introduces risk aversion and precautionary behavior in firms that affects their investment portfolio choices. These choices



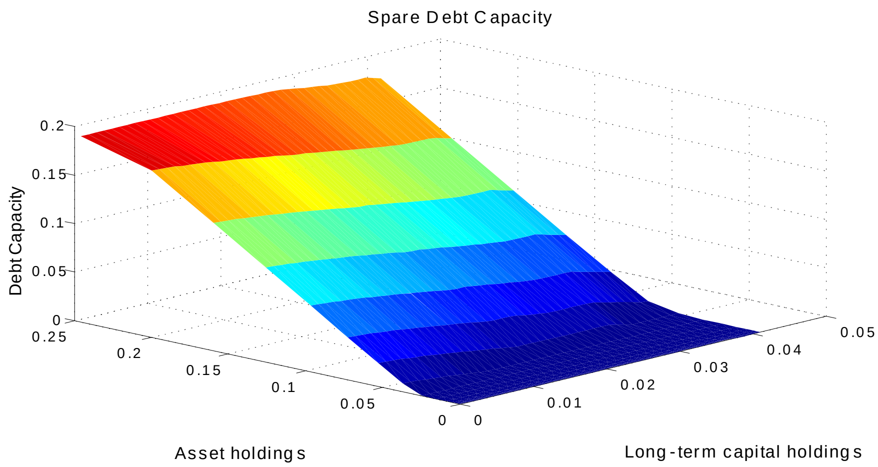


Figure 2: FIRM BORROWING CONSTRAINTS AND SPARE DEBT CAPACITY. The top figure displays the endogenous borrowing constraint of firms in the space of asset holdings $a_{f,t}$ and long-term capital holdings $k_{L,t}$. The lower figure shows the spare debt capacity, which is calculated as the difference between the borrowing constraint and actual borrowing, in the same space defined by asset holdings and long-term capital holdings.

are captured in the bottom graph of Figure 1, which displays long-term investment as a share of total investment in the same space defined by asset and long-term capital holdings. Long-term investment decreases as a share of total investment as asset holdings decrease. This is the result of two forces. On one hand, long-term investment generates less pledgeable output as a share of its total future output than short-term investment. Only next period returns are potentially pledgeable, and long-term investment generates most of its returns further in the future. This means that credit constrained firms can leverage long-term investment less than short-term investment, and this feature makes short-term investment relatively more interesting for a credit constrained firm. On top of this, long-term investment generates liquidity risk, which credit-constrained firms are keen to avoid. The increase in liquidity risk comes about because the size of the liquidity

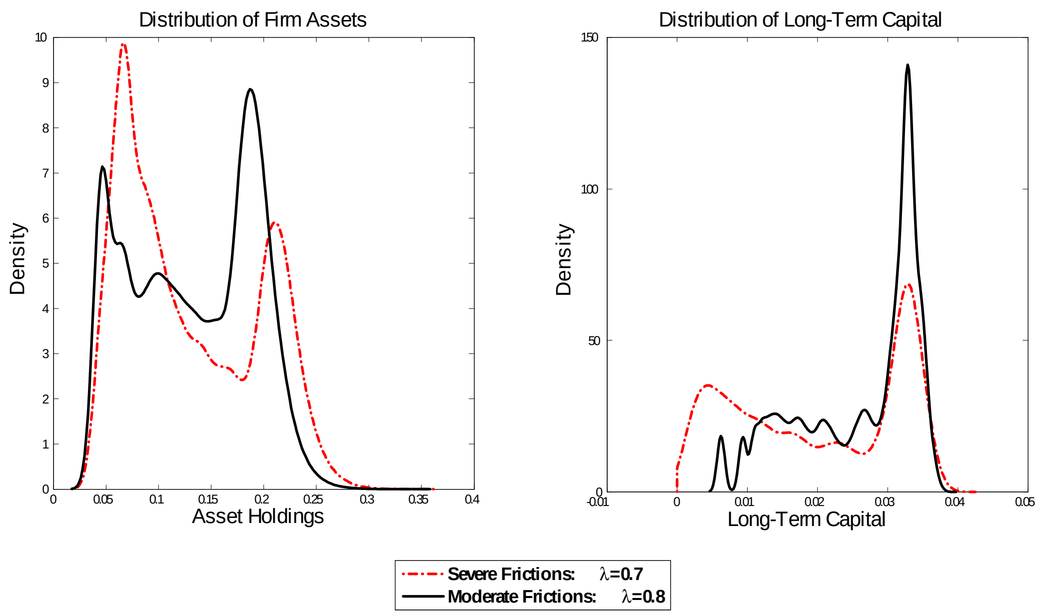


Figure 3: FIRM HETEROGENEITY. This figure displays the distribution of asset holdings $a_{f,t}$ and long-term capital holdings $k_{L,t}$ in the steady state equilibrium. The benchmark economy in which the pledgeable fraction of next period's revenues in the steady state is $\lambda_{SS} = 0.8$ is compared with an economy with more severe credit constraints in which $\lambda_{SS} = 0.7$.

shock is proportional to total capital (long and short-term), and long-term capital depreciates slowly. Increases in long-term investment today increase liquidity risk for several periods into the future. For these two reasons, firms with low asset holdings who are facing credit constraints or are likely to face them in the near future shift from long-term to short-term investment. Long-term investment also decreases with long-term capital holdings, but only very gradually given the convex costs of adjusting long-term capital.

The financing constraints and borrowing decisions of firms are captured in Figure 2. The top graph displays the endogenous equilibrium borrowing constraints of firms in the space of asset and long-term capital holdings. The maximum amount a firm can borrow is equal to a fraction $\lambda_{SS} = 0.8$ of its lowest possible output next period (the state in which it has suffered a liquidity shock). The relationship between long-term capital holdings and borrowing constraints is non-linear. Long-term capital generates output next period which is partially pledgeable, but on the other hand increases the size of the liquidity shock. The first effect dominates for low levels of long-term capital when the average return to capital is high, but is dominated by the liquidity shock effect when the average return to capital becomes low. The relationship between assets and equilibrium borrowing constraints is more straightforward. Low asset holdings enable low investment and low pledgeable output. In the bottom graph we observe the actual borrowing behavior of firms. It displays the spare debt capacity of firms; firms with no spare debt capacity suffer binding credit constraints. Firms with low asset holdings borrow up to capacity, while firms with higher levels of asset holdings have spare capacity.

Figure 3 displays the distribution of asset and long-term capital holdings, the two dimensions along which firms differ. To better understand the impact of financing constraints on firm heterogeneity, we include the equilibrium distribution of firms for the benchmark calibration and for one that features tighter borrowing constraints, captured by a fraction of next period's output that can be pledged, λ_{SS}, of 70%, instead of 80% as in the benchmark calibration. It is clear from the figure that borrowing constraints slow down both asset and long-term capital accumulation. The pattern of asset holdings shows that firms cluster around two groups forming a twin-peaked distribution. Firms with very low asset holdings, which are either young firms, or older firms that have suffered several negative liquidity shocks, are unable to invest large quantities and their assets grow very slowly. They face very tight borrowing constraints as was shown in Figure 2. Beyond a level of asset holdings, they are able to leverage their existing assets more and thus invest more and accumulate assets more quickly. Given the decreasing marginal return to both types of capital, eventually returns to investment decrease and so does asset accumulation, and another large grouping of firms arises with large asset holdings.

4 Amplification and Propagation of Productivity Shocks

In this section we analyze the transitional dynamics following an unexpected, temporary and persistent negative shock to aggregate productivity θ_t. We solve the model numerically using value function iteration and simulation techniques. A detailed explanation of the solution method for the transitional dynamics can be found in the Appendix.

We introduce a shock to aggregate productivity θ_t equivalent to -1% of its steady state value ($\theta = 1$) in $t = 1$. We assume that following the shock, θ_t evolves according to

$$\log \theta_t = \rho \log \theta_{t-1}$$

where $\rho = 0.9$. Agents are assumed not to learn in advance about the shock.[18] They have however a perfect foresight about the evolution of θ_t from then on. The exposition that follows is designed so that the different mechanisms in operation are clearly distinguished. We will first analyze an economy as described in the model of section 2, but in which the interest rate is assumed fixed at its steady state level throughout the transitional dynamics. This version with constant interest rates can be interpreted as a small open economy with perfect capital mobility in which the international interest rate is the one that holds in the steady state. This version of the model will allow us to study the direct

[18] In other words, they do not know, in period t when households and firms are making their optimal choices about consumption, savings, investment and borrowing, that a shock will hit the economy in period $t+1$ that will affect the return on savings and the output that results from this period's investment.

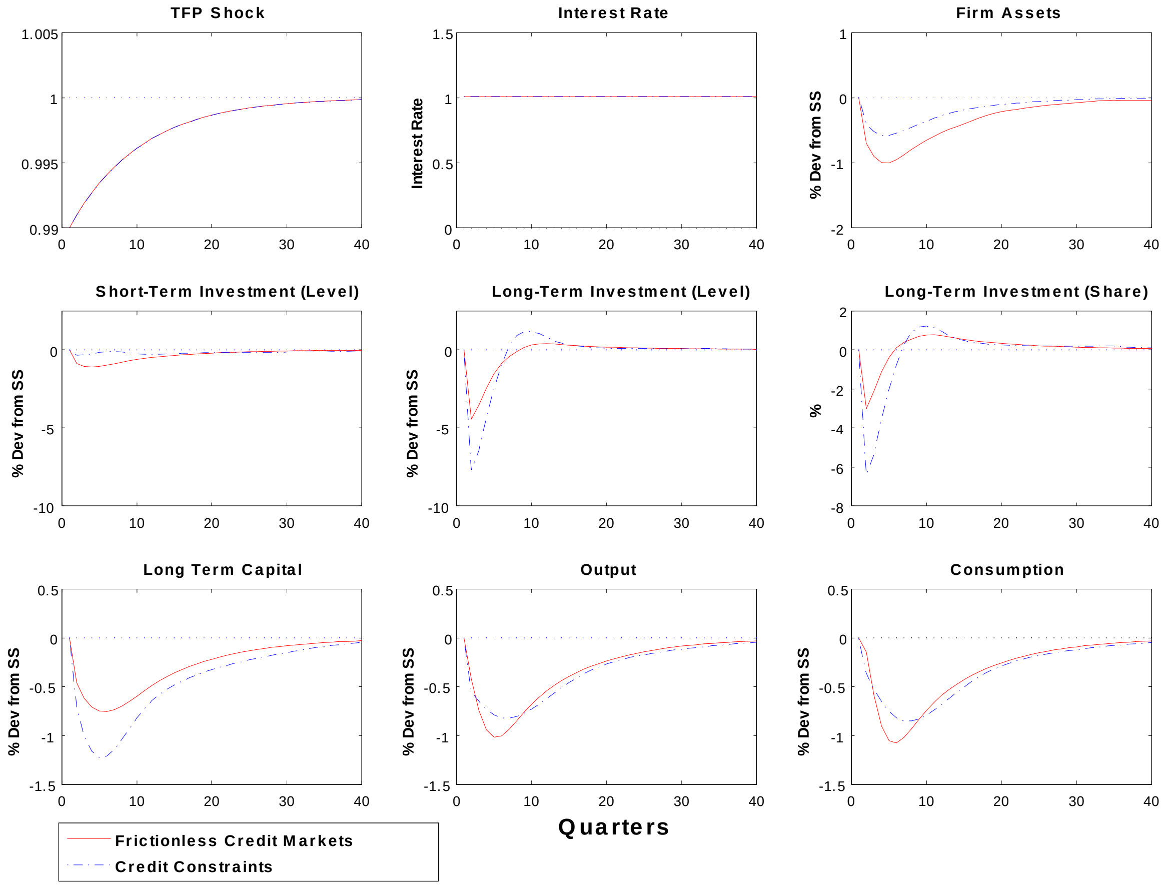


Figure 4: RESPONSE TO A NEGATIVE AGGREGATE PRODUCTIVITY SHOCK: FIXED INTEREST RATE CASE. The figure displays the response of nine key macroeconomic variables to a negative and persistent shock to aggregate productivity (TFP) θ_t equal to -1% of its steady state value $\theta_{SS} = 1$. The interest rate r_t is assumed to be fixed at its steady state value $r_{SS} = 1.01\%$. The responses of a model featuring frictionless credit markets are compared with those of one featuring credit constraints. The responses of all variables except the TFP shock, the interest rate, and long-term investment (share) are calculated as percent deviations from their steady state values. The response of the share of long-term investment is calculated as the difference between the share in each period and its steady state value.

impact of productivity changes on firms' investment choices. Next, we will return to the exact description of the economy described in section 2 in which the interest rate is endogenous to analyze the general equilibrium interactions that arise in the transitional dynamics. In both cases, we will compare the response of the benchmark economy with financing frictions with one in which financing frictions are absent.

4.1 Fixed Interest Rate

The responses of nine key aggregate variables to the productivity shock in the version of the model with no general equilibrium effects are shown in Figure 4. In that figure, the response of the benchmark economy with financial constraints and the response of an economy with financially unconstrained firms is compared.[19]

In the economy with no financial constraints, we find that both short and long-term investment drop moderately following the shock. Short term investment then recovers slowly towards the steady state level, while long term investment recovers faster and rises for several periods above the steady state level. Two forces are at play. The decrease in productivity discourages investment in both short and long-term investment, all else equal. On impact the effect is stronger for long-term investment however because most firms already hold some long-term capital, which depreciates slowly. The drop in short-term investment on the other hand is gradual and hump-shaped. The other force at play has to do with the change in the relative returns of long-term versus short-term investment. A temporary negative productivity shock decreases short term investment returns more than long-term investment returns, and this acts to encourage long-term investment as a share of total investment. This second force explains the more rapid recovery of long-term investment. Put together, this means that long-term investment as a share of total investment falls below the steady state level initially to rise above that level subsequently and stay high for around 20 quarters. The level of aggregate long-term investment eventually also rises above its steady state level after around 10 quarters. The effect on output and households' consumption is moderate, with both quantities roughly tracking the productivity process with little amplification or propagation.

The behavior of the composition of investment when firms face financing constraints changes substantially. For the initial quarters following the shock, long-term investment drops significantly. It falls by 7.66% relative to the steady state level in the first quarter, compared to only 4.42% in the unconstrained case. Short-term investment instead falls much less than in the unconstrained case. It bottoms at -0.35% in the third quarter, while it falls three times as much as (-1.11%) in the unconstrained case. Firms, facing a worsening of their financing constraints, react by cutting strongly on long-term investment while keeping short-term investment stable, despite the fact that it is now relatively less productive than long-term investment. As a result, the share of long-term investment in total investment falls by 6.4 percentage points relative to the steady state share, compared to 3 percentage points in the unconstrained case. This translates into a large drop in the stock of long-term capital which takes many periods to recover and propagates the effects of the shock.

Firms' investment dynamics translate into a dampening of the effects of a negative productivity shock in the short-run, and a moderate propagation in the medium and long-run. Output drops more and sooner in the unconstrained case; it bottoms at -1.02% relative to the steady state level in the fourth quarter following the shock, while the output drop in that quarter when firms face financing constraints is -0.79%. In the

[19] The calibration is kept unchanged for both versions of the model. The responses plotted in Figure 4 as deviations from steady state values are calculated relative to the relevant steady state for each case.

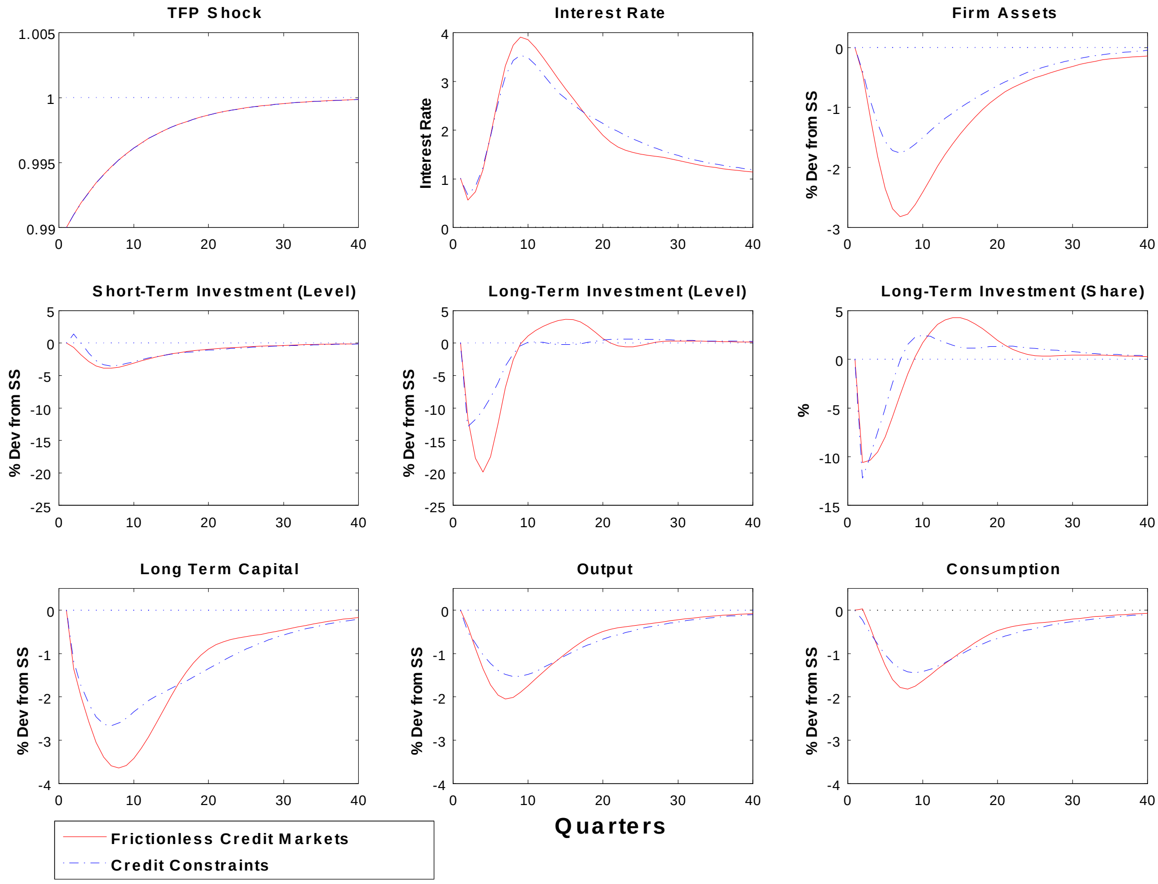


Figure 5: RESPONSE TO A NEGATIVE AGGREGATE PRODUCTIVITY SHOCK: ENDOGENOUS INTEREST RATE CASE. The figure displays the response of nine key macroeconomic variables to a negative and persistent shock to aggregate productivity (TFP) θ_t equal to -1% of its steady state value $\theta_{SS} = 1$. The interest rate r_t is assumed to be endogenous. The responses of a model featuring frictionless credit markets are compared with those of one featuring credit constraints. The responses of all variables except the TFP shock, the interest rate, and long-term investment (share) are calculated as percent deviations from their steady state values. The response of the share of long-term investment is calculated as the difference between the share in each period and its steady state value.

medium and long run, the large initial drop in long-term capital and its slow recovery means that the propagation of the effects of the shock is greater in the economy with financing constraints.

4.2 Endogenous Interest Rate

The general equilibrium implications in our model are of particular relevance given that we introduce firm-level heterogeneity and calibrate the model to generate a realistic distribution of the severity of firms' credit constraints. This setup can capture the possibility that, in a general equilibrium context, financially unconstrained firms react in such a way that they compensate for constrained firms' behavior and aggregate dynamics are either hardly affected by the presence of financing constraints, or affected in a significantly different manner. The transitional dynamics in this case are displayed in Figure 5.

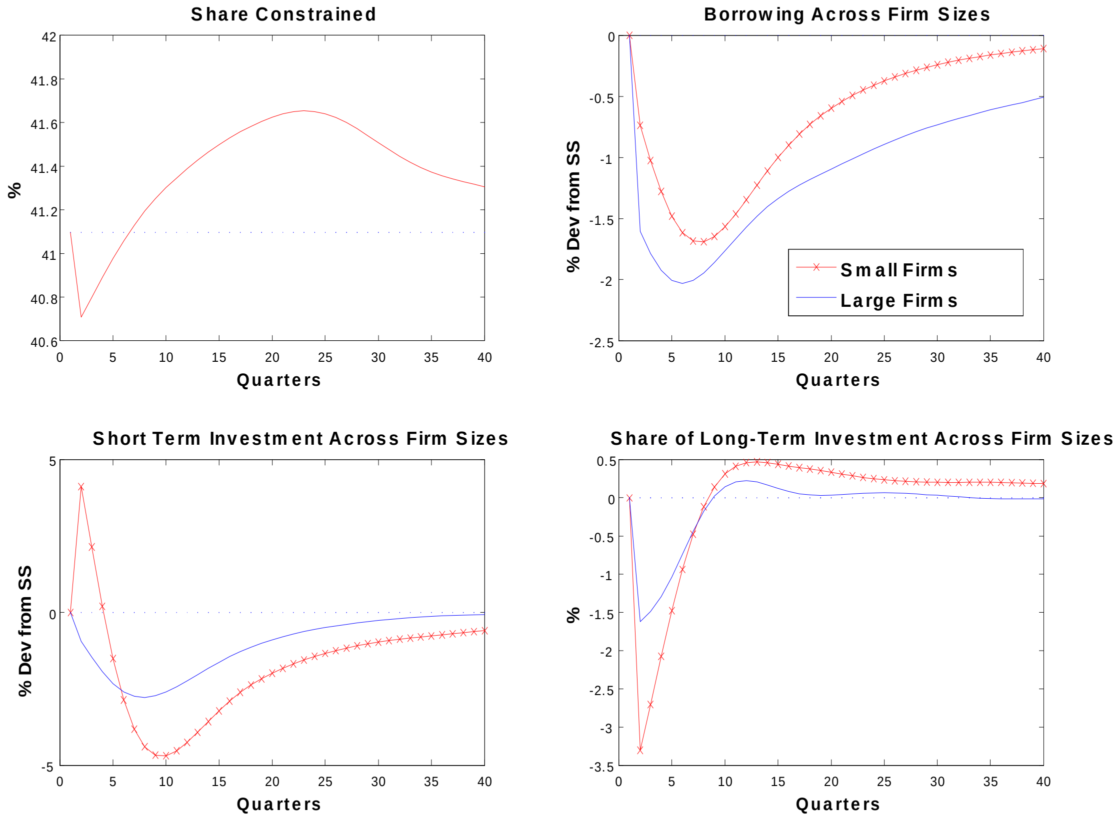


Figure 6: RESPONSE OF SELECTED FINANCIAL VARIABLES TO A NEGATIVE AGGREGATE PRODUCTIVITY SHOCK: ENDOGENOUS INTEREST RATE CASE. The figure displays the response of selected variables to a negative and persistent shock to aggregate productivity equal to -1% of its steady state value. The interest rate r_t is assumed to be endogenous. For some variables, the average values for firms in the lower and upper deciles of the distribution of asset holdings ("Small Firms" and "Large Firms", respectively) are displayed. The response of the share of long-term investment is calculated as the difference between the share in each period and its steady state value.

We focus again first on the economy in which credit market imperfections are absent to be able to compare it to the version in which they are present and understand their role. In this version, the interest rate r_t initially falls below its steady state level of 1.01% and subsequently rises above that level to peak at more or less 4% around 10 quarters following the shock. Interest rates then decrease very gradually back to their steady state level. This means that immediately following the shock, firms anticipate a sharp steepening of the yield curve. The decrease in interest rates in the short-run encourages short-term investment, while the steep yield curve discourages long-term investment. Eventually, interest rates rise and the slope of the yield curve decreases until it becomes downward sloping. The increase in short-run interest rates discourages both types of investment, but especially short-term investment, and the downward sloping yield curve encourages a shift towards long-term investment in the medium-term.

In the economy with financing constraints there are additional forces at play. Low interest rates in the early periods loosen borrowing constraints and encourage investment in both short and long term projects. Very high interest rates in the future however severely tighten borrowing constraints and induce a precautionary behavior in firms that encourages them to shift even more strongly into short-term investment. Both forces push short-term investment in the initial periods so that in equilibrium it is *above* the steady state level despite the decrease in productivity. Long-term investment dynamics are affected by yet another force. The interaction of convex costs of adjustment and borrowing constraints mean that firms have an incentive to spread out their investment in long-term capital. Anticipating very high interest rates and tight borrowing constraints that will restrict the amount that firms can invest in long-term they cut on long-term investment less following the shock than firms in the economy with no financing constraints.

The implications of firm' investment behavior for output and consumption are qualitatively similar as in the case with fixed r_t, but quantitatively significantly stronger. Output in the economy with no financial imperfections drops by as much as 2.6% relative to its steady state level in the seventh quarter, while it only drops as much as 1.8% in the eighth quarter in the economy with firm financing frictions. As it turns out, the general equilibrium forces enhance the financial decelerator effect and result in a further dampening in the short run of the effects of a temporary negative productivity shock.

4.3 Countercyclicality of financing constraints

It is worth inspecting more closely the dynamics of the severity of firm financing constraints and of firm heterogeneity. There are three mechanisms that influence how tight borrowing constraints are along the transitional dynamics. One is the decrease in firms' asset holdings following the shock. Another is the increase in interest rates which decreases the present value of pledgeable output and tightens borrowing constraints. These two sources of countercyclical financing constraints are standard and have been discussed at length in the literature.

A third, novel, source of countercyclicality of borrowing constraints arises from the coexistence of short and long-term investment opportunities. A temporary negative shock to productivity encourages a shift towards investing in long-term capital because its returns are relatively more acyclical. Long-term capital produces less pledgeable output per unit of investment, all else equal, so a shift towards long-term investment results in tighter borrowing constraints. Figure 6 shows some aspects of this mechanism. The share of borrowing constrained agents increases following a small initial drop. The initial drop can be explained by firms' lower demand for funds because of the combination of lower productivity and an existing large stock of long-term capital, and because of low interest rates. As long-term investment picks up and interest rates rise, so does the fraction of credit constrained firms. On the one hand, firms want to invest more in long-term capital in the medium-run (after around 10 quarters), which produces less pledgeable output

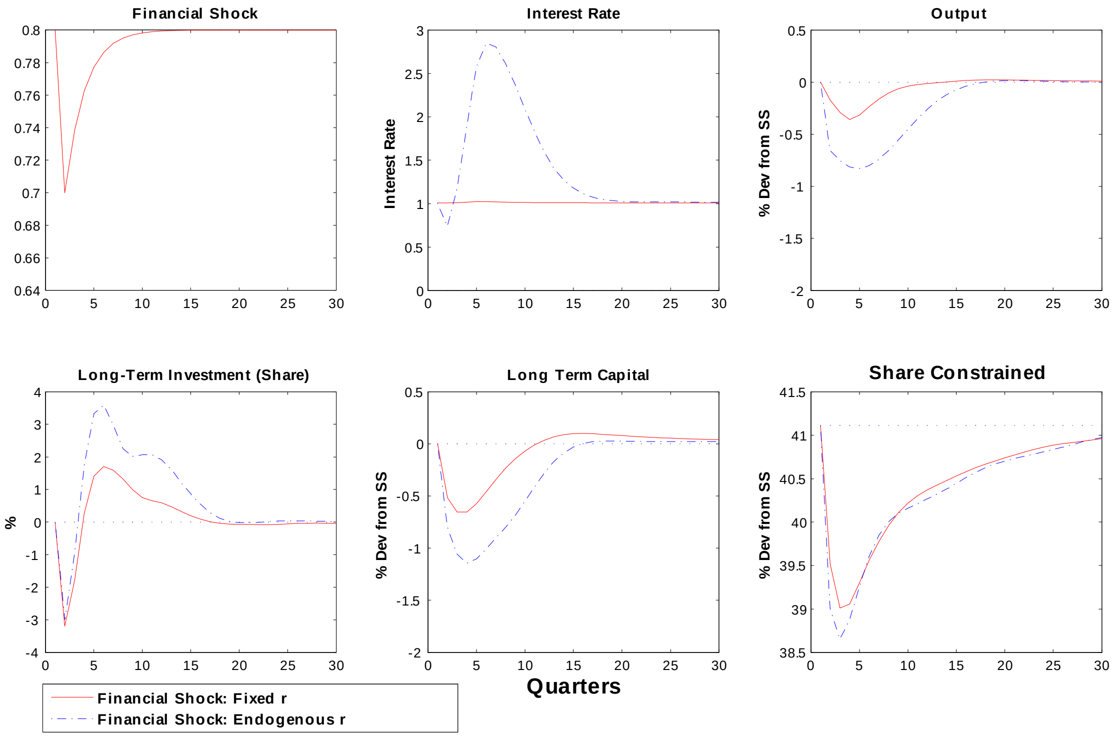


Figure 7: RESPONSE TO A NEGATIVE FINANCIAL SHOCK. The figure displays the response of six key macroeconomic variables to a negative shock to the severity of credit constraints (the pledgeability parameter λ_t) equal to -10%. The responses of models with a fixed and an endogenous interest rate r_t are displayed. Aggregate productivity (TFP) θ_t is assumed to be at its steady state value $\theta_{SS} = 1$. The responses of output and long-term capital are calculated as percent deviations from their steady state values. The response of the share of long-term investment is calculated as the difference between the share in each period and its steady state value.

than short-term investment and tightens borrowing constraints. Higher interest rates further decrease the present value of pledgeable output.

If we compare the behavior of small and large firms, which broadly capture financially constrained and unconstrained firms, respectively, we can observe that small firms react by strongly shifting their investment towards the short-term project while large firms do so to a much smaller extent. Small firms even *increase* their level of investment in the short-term project relative to the steady state level despite the decrease in productivity, because short-term investment produces more pledgeable output per unit of investment and thus requires a lower downpayment, and also as a precautionary reaction to the anticipation of future credit constraints.

5 Extensions

5.1 Amplification and Propagation of Financial Shocks

In this section we analyze the transitional dynamics following an unexpected, temporary and persistent negative shock to the severity of financing frictions. In particular, we introduce an unexpected shock to λ_t, the parameter that determines the share of next period's output that can be pledged to lenders today, equal to 10%. In other words, λ_t decreases in $t = 1$ unexpectedly from $\lambda_{t=0} = 0.8$ to $\lambda_{t=1} = 0.7$, and we assume that following the shock, for $t > 1$, λ_t evolves according to

$$\log \lambda_t = \rho \log \lambda_{t-1}$$

where $\rho = 0.6$.[20] As with the technology shock, agents are assumed not to learn in advance about the shock. They have however a perfect foresight about the evolution of λ_t from then on. The aggregate productivity factor θ_t is assumed to remain constant at its steady state value θ_{SS}. The resulting transitional dynamics are displayed in Figure 7, in which both the fixed and endogenous r_t cases are shown.

In the case of a fixed r_t, a tightening of financing constraints results in lower total investment, which lowers output. It also translates into a lower share of long-term investment. Two forces are at play. Given that on the margin long-term investment produces less pledgeable output than short-term investment, a tightening of credit constraints shifts investment towards the short-term project. Given, on the other hand, that long-term investment increases future liquidity risk more than short-term investment, this interacts with firms' precautionary motive to also induce a shift towards the short-term project. Firms' precautionary behavior is also clear from the pattern of firms' borrowing. They decrease their borrowing to leave spare debt capacity, and this results in a decrease in the share of credit constrained firms. This increase in spare debt capacity comes about because the increase in the severity of financing frictions increases the concavity of the value function in the dimension of asset holdings and increases the precautionary motive of firms.

In the endogenous r_t case, the combination of lower output and the desire of households to smooth consumption translates into significantly higher interest rates for much of the transition period, following an initial brief decrease. As in the case with a technology shock, the term structure adopts a hump-shape with interest rates peaking at around 2.85% following five quarters. The effects on the dynamics of the composition of investment are small in the short run, but the effects on the level of investment and output are strong. Output drops by as much as 0.83% four quarters after the impact of the shock in the endogenous r_t case, compared to a maximum fall of 0.36% in the fixed r_t case in the third quarter. In the long-term, the large fall in the long-term stock of capital following the impact of the shock translates into a slow recovery, and much more so in the case with an endogenous interest rate. In that case, the stock of long-term capital and the level of output remain significantly below the steady state level for around 13

[20] The value for the quarterly persistence of the financial shock is taken from Hall (2011), who estimates the persistence of the increase in spreads during the financial crisis of 2008-2009.

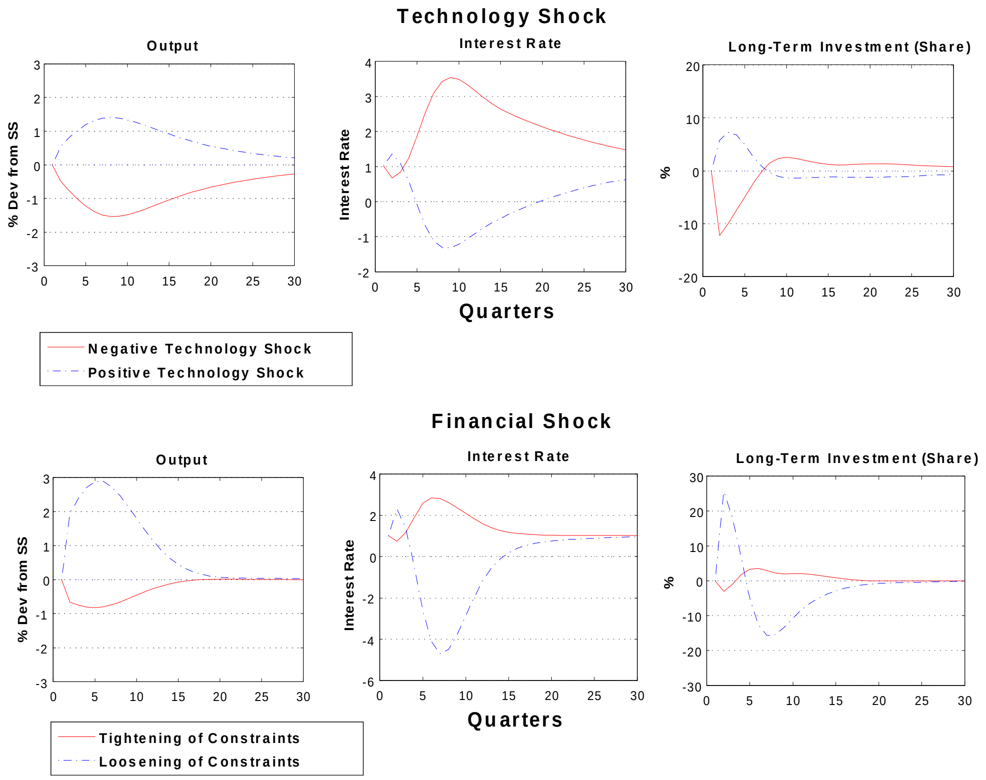


Figure 8: ASYMMETRY IN THE RESPONSE TO SHOCKS. The figure displays the response of three key macroeconomic variables to negative and positive shocks to aggregate productivity (θ_t) and to the severity of financing constraints (λ_t). The response of the share of long-term investment is calculated as the difference between the share in each period and its steady state value.

quarters, even though 95% of the financial shock has died out by the $6th$ quarter.

5.2 Asymmetries

The nature of our solution method allows for the sign of the shock to influence transitional dynamics, and in Figure 8 we analyze the results of such an exercise by comparing the transitional dynamics of shocks to technology and financing frictions of different signs. We focus first on the analysis of technology shocks, in which we compare the reaction to a +1% and a −1% shock to aggregate productivity θ.[21] The economy displays a moderately stronger reaction to negative technology shocks than to positive ones, but the differences are small. The difference is most obvious in interest rates, with a maximum fall of 232 basis points in the case of a positive shock, and a maximum increase of only 79 basis points in the case of a negative shock. The drop in long-term investment as a share of total investment in the negative shock case is nearly double the size as the increase in that ratio in the positive shock case. The differences in terms of the reaction of output are more moderate, with a maximum drop of 1.53% in the case of a negative shock compared against a maximum increase of 1.41% in the positive shock case.

[21] For both shocks, we use the same persistence parameter values as in the analysis of section 4.

In the case of financial shocks, the differences are much more stark, and the shock that is most amplified is the positive shock (a loosening of financing frictions). In this exercise, we compare a $+10\%$ with a -10% shock to λ_t, the parameter that determines the share of next period's output that can be pledged to lenders today. In other words, λ_t decreases in $t = 1$ unexpectedly from $\lambda_{t=0} = 0.8$ to $\lambda_{t=1} = 0.7$ in one case, and increases in $t = 1$ unexpectedly from $\lambda_{t=0} = 0.8$ to $\lambda_{t=1} = 0.9$ in another. Output, the interest rate, and the share of long-term investment in total investment react much more strongly to a loosening of credit constraints than to a tightening. Output falls by 0.83% relative to its steady state value at the trough following a shock that increases financing frictions, but increases 2.88% at its peak following a credit constrained loosening.

The intuition for this large asymmetry is that in equilibrium firms converge broadly into two groups, one in which firms hold few assets and find it hard to invest and grow out of their credit constraints because they are severely credit constrained, and a group of older firms that hold a large amount of assets and are relatively unconstrained.[22] A negative financial shock hurts credit constrained firms' total investment, which was already relatively small in size before the shock, and produces a moderate drop in output, but a positive shock enables these firms to overcome their constraints after a few periods and significantly increase their investment, resulting in a large output increase. This asymmetry in the response to a financial shock highlights the importance of allowing for credit constraints to bind occasionally and of carefully modelling firm heterogeneity in models that attempt to capture the macroeconomic implications of credit constraints.

[22] These patterns are displayed in Figure 3.

6 Conclusion

This paper introduces a model in which credit constrained firms can choose between investment projects that differ in their duration, pledgeability of their returns, and liquidity risk. The model provides a novel theoretical underpinning for the countercyclicality of financing constraints based on the procyclicality of investment return pledgeability. We use this framework to explore how the consideration of firms' investment choice problem in the presence of financing frictions might affect the role of such frictions in amplifying or dampening shocks to the macroeconomy. A novel dampening mechanism of technology shocks is identified, which is based on time-varying financing constraints that affects firms' preference for the duration profile of their portfolio of investment projects. The dampening effect is shown to be quantitatively large, especially in the presence of general equilibrium interactions. On the other hand, this framework is able to account for the empirically documented cyclical variation in the composition of investment, a feature which most existing models studying the macroeconomic implications of financial constraints cannot account for.

One limit of the analysis is that it only considers the impact of unexpected shocks. Introducing aggregate uncertainty in the form of stochastic productivity or financing constraints processes could influence the results obtained in this paper, particularly if firms are able to write financial contracts based on the outcomes of these aggregate shocks. A framework with multiple aggregate shocks and two-dimensional firm heterogeneity becomes computationally demanding, however, and whereas a body of methodological research has provided feasible and accurate approximations for a subset of environments featuring one-dimensional heterogeneity in the household sector and aggregate uncertainty, similar work has yet to appear in more general contexts that could be applicable to an environment like the one described in this paper.

This paper highlights the importance for models designed to study the macroeconomic implications of credit constraints of considering the type of investment firms carry out, in addition to the amount. While this paper focuses on a particular set of dimensions of project heterogeneity, several other important dimensions remain to be analyzed in the context of a general equilibrium heterogeneous agent framework like the one presented in this paper, such as minimum investment size, degree of investment irreversibility, or volatility of returns. Models that incorporate these or other dimensions of heterogeneity could be fruitful avenues of research.

A Appendix: Numerical Solution

A.1 Steady State

In the steady state, equilibrium interest rate r_{ss} will be equal to the households' rate of time preference given that they are financially unconstrained, so

$$r_{ss} = \frac{1}{\beta} - 1.$$

Firms' optimization in the steady state is solved taking r_{ss} as given, and assuming aggregate productivity θ_t and financing frictions λ_t are constant at their steady state level. Value function iteration is used, and the firm's idiosyncratic state space is discretized. Along the dimension of asset holdings $a_{f,t}$ the grid contains 20 points starting at $a_{f,t} = 0$, and given that important non-linearities arise in the value function for low values of asset holdings, the grid's density decreases exponentially with asset holdings. Along the dimension of long-term capital holdings $k_{L,t}$ the value function is moderately concave for most of the region, due to the presence of convex costs of adjustment, and for this reason 40 equally-spaced gridpoints are chosen, again starting at $k_{L,t} = 0$.

The steady state distribution of firm asset and long-term capital holdings is calculated using the firms' optimal policies by performing 600 iterations of a firm sector composed of 250,000 firms. From the converged distribution we obtain the steady state aggregate output, aggregate dividends, aggregate net corporate borrowing and wages. In equilibrium, firms with large asset holdings save and those with low asset holdings borrow. It is the case for all combinations of parameter values considered that net borrowing of the firm sector is positive.

In the steady state equilibrium households' consumption must be equal to the sum of wages, dividends and the return on savings:

$$C_{ss} = r_{ss}A_{ss} + W_{ss} + D_{ss},$$

where aggregate savings A_{ss} are equal to aggregate net corporate borrowing.

A.2 Transitional Dynamics Following Temporary Shocks

The solution method to obtain the transitional dynamics proceeds as follows. The process is broadly similar for technology and financial shocks, and the few differences are pointed out.

1. We first establish the shock size and persistence

2. We guess the number of periods it will take for convergence back to the initial steady state. We use 150 periods for technology shocks, and 80 periods for financial shocks, which are less persistent.

3. We guess a path for r_t and for C_t for all the transition periods

4. We start backwards and, taking the $t+1$ value function of firms as given, calculate optimal time t policies of firms and the resulting value function given (i) the path of the shock, and (ii) the conjectured path for $\{r_t\}$. We do this for every period, using the updated $t+1$ value function when calculating the time t policies and value function.

5. Starting from the steady state distribution of firms, we simulate the entire firm sector (250,000 firms) for the duration of the transitional dynamics using the optimal policies calculated in the previous step. For each period, we calculate aggregate dividends D_t, aggregate net corporate borrowing (which we will equate to A_{t+1}) and wages W_t. Next, for each period we obtain the r_{t+1} that clears the bond market. We do so by first calculating the household consumption C_t^{eq} that would be consistent with a level of household savings equal to net corporate borrowing

$$C_t^{eq} = (1+r_t)A_t + W_t + D_t - A_{t+1}^{eq},$$

taking r_t from our guess and A_t from the previous period, and W_t, D_t and A_{t+1}^{eq} from the simulation of firms. Finally, we calculate the equilibrium interest rate that clears the bond market

$$r^{eq} = \frac{1}{\beta}\left(\frac{C_t^{eq}}{C_{t+1}}\right)^{-\gamma} - 1,$$

taking C_{t+1} as given from the conjectured path.

6. Update the conjectured path for r_t and C_t using a small updating parameter (0.02 in our case), and repeat 4-5 until convergence.

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