

ABSTRACT

“North-South Convergence and the Allocation of CO₂ Emissions,”

by

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We postulate a two-region world, comprised of North (calibrated after the US) and South (calibrated after China). Our optimization results show the compatibility of the following three desiderata:

- (1) Global CO₂ emissions follow a conservative path that leads to the stabilization of concentrations at 450 ppm.
- (2) North and South converge to a path of sustained growth at 1% per year (28.2% per generation) in 2075.
- (3) During the transition to the steady state, North also grows at 1% per year while South's rates of growth are markedly higher.

The transition paths require a drastic reduction of the share of emissions allocated to North, large investments in knowledge, both in North and South, as well as very large investments in education in South. Surprisingly, in order to sustain North's utility growth rate, some output must be transferred from South to North during the transition.

Although undoubtedly subject to many caveats, our results support a degree of optimism by providing prima facie evidence of the possibility of tackling climate change in a way that is fair both across generations and across regions while allowing for positive rates of human development.

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1 Introduction

1.1 Motivation of Convergence in 75 Years

Two questions are central to a successful attack on the problem of global warming: (1) what is the magnitude of global emissions of greenhouse gases that should be set as the target, in order to approximately stabilize the concentration of carbon in the atmosphere at an acceptably low level, and (2) how should this budget of total emissions be allocated to regions of the world. Both issues are contentious. In this paper, we shall assume the first question has been answered – and we shall take that answer to be a path of emissions that will approximately stabilize atmospheric concentration at 450 ppm – and focus upon the second question.

A number of proposals are on offer that begin with an ethical premise. One is that each country should be allocated permits to emit in proportion to its population, and then a market in permits be used to re-allocate permits according to willingness to pay. The ethical postulate behind this proposal is clearly that each citizen of the world has an equal property right in the relevant global commons – the biosphere. Another proposal is that account must be taken of the fact that the currently industrialized countries (call these the North) are responsible for the lion’s share of existing carbon pollution, and so their budget should be correspondingly smaller than that of the ‘South.’ The North has effectively used a large share of its entitlement, and it is now South’s turn.

Our approach will be different: we wish to propose a solution which we believe is politically feasible, a solution that all countries might agree to. We do not think that either of the above proposals is politically feasible today. For the sake of argument, we shall heretofore simplify by assuming that the world consists of two regions: North, which we take to have the

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level of economic development of the United States, and South, which we take to have the level of economic development of China. It will not hurt very much if the reader wishes to consider our model one of how the actual nations of the US and China should allocate emission rights between them, assuming that they were the only countries in the world.

Indeed, digressing from the model, it is probably the case that an agreement between the US and China concerning how to constrain their emissions is probably both necessary and sufficient for a global agreement. It is obviously necessary, since these are the two largest emitters of greenhouse gases (GHG's). But it is also perhaps sufficient: it is likely that if these two giants can agree, the rest of the world will fall into line.

Our proposal is based upon a view of the kind of bargaining that will take place between China and the US about the allocation of emissions between them, assuming, as we said, that the total amount of emissions has already been agreed to. We believe that Thomas Schelling's idea of a salient focal point is key here, and we think the salient focal point is the *date at which Chinese GDP and US GDP per capita will converge*. (More specifically, we believe per capita *welfare* is the right equalisandum, but we ignore this distinction in this introduction.) We can view the convergence of the GDP's per capita of the North and South as something of both ethical and political importance: but here, we emphasize the political aspect. In short, we propose that the focus of bargaining over the allocation of emissions between the US and China will be to maintain the date of convergence of their respective GDP's per capita; that is, each country should reduce its growth factor in such a way that the global emissions constraint is satisfied, and the date of convergence remains what it would have been if the problem of global warming did not exist. Following the Stern Review (2007), we can call the latter the business-as-usual, or BAU, scenario.

To see why this is the case, suppose that under BAU, China and the US would converge in GDP per capita in 75 years. Suppose that a proposal were made for emissions control that implied that the two GDP's would converge in 100 years. Then we believe China would say, "This is unfair. You, US, are advantaged by this proposal, because you are remaining ahead of us for 25 years longer than would have been the case under BAU." In like manner, if a proposal were tendered in which convergence would occur in 60 years, the US could prosecute a similar case against China. The equilibrium in the bargaining game therefore should preserve the expected date of convergence under BAU.

It is simple to compute the implication of this proposal. Suppose that the current Chinese

and US GDP's per capita are y^{Ch} and y^{US} , and suppose their (annual) growth rates would be constant over the next century under BAU at values g^{Ch} and g^{US} , Then the date of convergence is the solution T of the equation:

$$\frac{(1+g^{Ch})^T y^{Ch}}{(1+g^{US})^T y^{US}} = 1.$$

For example, if we take $y^{Ch} = \$5,970$ and $y^{US} = \$47,400$ (2008 figures) and $g^{US} = 0.02$ and $g^{Ch} = 0.05$, then the solution is $T = 71.5$ years – about three generations, if a generation is 25 years. Now suppose that each country multiplies its growth factor by the same fraction r ; then the date of convergence is clearly maintained:

$$\frac{(r(1+g^{Ch}))^T y^{Ch}}{(r(1+g^{US}))^T y^{US}} = \frac{(1+g^{Ch})^T y^{Ch}}{(1+g^{US})^T y^{US}}.$$

So the focal-point bargaining we have proposed implies that each country should reduce its growth factor by the same fraction.

Now many details would have to be filled in concerning the bargaining between the US and China: the bargaining is probably going to take place quite often (not once for the next 75 years), there will be disagreements about what growth rates would have been under BAU, and so on. In this paper, we will not pursue these issues further, important as they may be. Our goal, rather, is to understand what the allocation of emissions to China and US should be if we would like them to converge (in welfare per capita) in three generations, where the convergence occurs in an optimal way. In other words, we wish to derive the path of emissions that each region should follow as a consequence of two premises: that they converge in 75 years, and that the growth paths of the two countries be somehow optimal, subject to not exceeding the constraint that we impose on total emissions.

1.2 Optimality

Our previous papers on intergenerational ethics and climate change (see Llavador, Roemer and Silvestre, 2010, and In Press, to be referred to as LRS 2010 and LRS In Press respectively) have studied the problem of optimizing human welfare across many generations, in a warming planet, when there is a single representative family (adult and child) at each generation. The

innovation of the present paper is the introduction of *two* representative families at each date, one from a Southern economy and one from a Northern one. Thus, we here introduce intragenerational considerations into the intergenerational problem.

In the above papers, we advocated *sustainability* as the ethical objective. We interpreted sustainability in two ways: (A) sustaining the level of human welfare at the highest possible level forever, or (B) sustaining the *growth rate* of human welfare forever, for some specified growth rate ρ . Suppose a utility path for this society of an infinite number of generations, with one representative adult at each date, is $u = (u_1, u_2, u_3, \dots)$. Let \mathfrak{S} be the set of utility paths that are feasible, beginning with the present technology and endowments, *and* which maintain global carbon concentration at – say – 450 ppm by constraining emissions appropriately. Then the problem of sustaining the highest possible level of human welfare forever, problem (A) above, can be stated as:

$$\begin{aligned} & \max \Lambda \\ & \text{s.t. } u \in \mathfrak{S}, \\ & \quad u_t \geq \Lambda, \quad t = 1, 2, \dots \end{aligned} \quad (\text{SUS})$$

We have defined the set \mathfrak{S} explicitly by specifying three sectors of production, and their technologies, as shall do below in Section 2. The solution to (SUS) for our calibration of the model (see LRS 2010) entailed that, indeed, the sustainability constraint held with equality for all t in the optimum: $u_t = \Lambda$ for all t . We call this *pure sustainability*.

Clearly, pure sustainability is another name for Rawlsian maximin applied to an intergenerational world. Indeed, an ethical justification of pure sustainability is that the date at which a person is born is morally arbitrary, and so each person, regardless of her birth date, has a right to as much welfare as a person born at any later date.

However, individuals may wish to abdicate this right, if they view the growth of human welfare as desirable. If all humans, say, would be willing to sacrifice some welfare if doing so enabled future generations to be better off than they, they would abdicate the right they putatively hold, expressed just above. Then the right problem to solve would be of the form:

$$\begin{aligned} & \max \Lambda \\ & \text{s.t. } u \in \mathfrak{S}, \\ & \quad u_t \geq (1 + \rho)^{t-1} \Lambda, \end{aligned} \quad (\rho\text{-SUS})$$

for some (perhaps small, positive) growth rate ρ . In particular, at least the first generation will

have lower welfare in the solution to (ρ -SUS) than in the solution to (SUS), but since (as we said) welfare is constant at the solution of (SUS) for our model, eventually generations will be much better off in the solution to (ρ -SUS). Note that when we here say ‘welfare,’ it would now be better to say ‘standard of living,’ as welfare evidently includes a desire for a positive rate of growth of standard of living.

In LRS 2010 and in Roemer (In Press) we defended the sustainability approach against the more common discounted utilitarian approach, which instead would solve the problem

$$\begin{aligned} \max \sum_1^{\infty} \varphi^{t-1} u_t & \quad (\text{DU}) \\ \text{s.t. } u \in \mathfrak{S}, & \end{aligned}$$

for some φ in the interval $(0,1)$. We will not repeat these arguments here, although, it must be noted, it is principally our choice of a sustainability objective that differentiates our work from that of many other economists, who prefer the specification of the problem in its discounted-utilitarian form.

The question we now must address is: How shall we adapt our sustainability approach to the problem where there are two representative agents (or households) at each date – one from China, one from the US? The answer is based upon a turnpike theorem that we prove in LRS In Press. In the fleshed-out economic model of which the program (SUS) is an abstract version, we have an economy that begins with a vector of endowments of capital, knowledge, and labor. We fix a path of emissions that converges to the desired atmospheric concentration of carbon as one of the constraints defining \mathfrak{S} . (Emissions in turn constrain the production of commodities used for consumption and investment.) The turnpike theorem makes two claims: first, if emissions are such as to maintain a constant level of atmospheric carbon concentration, then there exists a ray in \mathfrak{R}_+^3 , such that if the initial endowment vector of labor, capital and knowledge lies on the ray, then the solution of (SUS) is constant in all variables (investment, capital stock, consumption, education, labor expended in three sectors, etc.). The second claim is that if we begin with an endowment vector off this ray, then the endowments of the economy in optimal solution of (SUS) converge to it, and so the solution path converges to the constant path. If, for a specified $\rho > 0$ there is a feasible solution to program (ρ -SUS), then we also prove the existence of a ray $\Gamma(\rho, \cdot)$ with the analogous first property: that if the initial endowment lies on $\Gamma(\rho, \cdot)$, the solution to (ρ -SUS) is a

path in which all variables grow forever at rate ρ . We have not proved the second part of the growth-turnpike theorem, but we suppose that it is true.

Motivated by the turnpike property, we shall model the problem of North-South emissions-sharing as one where the Northern and Southern representative households begin with different endowments, and we study the set of paths of resource use under which both representative agents converge to the same point on the ray $\Gamma(\rho, \cdot)$ in 75 years (or three generations): the assumption is that both economies then enjoy balanced growth at rate ρ from that date on.

But there are many paths upon which the North and the South will converge to the same point on the ray $\Gamma(\rho, \cdot)$ (for some fixed ρ) in 75 years. Among these we choose an optimal path. We shall describe below exactly what we optimize.

In Section 2, we present our model of two representative agents at each date, one with Chinese and the other with US characteristics. In Section 3, we discuss what to optimize during the three generations in which convergence to $\Gamma(\rho, \cdot)$ takes place: inter alia, this will determine the point on the ray $\Gamma(\rho, \cdot)$ to which the two households converge. In Section 4, we present optimal paths, and discuss and interpret our results.

2 The Model

We adapt the model in LRS 2010 to a world comprised of two regions, namely North (N) and South (S). Generations are indexed by $t \geq 1$, and understood to live for 25 years. The population of Generation t in Region $J = N, S$, denoted by N_t^J , is exogenously given in accordance with United Nations projections. (See Table 2 below.) A zero subscript indicates year-2000 reference values.

2.1 Utility

The utility functions in North and South are identical and as defined in LRS 2010. As in there, consumption, educated leisure, the stock of human knowledge, and the quality of the biosphere are the arguments in the utility function. The first two arguments are private goods, and the last two are public goods.

We assume a representative agent in each generation and region. We assume that a

generation lives for 25 years, and we formally postulate the following utility function of Generation t , $t \geq 1$ in region J , $J = N, S$:

$$(c_t^J)^{\alpha_c} (x_t^{lJ})^{\alpha_l} (S_t^{nJ})^{\alpha_n} (\hat{S}^m - S_t^m)^{\alpha_m}, \quad (1)$$

where the exponents are positive and normalized such that $\alpha_c + \alpha_l + \alpha_n + \alpha_m = 1$ and where:

- c_t^J = annual average consumption per capita by Generation t in region J ;
- x_t^{lJ} = annual average leisure per capita, in efficiency units, by Generation t in region J ;
- S_t^{nJ} = stock of knowledge per capita in region J , which enters Generation t 's utility function and production function, understood as located in the last year of life of Generation t ;
- S_t^m = total CO₂ in the atmosphere above the equilibrium pre-industrial level, in GtC, which is understood as located in the last year of life of Generation t ;² and
- \hat{S}^m = “catastrophic” level of CO₂ in the atmosphere above the pre-industrial level.

2.2 Production Function

We postulate that North and South have the same technology, but different initial education levels and stocks of knowledge and physical capital. As in LRS 2010, the production function of output in Region J ($J = N, S$) is

$$f(x_t^{cJ}, S_t^{kJ}, S_t^{nJ}, e_t^J, S_t^m) \equiv k_1 (x_t^{cJ})^{\theta_c} (S_t^{kJ})^{\theta_k} (S_t^{nJ})^{\theta_n} (e_t^J)^{\theta_e} (S_t^m)^{\theta_m}, \quad t \geq 1, \quad (2)$$

where $k_1 > 0$, $\theta_c > 0$, $\theta_k > 0$, $\theta_n > 0$, $\theta_e > 0$, $\theta_c + \theta_k + \theta_n + \theta_e = 1$, $\theta_m < 0$, where S_t^m and S_t^{nJ} have been defined above, and where:

- x_t^{cJ} = average annual efficiency units of labor per capita devoted to the production of output by Generation t in region J ;
- S_t^{kJ} = capital stock per capita available to Generation t in Region J ;
- e_t^J = average annual emissions of CO₂ in GtC by Generation t in Region J .

² The preindustrial values for the CO₂ stock are taken to be 595.5 GtC or 280 ppm. To convert our S_t^m into CO₂ ppm, add 595.5 to S_t^m and multiply by 0.47. To convert a number of CO₂ ppm into our S_t^m , subtract 280 from it and multiply by 2.13. The presence of the stock of CO₂ in the utility function captures our view that environmental deterioration is a public bad in consumption (as well as in production).

We call emissions e_t^J and concentrations S_t^m *environmental variables*, whereas all other variables will be called *economic*.

Remark 1. The labor input in production, x_t^{cJ} , is measured in efficiency units of labor, which may be viewed as the number of labor-time units (“hours”) multiplied by the amount of human capital embodied in one time unit.

2.3 Emissions and Concentrations

As in LRS 2010, because of the complexity of the climate models proposed and the lack of a canonical physical model of the current state of climate science, we shun false precision and do not attempt to specify the set of climatologically feasible flow-stock sequences. Instead, we adopt a simple emission path, inspired by Meehl *et al.* (2007, Section 10.4, Figure 10.21(a)), that leads to a target stabilization level of 450 ppm. These paths involve increasing emissions in the near future, and drastically reducing emissions in the more distant future. We adopt this general pattern, but we simplify the path by postulating only three levels of emissions and stock, which average over each generation the above-mentioned lifetime paths for emissions, while taking as stock values those dated at the end of the life of the generation. Hence, the Meehl *et al.* (2007) analysis justifies the feasibility of our paths given the initial values $(e_0^W, S_0^m) = (6.58, 177.1)$ at year 2000 (World Resources Institute, 2009).³ Writing e_t^{W*} for annual world emissions by Generation t in GtC, our postulated (emission, atmospheric stock) pairs are:

$$(e_1^{W*}, S_1^{m*}) = (6.97, 303) \text{ for Generation 1,}$$

$$(e_2^{W*}, S_2^{m*}) = (4.41, 354) \text{ for Generation 2,}$$

$$\text{and } (e_t^{W*}, S_t^{m*}) = (e^{W*}, S^{m*}) = (0.4, 363) \text{ for Generation } t, t \geq 3,$$

see Table 1 and Figure 2 in Section 4 below for a graphical representation. Table 2 adds population data for North and South, with the notation $N_t = N_t^N + N_t^S, t = 0, 1, 2$, and

³ We take $S_0^m = 177.1$ GtC (or 83 ppm) as the year 2000 atmospheric CO₂ concentration above pre-industrial level (of approximately 595.5GtC in 1850) from the CAIT Indicator Framework Paper (World Resources Institute, WRI, 2009). Total annual world emissions from energy (fossil fuels and cement) are 6.58 GtC. Once we include CO₂ emissions from land use change (7.62 GtCO₂e) and from other Kyoto gases (9.72 GtCO₂e), total emissions (41.42 GtCO₂e) are consistent with the 42 GtCO₂e total GHG emissions in 2000 reported in the Stern Review (page 170).

$n_t^J \equiv \frac{N_t^J}{N_t}$, $t = 0, 1, 2$, $J = N, S$, as well as the world per capita emissions, which correspond to the

totals of column 1 in Table 1, with the notation $e_t^* \equiv \frac{e_t^{W*}}{N_t^N + N_t^S}$, $t = 0, 1, 2$.

Our choices for (e_1^{W*}, S_1^{m*}) , (e_2^{W*}, S_2^{m*}) and (e^{W*}, S^{m*}) imply that, in 2075, the concentration of CO₂ in the atmosphere will be of 450 ppm: this corresponds to our value of $S^{m*} = 363$ GtC for the atmospheric stock of CO₂ above the preindustrial stock, see Footnote 2 above.

In a nutshell, our approach to emissions consists in postulating a given path of world emissions $\{e_t^W, t \geq 1\}$ while endogeneizing the allocation of these emissions between North and South by solving the relevant optimization problem.

	World Total CO ₂ Emissions (GtC)	North's Total CO ₂ Emissions (GtC)	South's Total CO ₂ Emissions (GtC)	Stock of CO ₂ in (World) Atmosphere (GtC)
Year 2000	$e_0^W = 6.58$	3.82	2.76	$S_0^m = 177.1$
Generation 1	$e_1^{W*} = 6.97$	Endogenous	Endogenous	$S_1^{m*} = 303$
Generation 2	$e_2^{W*} = 4.41$	Endogenous	Endogenous	$S_2^{m*} = 354$
Generation t , $t \geq 3$	$e^{W*} = 0.4$	Endogenous	Endogenous	$S^{m*} = 363$

Table 1. Our postulated paths for the world annual CO₂ emissions and stocks.

$t = 0$	$t = 1$	$t = 2$		Units
N_0^N	N_1^N	N_2^N		
1,194,199	1,258,970	1,245,247		thousands
N_0^S	N_1^S	N_2^S		
4,929,924	6,751,540	7,946,040		thousands
n_0^N	n_1^N	n_2^N		
0.195	0.157	0.135		proportion of total pop.
n_0^S	n_1^S	n_2^S		
0.805	0.843	0.865		proportion of total pop.
N_0	N_1	N_2		
6,124,123	8,010,509	9,191,287		thousands
e_0^*	e_1^*	e_2^*	e^*	
1.075	0.87	0.48	0.04	tC per capita

Table 2. Population and world per capita emissions.

2.4 Laws of Motion and Initial Conditions

Two new features appear in this model that had no place in the model of LRS 2010: there is *diffusion* of knowledge from North to South (that is, knowledge is an interregional public good), and there is the possibility of *transferring output* between the two regions.

Recall that we take as exogenous the paths for world emissions and the stock of CO₂. Our model displays three additional intergenerational links given by the stock of physical capital, the stock of knowledge and the level of education. Our per capita variables must be consistent with the population data $N_t^J, t \geq 0, J = N, S$.

The *law of motion of physical capital* in each region is standard, namely

$$(1 - \delta^k) S_{t-1}^{kJ} \frac{N_{t-1}^J}{N_t^J} + k_2 i_t^J \geq S_t^{kJ}, t \geq 1, J = N, S, \quad (3)$$

where $k_2 > 0$, $\delta^k \in (0, 1)$, and $i_t^J \geq 0$ is the average annual investment in physical capital (units of output per capita) by Generation t in Region J .

We assume that, in North, the creation of new knowledge requires only efficiency labor (dedicated to R&D, or to “learning by not doing”), but that knowledge depreciates at a positive rate, i. e., the *law of motion of the stock of knowledge in North* is

$$(1 - \delta^n) S_{t-1}^{nN} \frac{N_{t-1}^N}{N_t^N} + k_3 x_t^{nN} - S_t^{nN} \geq 0, t \geq 1, \quad (4)$$

where $k_3 > 0$, $\delta^n \in (0,1)$, and $x_t^{nN} \geq 0$ is the average annual number of efficiency units of labor per capita devoted to the creation of knowledge by Generation t in North.

As long as North’s stock of knowledge per capita is larger than that of South’s, we postulate that North’s knowledge spills over to South, which in addition can devote a fraction of its efficiency labor to the creation of knowledge. Hence, the law of motion for the stock of knowledge in South captures the presence of international technological diffusion (Eaton and Kortum 1999, Keller, 2004). Our formulation starts from a year-to-year equation for knowledge diffusion that after some manipulation (see Appendix C below) yields the following generational *law of motion for the stock of knowledge in South whenever $S_{t-1}^{nN} > S_{t-1}^{nS}$* :

$$(1 - \delta^n)(1 - \lambda) \frac{N_{t-1}^S}{N_t^S} S_{t-1}^{nS} + (1 - \delta^n) \lambda \frac{N_{t-1}^N}{N_t^N} \frac{N_{t-1}^N}{N_t^S} S_{t-1}^{nN} + k_{3N} \frac{N_{t-1}^N}{N_t^S} x_t^{nN} + k_{3S} x_t^{nS} \geq S_t^{nS}, \quad (5)$$

$$t \geq 1, S_{t-1}^{nN} > S_{t-1}^{nS},$$

where $\lambda \in [0,1]$ is the generational technological diffusion parameter, $k_{3N} > 0$, $k_{3S} > 0$, and x_t^{nS} is the average annual efficiency units of labor per capita devoted to the production of knowledge by Generation t in South.

The *law of motion of the stock of knowledge in South for $S_{t-1}^{nN} = S_{t-1}^{nS}$* parallels (4), i. e.,

$$(1 - \delta^n) S_{t-1}^{nS} \frac{N_{t-1}^S}{N_t^S} + k_3 x_t^{nS} - S_t^{nS} \geq 0, t \geq 1, S_{t-1}^{nS} = S_{t-1}^{nN} \quad (6)$$

LRS 2010 justifies our education production function, which transforms labor-leisure time in efficiency units of labor and leisure. It states that the education of a young generation requires only efficiency labor of the previous generation. Formally, the *education production function* is given by

$$x_t^J \leq k_4 x_{t-1}^{eJ} \frac{N_{t-1}^J}{N_t^J}, t \geq 1, J = N, S, \quad (7)$$

where $k_4 > 0$, and $x_t^{eJ} \geq 0$ (resp. $x_t^J \geq 0$) is the annual average number of efficiency units of labor

per capita devoted to education by Generation t (resp., per capita units of time, labor plus leisure, available to Generation t) in Region J . If we normalize to unity the total labor-leisure time available to Generation t , then x_t can be interpreted as the amount of human capital per unit time in Generation t .

The available amount of efficiency units of labor is allocated into four uses, namely

$$x_t^{eJ} + x_t^{cJ} + x_t^{nJ} + x_t^{lJ} \equiv x_t^J, \quad t \geq 1, J = N, S. \quad (8)$$

The *initial conditions* are $(x_0^{eJ}, S_0^{kJ}, S_0^{nJ}) \in \mathfrak{R}_{++}^3$, $J = N, S$.

2.5 Calibrated Values

Appendix B below details our calibration procedures, which yield the following values for the parameters and initial conditions of our model.

Parameter	Value
α_c	0.32
α_l	0.65
α_n	0.02
α_m	0.01
k_1	21.5861
k_2	13.1182
k_3	649.349
k_{3N}	133.267
k_{3S}	516.082
k_4	35.45
λ	0.928
δ_k	0.787
δ_n	0.787
\hat{S}^m	781.55
θ_c	0.6667
θ_k	0.2019
θ_n	0.0404
θ_e	0.0910
θ_m	-0.0075

Table 3. Calibrated parameter values.

Stocks	Value	Units
S_0^{kN}	73.65	thousands of 2000 dollars per capita
S_0^{kS}	0.50	thousands of 2000 dollars per capita
S_0^{nN}	15.64	thousands of 2000 dollars per capita
S_0^{nS}	1.103	thousands of 2000 dollars per capita
S_0^m	177.1	GtC above pre-industrial level
x_0^{eN}	0.0461	1950-US efficiency units per capita
x_0^{eS}	0.0206	1950-US efficiency units per capita

Table 4. Initial values of the stocks in the reference year (2000).

3 Optimization

As explained in the introduction, we impose the convergence of North's and South's utility at date 3, i. e., the utility of the representative citizen of South is the same as that of a contemporaneous citizen of North for all generations t greater than or equal to 3. Because South starts at a lower utility level, South must grow faster than North in the transition period $t = 1, 2$.

But, once convergence is achieved, we aim at steady growth at a common given rate. The Ray Optimization Theorem of LRS In Press, in the spirit of turnpike theory, underpins our analysis. Consider a given triple (ρ, e^*, S^{m*}) , where ρ is a rate of utility growth per generation, and (e^*, S^{m*}) is a CO₂ emission-stock pair. Consider also the program of maximizing the utility of Generation 1 subject to the condition that utility subsequently grows at rate ρ . The Ray Optimization Theorem asserts the existence of a ray $\Gamma(\rho, e^*, S^{m*})$ in \mathfrak{R}_+^3 such that, if the initial conditions (x_0^e, S_0^k, S_0^n) belong to this ray, then at the solution of the program utility grows at rate ρ , the environmental variables remain constant at (e^*, S^{m*}) and all economic variables grow at rate g (per generation), where $1 + \rho = (1 + g)^{1 - \alpha_m}$. Denote by $\gamma_1(\rho, e^*, S^{m*})$ (resp. $\gamma_2(\rho, e^*, S^{m*})$) the ratio of consumption at $t = 1$ to x_0^e (resp. leisure at $t = 1$ to x_0^e) at the solution.

Accordingly, we choose a rate of utility growth ρ per generation (in fact, we choose $\rho = 28.2\%$ per generation, which corresponds to 1% per annum) and put the stocks of labor,

capital and knowledge on the ray $\Gamma(\rho, e^*, S^{m*})$ at $t = 2$. It follows that, for $t \geq 3$, both North and South are in steady state, their utilities equal and growing together at rate ρ . The utility levels of North's and South's year-2000 reference values are historically given, and Generations 1 and 2 in North and South are the *transition generations* from the reference values to the state of steady growth.

We constrain North to grow at rate ρ during the transition. It follows that the utility level of North's Generation t is North's year-2000 reference value multiplied by $(1+\rho)^t$. On the other hand, the utilities of South's transition generations 1 and 2 are endogenized by the optimization program.

The worst off generation (for $t > 0$) is then South's Generation 1, and the Maximin principle requires the maximization of its utility. But it turns out that this requires Generation 1 in South to invest little in education, which imposes low utility in South's Generation 2. The resulting path entails, in South, little growth from $t = 1$ to $t = 2$, together with a big jump from $t = 2$ to $t = 3$. On the other hand, maximizing the utility of Generation 2 in South subject to Generation 1 growing at rate of at least ρ gives a smoother path *with little loss of utility* for Generation 1. For these reasons, we choose to maximize the utility of Generation 2 in South subject to all the stated conditions, evidently including the constraints imposed by technology (2), the laws of motion of stocks (3)-(6) and the labor-resource constraint (8).

We now explicitly write our formal optimization program. Because we seek convergence of the economic stocks at $t = 2$, and of flows at $t = 3$, we use the notation $S_2^j \equiv S_2^{jN} = S_2^{jS}$, $j = k, n$, $S_3^n \equiv S_3^{nN} = S_3^{nS}$, $x_2^e \equiv x_2^{eN} = x_2^{eS}$, $c_3 \equiv c_3^N = c_3^S$, and $x_3^l \equiv x_3^{lN} = x_3^{lS}$. Moreover, as indicated above, we assume that output is transferable between regions during the transition: for $t = 1, 2$, T_t denotes the number of units of output per capita in North that North transfers to South. A negative T_t indicates a transfer from South to North.

Optimization Program

Choose , $S_1^{kN}, S_1^{kS}, S_2^k, S_1^{nN}, S_1^{nS}, S_2^n, i_1^N, i_1^S, i_2^N, i_2^S, c_1^N, c_1^S, c_2^N, c_2^S, e_1^N, e_1^S, e_2^N, e_2^S$,
 $x_1^{lN}, x_1^{lS}, x_1^{eN}, x_1^{eS}, x_1^{nN}, x_1^{nS}, x_2^{lN}, x_2^{lS}, x_2^{eN}, x_2^{eS}, x_1^{lS}, x_1^{eS}, x_1^{nS}, x_2^{lS}, x_2^{eS}, x_2^{nS}, T_1, T_2, c_3, x_3^l$ and S_3^n
in order to maximize Λ_2^S subject to:

$$\begin{aligned}
& (c_2^S)^{\alpha_c} (x_2^{IS})^{\alpha_l} (S_2^n)^{\alpha_n} (\hat{S}^m - S_2^{m*})^{\alpha_m} - \Lambda_2^S \geq 0, \\
& (c_1^S)^{\alpha_c} (x_1^{IS})^{\alpha_l} (S_1^{nS})^{\alpha_n} (\hat{S}^m - S_1^{m*})^{\alpha_m} - (1+\rho)\Lambda_0^S \geq 0, \\
& (c_1^N)^{\alpha_c} (x_1^{IN})^{\alpha_l} (S_1^{nN})^{\alpha_n} (\hat{S}^m - S_1^{m*})^{\alpha_m} - (1+\rho)\Lambda_0^N \geq 0, \\
& (c_2^N)^{\alpha_c} (x_2^{IN})^{\alpha_l} (S_2^n)^{\alpha_n} (\hat{S}^m - S_2^{m*})^{\alpha_m} - (1+\rho)^2 \Lambda_0^N \geq 0, \\
& (c_3)^{\alpha_c} (x_3^I)^{\alpha_l} (S_3^n)^{\alpha_n} (\hat{S}^m - S^{m*})^{\alpha_m} - (1+\rho)^3 \Lambda_0^N \geq 0, \\
& (1-\delta^k) S_0^{kN} \frac{N_0^N}{N_1^N} + k_2 i_1^N - S_1^{kN} \geq 0, \\
& (1-\delta^k) S_1^{kN} \frac{N_1^N}{N_2^N} + k_2 i_2^N - S_2^k \geq 0, \\
& (1-\delta^k) S_0^{kS} \frac{N_0^S}{N_1^S} + k_2 i_1^S - S_1^{kS} \geq 0, \\
& (1-\delta^k) S_1^{kS} \frac{N_1^S}{N_2^S} + k_2 i_2^S - S_2^k \geq 0, \\
& (1-\delta^n) \left((1-\lambda) \frac{N_0^S}{N_1^S} S_0^{nS} + \lambda \frac{N_0^N}{N_1^N} \frac{N_0^N}{N_1^S} S_0^{nN} \right) + k_{3N} \frac{N_0^N}{N_1^S} x_1^{nN} + k_{3S} x_1^{nS} - S_1^{nS} \geq 0, \\
& (1-\delta^n) \left((1-\lambda) \frac{N_1^S}{N_2^S} S_1^{nS} + \lambda \frac{N_1^N}{N_2^N} \frac{N_1^N}{N_2^S} S_1^{nN} \right) + k_{3N} \frac{N_1^N}{N_2^S} x_2^{nN} + k_{3S} x_2^{nS} - S_2^n \geq 0, \\
& (1-\delta^n) S_0^{nN} \frac{N_0^N}{N_1^N} + k_3 x_1^{nN} - S_1^{nN} \geq 0, \\
& (1-\delta^n) S_1^{nN} \frac{N_1^N}{N_2^N} + k_3 x_2^{nN} - S_2^n \geq 0, \\
& k_4 x_0^{eN} \frac{N_0^N}{N_1^N} - x_1^{IN} - x_1^{cN} - x_1^{eN} - x_1^{nN} \geq 0, \\
& k_4 x_1^{eN} \frac{N_1^N}{N_2^N} - x_2^{IN} - x_2^{c,N} - x_2^e - x_2^{nN} \geq 0, \\
& k_4 x_0^{eS} \frac{N_0^S}{N_1^S} - x_1^{IS} - x_1^{cS} - x_1^{eS} - x_1^{nS} \geq 0, \\
& k_4 x_1^{eS} \frac{N_1^S}{N_2^S} - x_2^{IS} - x_2^{cS} - x_2^e - x_2^{nS} \geq 0, \\
& e_1^* - \frac{N_1^N}{N_1^N + N_1^S} e_1^N - \frac{N_1^S}{N_1^N + N_1^S} e_1^S \geq 0, \\
& e_2^* - \frac{N_2^N}{N_2^N + N_2^S} e_2^N - \frac{N_2^S}{N_2^N + N_2^S} e_2^S \geq 0,
\end{aligned}$$

$$\begin{aligned}
& -T_1 + k_1 (x_1^{cN})^{\theta_c} (S_1^{kN})^{\theta_k} (S_1^{nN})^{\theta_n} (S_1^{m*})^{\theta_m} (e_1^N)^{\theta_e} - c_1^N - i_1^N \geq 0, \\
& -T_2 + k_1 (x_2^{cN})^{\theta_c} (S_2^k)^{\theta_k} (S_2^n)^{\theta_n} (S_2^{m*})^{\theta_m} (e_2^N)^{\theta_e} - c_2^N - i_2^N \geq 0, \\
& T_1 \frac{N_1^N}{N_1^S} + k_1 (x_1^{cS})^{\theta_c} (S_1^{kS})^{\theta_k} (S_1^{nS})^{\theta_n} (S_1^{m*})^{\theta_m} (e_1^S)^{\theta_e} - c_1^S - i_1^S \geq 0, \\
& T_2 \frac{N_2^N}{N_2^S} + k_1 (x_2^{cS})^{\theta_c} (S_2^k)^{\theta_k} (S_2^n)^{\theta_n} (S_2^{m*})^{\theta_m} (e_2^S)^{\theta_e} - c_2^S - i_2^S \geq 0, \\
& (x_2^e, S_2^k, S_2^n) \in \Gamma(\rho, e^*, S^{m*}), \\
& S_3^n - (1+g)S_2^n \geq 0, \\
& c_3 - \gamma_1(\rho, e^*, S^{m*}) \geq 0, \\
& x_3^l - \gamma_2(\rho, e^*, S^{m*}) \geq 0,
\end{aligned}$$

with initial conditions $(x_0^{eN}, S_0^{kN}, S_0^{nN}), (x_0^{eS}, S_0^{kS}, S_0^{nS}), \Lambda_0^N$ and Λ_0^S .

Remark 2. The last three inequalities, involving S_3^n , c_3 and x_3^l , require both regions to be on the steady state defined by the ray $\Gamma(\rho, e^*, S^{m*})$ at the beginning of date 3.

4 Results

Recall that we:

- Take the annual rate of growth of utility to be 1 %, which corresponds to a generational growth rate of utility of $\rho = 28.2\%$, and a generational growth rate of the economic variables of $g = 28.6\%$ (recall that $1 + \rho = (1 + g)^{1-\alpha_m}$);
- Postulate the path $((e_1^{W*}, S_1^{m*}), (e_2^{W*}, S_2^{m*}), (e^{W*}, S^{m*}))$ of global emissions and CO₂ stocks;
- Constrain North's utility to grow at rate ρ starting from the reference level Λ_0^N ;
- Constrain South's utility to grow at least at rate ρ starting from the reference level Λ_0^S ;
- Constrain stocks in both North and South to converge to the same point in ray $\Gamma(\rho, e^*, S^{m*})$ at $t = 2$, and therefore, by the Ray Optimization Theorem, for $t \geq 3$ North and South can reach same utility and grow together at rate ρ .

4.1 Utility Growth, the Steady State and the Transition

We numerically solve the optimization program for our calibrated model using *Mathematica*'s 'NMaximize' routine. Recall that, subject to the above conditions, we choose all economic variables as well as the allocation of global emissions $(e_1^{W*}, e_2^{W*}, e^{W*})$ between North and South in order to maximize the utility of South's Generation 2.

Result 1. *A solution exists for a generational utility growth rate of 28.2% (1% per year). The above conditions imply that, at the solution, utility in both North and South always grows at least at rate 28.2%, starting from the historical reference level.*

Result 2. *A steady state is reached for $t \geq 3$, where utilities in North and South converge and grow at rate $\rho = 28.2\%$ per generation, whereas the per capita economic variables converge and grow at rate $g = 28.6\%$ per generation. In particular, the per capita values for the stock of knowledge and for investment in knowledge are equalized. Moreover, per capita emissions are equalized in the steady state.*

Tables 5-7 present the quantitative results of our optimization: Figure 1 depicts the utility paths for North and South.

Generations 1 and 2 must implement the transition from the initial reference conditions, where the endowments of South are low, to the common steady state path while allowing North's utility to grow at 28.2%. In broad terms, South's jump to the steady state is accomplished by partially shifting its share of emissions to North, and by benefiting from the spillover of North's higher levels of knowledge. These are interregional factors. Domestically, South invests heavily in all sorts of capital, particularly in knowledge and in education.

The constraint that North must grow at the prespecified generational rate of 28.2% is no doubt the source of some peculiarities during the transition, see Section 4.5 below.

4.2 Allocation of Emissions

The optimal values for the allocation of emissions are presented in Table 7 and figures 2 and 3. Recall that the postulated path of global emissions decreases to a low value in the steady state, and, accordingly, both emissions per capita and the emissions-to-output ratio ("GHG

intensity,” in the IPCC parlance) must eventually decrease.

Result 3. *Because output can be freely transferred, the (endogenous) allocation of the postulated global emissions between North and South equalizes the marginal product of emissions across regions. In our Cobb-Douglas model, this marginal equality implies that the emissions-to-output ratio is equalized across regions both at the steady state and during the transition. Compare with the reference values, where the emissions-to-output ratio is a 257% larger in South than in the North.*

	$\frac{\Lambda_t^J}{\Lambda_0^J}$	$\frac{\Lambda_t^J}{\Lambda_{t-1}^J}$	c_t^J	$\frac{c_t^J}{c_0^J}$	$\frac{c_t^J}{c_{t-1}^J}$	i_t^J	S_t^{kJ}	S_t^{nJ}	T_t^J
Gen	NORTH								
0	1	-	27.78	1	-	6.83	73.65	15.64	0
1	1.282	1.282	35.312	1.271	1.271	3.159	56.308	37.762	-17.349
2	1.645	1.282	42.950	1.546	1.216	10.243	146.490	49.415	-18.898
3	2.109	1.282	58.698	2.113	1.367	11.978	188.324	63.526	0
4	2.705	1.282	75.461	2.716	1.286	15.399	242.104	81.667	0
Gen	SOUTH								
0	1	-	1.51	1	-	0.85	0.5	1.103	0
1	1.282	1.282	8.042	5.326	5.326	2.869	37.714	9.071	-3.235
2	4.538	3.539	27.081	17.935	3.368	10.647	146.490	49.415	-2.962
3	8.760	1.930	58.698	38.873	1.367	11.978	188.324	63.526	0
4	11.234	1.282	75.461	49.974	1.286	15.399	242.104	81.667	0

Table 5. Values along the optimal path sustaining a 1% annual growth ($\hat{\rho} = 0.01$).

	x_t^J	x_t^{lJ}	x_t^{cJ}	x_t^{nJ}	x_t^{eJ}	$x_t^{lJ}(\%)$	$x_t^{cJ}(\%)$	$x_t^{nJ}(\%)$	$x_t^{eJ}(\%)$
Gen.	NORTH								
0	1.396	0.935	0.392	0.023	0.046	0.670	0.281	0.017	0.033
1	1.546	1.195	0.238	0.053	0.060	0.773	0.154	0.034	0.039
2	2.148	1.581	0.421	0.064	0.083	0.736	0.196	0.030	0.039
3	2.952	1.972	0.791	0.082	0.107	0.668	0.268	0.028	0.036
4	3.795	2.535	1.017	0.105	0.138	0.668	0.268	0.028	0.036
Gen.	SOUTH								
0	0.736	0.478	0.234	0.003	0.021	0.650	0.317	0.005	0.028
1	0.532	0.289	0.170	0.014	0.059	0.544	0.319	0.027	0.110
2	1.766	1.061	0.531	0.090	0.083	0.601	0.301	0.051	0.047
3	2.952	1.972	0.791	0.082	0.107	0.668	0.268	0.028	0.036
4	3.795	2.535	1.017	0.105	0.138	0.668	0.268	0.028	0.036

Table 6. Labor allocation along the optimal path sustaining a 1% annual growth ($\hat{p} = 0.01$).

	Total Emissions <i>GtC</i>	Emissions per capita <i>tC per capita</i>	Emissions to Output <i>tC per \$'000</i>
North			
Year 2000	3.821	3.200	0.092
Generation 1	1.518	1.206	0.057
Generation 2	0.515	0.413	0.012
Generation 3	0.050	0.040	0.001
South			
Year 2000	2.761	0.560	0.237
Generation 1	5.451	0.807	0.057
Generation 2	3.897	0.490	0.012
Generation 3	0.318	0.040	0.001
World			
Year 2000	6.583	1.0748	0.124
Generation 1	6.969	0.87	0.057
Generation 2	4.412	0.48	0.012
Generation 3	0.368	0.04	0.001

Table 7. Emissions

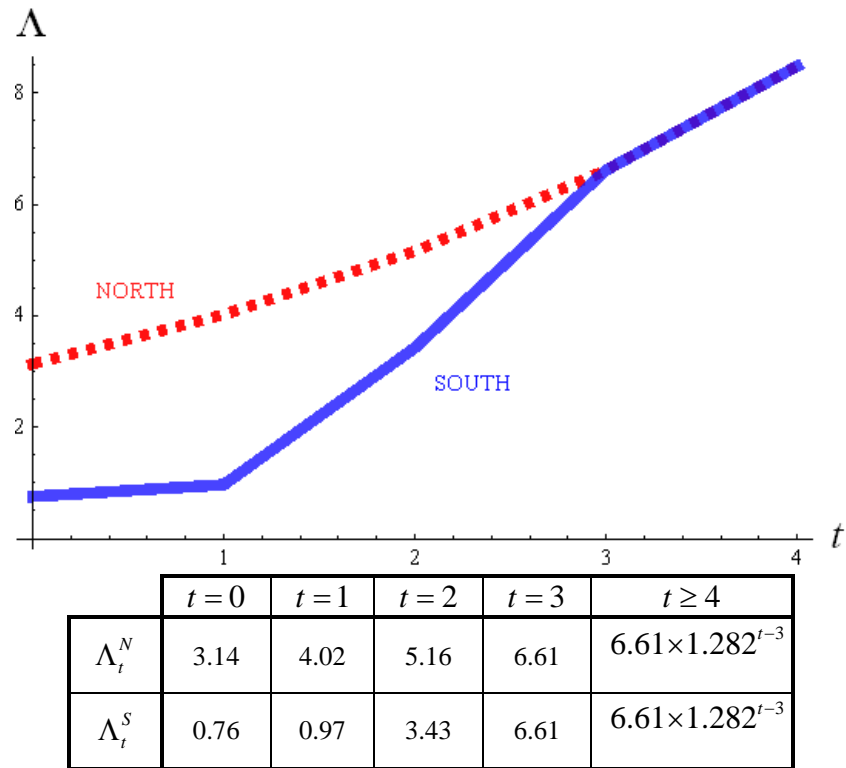


Figure 1 . Optimal paths for a 1% sustained annual growth

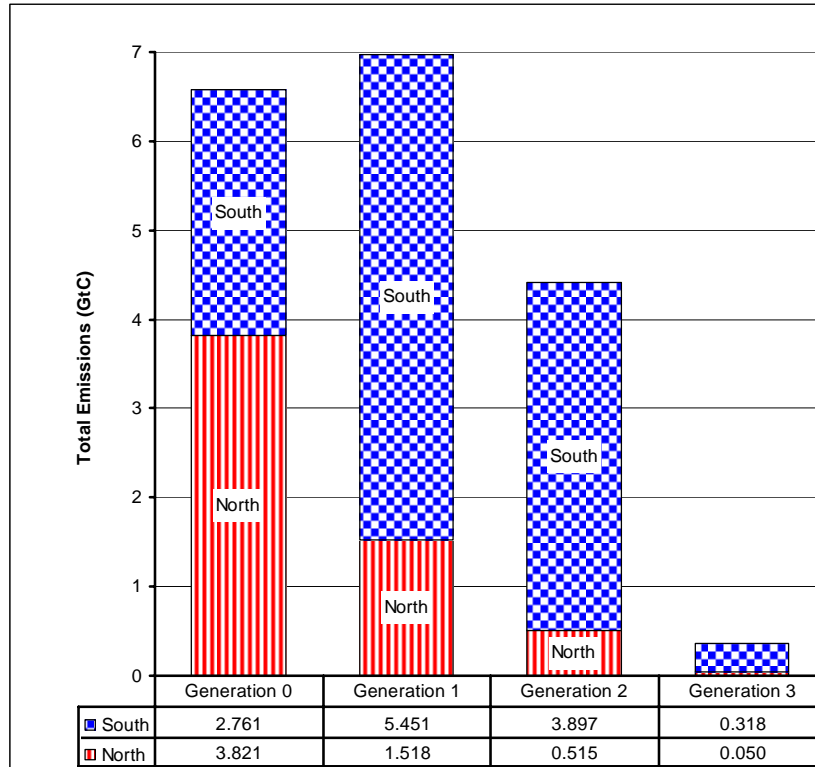


Figure 2. Total Emissions (North, South and World)

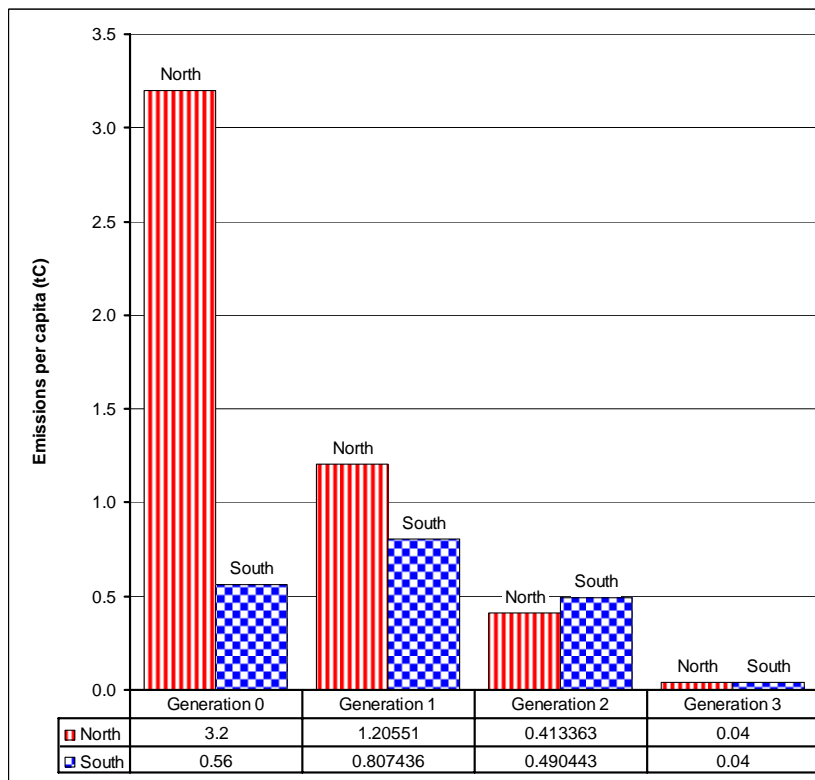


Figure 3. Emissions per capita for a 1% sustained annual growth

Result 4. *At the year-2000 reference values, emissions per capita in South are substantially lower than in North. At the steady state, however, they must be equalized. In fact, the ratio of emissions per capita in South to that in North increases from the reference value of 17.5% to 118% at $t = 2$, eventually falling down to the steady state 100% (see Table 7). North's emissions per capita decrease, and they do so monotonically (see Figure 3). For South, they first rise moderately, and then they fall.*

The emissions-to-output ratios of the last column of Table 7 and our calibrated value of the emissions elasticity parameter $\theta_e = 0.091$ (see Table 3 above) determine the marginal product of emissions in terms of output, implying a shadow price of carbon. For Generation 1, Table 7 gives the emissions/output ratio as .057. Therefore the marginal product of emissions in output is $\theta_e (1 / .057) = .091 / .057 = 1.596$ or \$1596 per ton of carbon. What should be the tax on a gallon of gasoline? Burning one gallon of gasoline emits 19.4 pounds of CO₂, or $19.4 / 3.664 = 5.29$ pounds of carbon (<http://www.epa.gov/oms/climate/420f05001.htm#carbon>), implying a Pigovian tax of $\frac{1596}{2000} \times 5.29 = \4.22 per gallon of gasoline. This is substantially higher than other policy proposals for the US (see e. g., Table 5-4 in Nordhaus, 2008), but not out of line with European gasoline taxes.⁴ Of course, the price of carbon implied by Table 7 increases drastically for Generation t , $t > 2$, provided that the exponent θ_e stays at 0.091. Note that technical change is neutral in our model; in reality, however, technical progress will in all likelihood increase the elasticity θ_e , allowing for more output per ton of emissions. If this happens, the optimal paths will be different and we cannot a priori say what the marginal product of carbon would be.

4.3 Flows of Consumption and Leisure

Result 5. *Not surprisingly, in order to catch up with North, South's consumption of output has to grow quite fast. Specifically, South's consumption at the convergence point $t = 3$ is 39 times its reference level, while North just doubles its consumption reference level. (See the fourth column in Table 5.)*

⁴ Wikipedia: <http://en.wikipedia.org/wiki/Fueltax> notes that that the tax per gallon of gas in Europe is currently about \$3.50.

Result 6. *At the steady state, the share of labor resources that either North or South devote to leisure does not significantly differ from the reference values. During the transition, North (resp. South) moderately increases (resp. reduces) its share of time devoted to leisure. (See the sixth column in Table 6.)*

4.4 Investment in Knowledge, Education and Physical Capital

Transitioning to the steady state requires substantial increases in the stock of knowledge and the level of education in South. Note that North's knowledge has the character of an interregional public good during the transition, whereas the direct benefits of investment in education only help future generations in the same region.

Result 7. *The optimal path requires substantial changes in the creation of knowledge in both North and South, and in education in South. The two last columns of Table 6 show that the fraction of labor devoted to the creation of knowledge jumps to the steady state value of 2.8 % from the reference values of 1.7% in North and 0.5% in South. South's education at the steady state absorbs a 3.6% of the labor resource, versus 2.8% in the reference year. During the transition, the most dramatic changes are the doubling of the fraction of the labor resource devoted to knowledge in North at $t = 1$ relative to the reference value (next to last column in Table 6) and the quadrupling of the fraction of the labor resource devoted to education in South at $t = 1$ relative to the reference value (last column in Table 6).*

Result 8. *At the steady state, both North and South devote to investment in physical capital 4.5% of their labor resources, a figure in line with the reference value of 4.3% in North, but substantially lower than the reference 11.4% in South.*

4.5 Transfers in the Transition

An unexpected result of our optimization is the need for South to transfer output to North during the transition ($t = 1, 2$, see the last column of Table 5: recall that a negative number indicates a transfer from South to North, per capita in the relevant region, in the amount of its absolute value). Several features of the modeling contribute to this result. First, during the transition the stock of knowledge of North spills over that of South, which implies that at the

optimal solution North must devote to the creation of knowledge a relatively large fraction of resources. Second, as noted, the optimal solution imposes a relatively small allocation of emissions to North during the transition. It turns out that these sacrifices by North are counterbalanced by the South-to-North transfers in order to satisfy the constraint, which we impose, that North's utility grow at rate of at least ρ starting from the reference level. In a sense, South has comparative advantage in the production of output during the transition. We conjecture that letting North grow at a slower rate during the transition would reduce or eliminate these transfers, but we feel that such a reduction in North's growth rate would be politically unfeasible. Moreover, it would force a lower utility to both North and South for $t \geq 3$.

5 Conclusion

We postulate a path of world CO₂ emissions that will approximately stabilize atmospheric concentration at 450 ppm, and we inquire how should this budget of total emissions be allocated to the regions of the world, that we simplify to two: North, calibrated after the US, and South, calibrated after China. We assume that North and South begin with different endowments of education, physical capital and knowledge, and we study paths under which both regions converge in utility and per capita variables in 75 years (or three generations), subsequently enjoying steady growth at 1% annual rate (28.2% per generation).

Preliminary work leading to the present paper explored the maximization of the utility of the worst-off generation, namely South's Generation 1, and found that then this generation invested very little in education, imposing low utility on South's Generation 2 together with a big leap from generations 2 to 3. On the other hand, maximizing the utility of Generation 2 in South subject to Generation 1 growing at rate of at least ρ gives a smoother path *with little loss of utility for Generation 1*. For these reasons, we choose to maximize the utility of Generation 2 in South subject to all the stated conditions.

Our optimization results show the compatibility of the following three desiderata:

- (1). Global CO₂ emissions follow a conservative path that leads to the stabilization of concentrations at 450 ppm.
- (2). North and South converge to a path of sustained growth at 1% per year (28.2% per generation) in 2075.
- (3). During the transition to the steady state, North also grows at 1% per year while South's

rates of growth are markedly higher.

The transition paths require a drastic reduction of the share of emissions allocated to North, large investments in knowledge, both in North and South, as well as very large investments in education in South. Surprisingly, in order to sustain North's growth rate, some output must be transferred from South to North during the transition.

Our results support a degree of optimism. Of course, many caveats are warranted in our complex, calibrated model. Among them, we must emphasize the normative nature of our analysis: we aim at ascertaining what an ethical observer would recommend, abstracting from all real life issues involving international bargaining, the behavior of markets, coordination and incentives. Second, this paper differs from LRS In Press and from our current research in disregarding uncertainty, an obvious attribute of climate change. Moreover, some of our modeling options, such as the Cobb-Douglas functional forms or the central role of education and knowledge, may well be challenged. But we do feel that our analysis provides prima facie evidence of the possibility of tackling climate change in way that is fair both across generations and across regions while allowing for modest, yet positive, rates of human development,

APPENDIX

A. CALIBRATED VALUES

Section 2.5 in the main text displays the calibrated values for the parameters of the model and for the initial values of the stocks in the reference year (2000). In addition, the following calibrated values are used in our computations

Flows	Value	Units
\bar{y}_0^N	34.61	thousands of 2000 dollars per capita
\bar{y}_0^S	2.36	thousands of 2000 dollars per capita
\bar{c}_0^N	27.78	thousands of 2000 dollars per capita
\bar{c}_0^S	1.51	thousands of 2000 dollars per capita
\bar{l}_0^N	6.83	thousands of 2000 dollars per capita
\bar{l}_0^S	0.85	thousands of 2000 dollars per capita
\bar{e}_0^N	3.2	tC per capita
\bar{e}_0^S	0.5	tC per capita

Table A1. Initial values of the flows in the reference year (2000).

Variable	Value	Units
x_0^N	1.396	1950-US efficiency units per capita
x_0^S	0.736	1950-US efficiency units per capita
x_0^{IN}	0.9353	1950-US efficiency units per capita
x_0^{IS}	0.4784	1950-US efficiency units per capita
x_0^{cN}	0.3916	1950-US efficiency units per capita
x_0^{cS}	0.2336	1950-US efficiency units per capita
x_0^{eN}	0.0461	1950-US efficiency units per capita
x_0^{eS}	0.0206	1950-US efficiency units per capita
x_0^{nN}	0.0230	1950-US efficiency units per capita
x_0^{nS}	0.0033	1950-US efficiency units per capita

Table A.2. Labor allocation in the reference year (2000).

Variable	Value	Units
N_{2000}^{USA}	284,257	thousands
N_{2000}^{China}	1,269,962	thousands
\bar{e}_{2000}^{USA}	5.6	tC per capita
\bar{e}_{2000}^{China}	0.7	tC per capita

Table A.3. USA's and China's population and emissions in the reference year (2000).

Variable	Value	Units
N_{2000}^{USA}	284,257	thousands
N_{2000}^{China}	1,269,962	thousands
\bar{e}_{2000}^{USA}	5.6	tC per capita
\bar{e}_{2000}^{China}	0.7	tC per capita

Table A.4. USA's and China's population and emissions in the reference year (2000).

B. CALIBRATIONS

We interpret that generations live for 25 years. In this appendix, flow variables are typically defined as per year averages, and it is understood that stocks are located in the last year of life of a generation. We group countries in two regions (*North* and *South*) following the United Nations classification of “more developed regions” (Europe, Northern America, Australia/New Zealand, and Japan) and “less developed regions” (Africa, Asia (excluding Japan), Latin America and the Caribbean plus Melanesia, Micronesia and Polynesia). The calibrated values that we obtain are reported in Section 2.5 of the main text and Appendix A above.

B.1. Variables

S_t^{kj} = capital stock available to Generation t in region J (in thousands of int. dollars per capita).

S_t^{nj} = stock of knowledge available to Generation t in region J (in thousands of int. dollars per capita).

S_t^m = CO₂ concentration in the atmosphere above the equilibrium pre-industrial level at the end of Generation t 's life (in GtC).

x_t^j = average annual efficiency units of time (labor and leisure) available to Generation t in region J (in efficiency units per capita).

x_t^{ej} = average annual labor devoted to education by Generation t in region J (in efficiency units per capita).

x_t^{cj} = average annual labor devoted to the production of output by Generation t in region J (in efficiency units per capita).

x_t^{lj} = annual average leisure by Generation t in region J (in efficiency units per capita).

x_t^{nj} = average annual labor devoted to the production of knowledge by Generation t in region J (in efficiency units per capita).

c_t^j = annual average consumption by Generation t in region J (in thousands of int. dollars per capita).

i_t^j = average annual investment by Generation t in region J (in thousands of int. dollars per capita).

e_t^j = average annual emissions per capita of CO₂ from fuel and cement by Generation t in region J (in tC per capita).

B.2. Parameters

α_j = exponents of the utility function for $j \in \{c$ (consumption), l (leisure), n (stock of knowledge), and m (quality of the biosphere)}.

k_1 = parameter of the production function f .

k_2 = parameter of the law of motion of capital.

k_3 = parameter of the law of motion of the stock of knowledge in North.

k_{3N}, k_{3S} = parameters of the law of motion of the stock of knowledge with technological diffusion from North to South.

k_4 = parameter of the education production function.

λ = rate of technological transfer from North to South (per generation)

θ_j = exponents of the inputs in the production function f for $j \in \{c$ (labor), k (stock of capital), n (stock of knowledge), e (emissions of CO₂), m (atmospheric carbon concentration)}.

δ^k = depreciation rate of the stock of capital (per generation).

δ^n = depreciation rate of the stock of knowledge (per generation).

e_t^* = average annual world emissions per capita of CO₂ from fuel and cement by Generation t (in tC per capita).

S_t^m = carbon concentration in the atmosphere above the equilibrium pre-industrial level at the end of Generation t (in GtC).

\hat{S}^m = catastrophic level of carbon concentration in the atmosphere above the equilibrium pre-industrial level (in GtC).

$\hat{\rho}$ = annual rate of growth of utility.

ρ = generational rate of growth of utility ($\rho = (1 + \hat{\rho})^{25}$).

B.3. Functions

Utility function: $(c_t^J)^{\alpha_c} (x_t^{kJ})^{\alpha_k} (S_t^{nJ})^{\alpha_n} (\hat{S}^m - S_t^m)^{\alpha_m}$.

Production function:

$$f(x_t^{cJ}, S_t^{kJ}, S_t^{nJ}, e_t^J, S_t^m) \equiv k_1 (x_t^{cJ})^{\theta_c} (S_t^{kJ})^{\theta_k} (S_t^{nJ})^{\theta_n} (e_t^J)^{\theta_e} (S_t^m)^{\theta_m}, \theta_c + \theta_k + \theta_n + \theta_e = 1.$$

Law of motion of physical capital: $S_t^{kJ} \leq (1 - \delta^k) \frac{N_{t-1}^J}{N_t^J} S_{t-1}^{kJ} + k_2 i_t^J$.

Law of motion of the stock of knowledge without technological diffusion:

$$S_t^{nJ} \leq (1 - \delta^n) \frac{N_{t-1}^J}{N_t^J} S_{t-1}^{nJ} + k_3 x_t^{nJ}.$$

Law of motion of the stock of knowledge with technological diffusion from North to South:

$$S_t^{nS} \leq (1 - \delta^n) (1 - \lambda) \frac{N_{t-1}^S}{N_t^S} S_{t-1}^{nS} + (1 - \delta^n) \lambda \frac{N_{t-1}^N}{N_t^N} \frac{N_{t-1}^N}{N_t^S} S_{t-1}^{nN} + k_{3N} \frac{N_{t-1}^N}{N_t^S} x_t^{nN} + k_{3S} x_t^{nS}.$$

Education production function: $x_t^J \leq k_4 \frac{N_{t-1}^J}{N_t^J} x_{t-1}^{eJ}$.

B.4. Population

We follow the United Nations (2008) forecast and assign to each generation the average population of its 25 years of existence. World population is 6.1 billion people in 2000, increases to 8.01 billion people for Generation 1, and stabilizes at 9.2 billion people from Generation 2 and on. Table B.1 reports the specific paths for North and South.

	North		South		World
	Total population (thousand people)	Percentage of world pop.	Total population (thousand people)	Percentage of world pop.	Total population (thousand people)
Year 2000	1,194,199	19.5%	4,929,924	80.5%	6,124,123
Generation 1	1,258,970	15.7%	6,751,540	84.3%	8,010,509
Generation 2	1,245,247	13.5%	7,946,040	86.5%	9,191,287

Table B.1 Population paths

B.5. The calibration of the utility function

We take the exponent of leisure to be twice that of consumption ($\alpha_l = 2\alpha_c$) and calibrate $\alpha_n/\alpha_c = 0.05$ as the average ratio of expenditure in knowledge (R&D expenditure plus investment in computer components and software) over expenditure in consumption during the period 1953-2000.⁵

Next, we calibrate the ratio α_m/α_c by the Stern Review (2007) statement that a 5°C increase in the global temperature over the pre-industrial level would imply a health related damage equivalent to a 5% loss of global GDP (page x).⁶ We can read the statement of the Stern Review as saying that a 5% decrease in consumption is equivalent to suffering an atmospheric CO₂ concentration of \tilde{S}^m , yielding

$$(0.95c)^{\alpha_c} (x^l)^{\alpha_l} (S^n)^{\alpha_n} (\hat{S}^m - S^m)^{\alpha_m} = (c)^{\alpha_c} (x^l)^{\alpha_l} (S^n)^{\alpha_n} (\hat{S}^m - \tilde{S}^m)^{\alpha_m},$$

that is,

$$(0.95)^{\alpha_c} (\hat{S}^m - S^m)^{\alpha_m} = (\hat{S}^m - \tilde{S}^m)^{\alpha_m}.$$

Taking logs,

$$\alpha_c \ln(0.95) = \alpha_m \left(\ln(\hat{S}^m - \tilde{S}^m) - \ln(\hat{S}^m - S^m) \right).$$

We take a 5°C increase in temperature to be associated with CO₂ equivalent (CO₂e) concentrations of 1470 GtC (Stern Review, 2007, Figure 2 in page v). Because we only consider CO₂ emissions (which account for 84% of all GHG) and we compute values above pre-industrial level (595.5 GtC), we adopt the value $\tilde{S}^m = 1470/1.16 - 595.5 = 671.64$ GtC.

⁵ Data on R&D is derived from Research and Development in Industry, Academic Research and Development Expenditures, Federal Funds for Research and Development, and the Survey of Research and Development Funding and Performance by Nonprofit Organizations (National Science Foundation, 2003). Data on public investment in software are constructed taking the value of public investment in equipment and software (U.S. Bureau of Economic Analysis 2007) and assuming the same share of software in private and public investment.

⁶ This is also in line with Nordhaus and Boyer (2000) who estimate a total cost (market and non-market) of between 9 to 11% of global GDP for a 6°C warming (as quoted in the Stern Review, 2007, p.148).

We consider that an increase in temperature of 6°-8°C (relative to pre-industrial level) would have catastrophic impacts.⁷ We take this temperature increases to be associated with CO₂ equivalent concentrations of 750 ppm (or 1597.5 GtC), the lower bound of the studies reported in the Stern Review (2007, p.12). As before, adjusting for all gases and subtracting pre-industrial levels, we obtain $\hat{S}^m = 1597.5/1.16 - 595.5 = 781.55$.

It follows that

$$\frac{\alpha_m}{\alpha_c} = \frac{\ln 0.95}{\ln(781.55 - 671.64) - \ln(781.55 - 177.1)} = 0.03.^8$$

Finally, we normalize $\alpha_c + \alpha_l + \alpha_m + \alpha_n = 1$.

B.6. The calibration of the production function

We calibrate the production function

$$f(x_t^c, S_t^k, S_t^n, e_t, S_t^m) \equiv k_1 (x_t^c)^{\theta_c} (S_t^k)^{\theta_k} (S_t^n)^{\theta_n} (e_t)^{\theta_e} (S_t^m)^{\theta_m}$$

in the following inputs: first the more usual labor, physical capital and knowledge, to which we add the environmental stock and emissions.

We assume constant returns to scale in the first four inputs, that is, $\theta_c + \theta_k + \theta_n + \theta_e = 1$.

We calibrate $\theta_e = 0.091$ as the “elasticity of output with respect to carbon services” from RICE99 in Nordhaus and Boyer (2000).

We construct time series for the stocks of physical capital, knowledge, and human capital, see sections B.7 and B.8 below. We take the labor income share to be 2/3, and compute the average share of physical capital and knowledge in the total stock of capital for the period 1960-2000, corresponding to 5/6 and 1/6, respectively. Hence, $\theta_c = 0.6667$, $\theta_k = 0.2019$ and $\theta_n = 0.0404$, representing the income share of each input.

For the calibration of θ_m , the elasticity of output to the CO₂ concentration in the atmosphere, we assume that doubling the CO₂ concentration from pre-industrial levels would increase temperature by 2.5°C (Stern Review, 2007, p.7),⁹ and that a 2.5°C increase in temperature is associated with a 1.5% loss of total GDP (Nordhaus and Boyer, 2000, p.91). Hence,

$$\theta_m = \frac{\% \Delta y}{\% \Delta S^m} = \frac{\% \Delta y}{\% \Delta T} \frac{\% \Delta T}{\% \Delta S^m} = -\frac{.015}{2} = -.0075,$$

where y is GDP per capita and T is global temperature.

Finally, we compute k_1 as the TFP of the US economy calibrated to year 2000 values:¹⁰

⁷ The Stern Review consistently associates catastrophic consequences to temperature increases of 6-8°C, like, for example, sea level rise threatening major world cities (including London, Shanghai, New York, Tokyo and Hong Kong), entire regions experiencing major declines in crop yields and high risk of abrupt, large scale shifts in the climate system (Figure 2 in page v), and catastrophic major disruptions and large-scale movements of population (Table 3.1 in p. 57).

⁸ As a reference, the US currently devotes approximately 2% of its gross domestic product to all forms of environmental protection.

⁹ The Stern Review asserts that temperature would increase 1.5°-4.5°C (if we consider feedback effects) and 1°C as direct effects.

¹⁰ GDP is denoted in thousands of constant 2000 dollars per capita from the World Development Indicator (World

$$k_1 = \frac{y_{2000}^{USA}}{\left(x_{2000}^{c,USA}\right)^{\theta_c} \left(S_{2000}^{k,USA}\right)^{\theta_k} \left(S_{2000}^{n,USA}\right)^{\theta_n} \left(e_{2000}^{USA}\right)^{\theta_e} \left(S_{2000}^m\right)^{\theta_m}} = \frac{34.61}{0.3916^{0.66667} 73.65^{0.20192} 15.64^{0.04038} 5.6^{0.09100} 177.1^{-0.0075}} = 21.5861.$$

B.7. The stock of physical capital

Law of motion of the stock of physical capital

We take $\hat{\delta}^k = 0.06$ as the annual rate of depreciation (Cooley and Prescott, 1995). In generational terms, $\delta^k = 0.787$.

To approximate the year-to-year discounting, we take i_t as the average investment in physical capital per year of a given generation, and compute that, at the end of the generation's life, the accumulated investment amounts are

$$i + i \times (1 - \hat{\delta}^k) + i \times (1 - \hat{\delta}^k)^2 + \dots + i \times (1 - \hat{\delta}^k)^{24} = \frac{1 - (1 - \hat{\delta}^k)^{25}}{1 - (1 - \hat{\delta}^k)} i.$$

Thus, since $1 - \hat{\delta}^k = 0.94$, the parameter $k_2 = \frac{1 - (1 - \hat{\delta}^k)^{25}}{1 - (1 - \hat{\delta}^k)} = 13.1182$.

Initial stock of physical capital

We assign North the stock of physical capital per capita in USA. The time series of the stock of physical capital is constructed by the perpetual inventory method, using US data for 1960-2000 and taking 1960 as initial value. For the initial year,

$$S_{1960}^{kN} = i_{1960}^{kN} / (\hat{\delta}^k + g^{kN}) = 2.51 / (0.06 + 0.038) = 25.63 \text{ thousands of constant 2000 dollars per capita,}$$

where i^{kN} represents total (private and public) investment per capita minus expenditure in software, and g^{kN} represents the average yearly growth rate of investment between 1960-1970 (set at 3.8%). The value for the stock of physical capital in the year 2000 is $\bar{S}_0^{kN} = 73.65$ thousands of 2000 dollars per capita.

We assign South the stock of physical capital per capita in China. From Table A.2 in the Appendix to "Forecasting China's Economic Growth to 2025," Version June 10, 2007 (Perkins and Rawski, 2008), we get a stock of capital for 2000 of 2,128.01 billions of 2000-Yuans. Dividing by population and using the PPP from the World Bank, we obtain $\bar{S}_0^{kS} = 0.5$ thousand international dollars per capita.¹¹

Bank, 2007). USA emissions are obtained from the World Resources Institute (2009). See Sections B.7, B.8 and B.10 below for the values of the other stocks and flows in the year 2000.

¹¹ $S_0^{kN} = 21,280.1 \times 10^8 (2000\text{-Yuan}) / 1,269,962 \times 10^3 = 1.67565$ thousands of 2000-Yuan per

B.8. The stock of knowledge.

Law of motion of the stock of knowledge

We calibrate the law of motion of the stock of knowledge with technological diffusion from North to South

$$S_t^{nS} = (1 - \delta^n)(1 - \lambda) \frac{N_{t-1}^S}{N_t^S} S_{t-1}^{nS} + (1 - \delta^n) \lambda \frac{N_{t-1}^N}{N_t^N} \frac{N_{t-1}^N}{N_t^S} S_{t-1}^{nN} + k_{3N} \frac{N_{t-1}^N}{N_t^S} x_t^{n,N} + k_{3S} x_t^{nS}, \quad (\text{A1})$$

where

$$k_{3N} = \left((1 - \hat{\delta}^n) \hat{\lambda} \frac{1 - (1 - \hat{\delta}^n)^{25} (1 - \lambda)}{1 - (1 - \hat{\delta}^n)(1 - \hat{\lambda})} - (1 - \hat{\delta}^n)^{25} \lambda \right) \frac{\bar{w}^{nN}}{\hat{\delta}^n}, \quad (\text{A2})$$

$$k_{3S} = \left(\frac{1 - (1 - \hat{\delta}^n)^{25} (1 - \lambda)}{1 - (1 - \hat{\delta}^n)(1 - \hat{\lambda})} \right) \bar{w}^{nN},$$

and \hat{w}^{nN} is the average wage of an efficient unit of labor. Appendix C presents the derivation of these expressions.

Observe that in the absence of technological diffusion the law of motion of the stock of knowledge naturally coincides with the law of motion of knowledge for North:

$$S_t^{kN} \leq (1 - \delta^k) \frac{N_{t-1}^N}{N_t^N} S_{t-1}^{kN} + k_2 i_t^N,$$

since,

$$\text{for } \lambda = 0, \quad k_{3N} = 0 \quad \text{and} \quad k_{3S} = k_3 = \frac{1 - (1 - \hat{\delta}^n)^{25}}{1 - (1 - \hat{\delta}^n)} \bar{w}^{nN}. \quad (\text{A3})$$

Therefore, the calibration of the laws of motion of the stock of knowledge only requires the estimation of three values: the annual depreciation rate of knowledge ($\hat{\delta}^n$), the diffusion rate of knowledge from North to South per year ($\hat{\lambda}$), and the average wage of an efficient unit of labor (\hat{w}^{nN}).

The yearly depreciation rate for knowledge commonly used is much lower than the one for capital (e. g., the Bank of Spain uses 15%, which would mean that knowledge dissipates almost entirely in one generation). We believe that the discount factor should be higher because of the intergenerational-public-good character of knowledge. A dollar invested in R&D by a firm may well generate no returns to the firm 25 years later, yet its impact to the accumulation of social knowledge capital may be substantial. Thus, as an approximation we take the depreciation rate of the stock of knowledge to be the same as that of physical capital, that is, $\hat{\delta}^n = \hat{\delta}^k = 0.06$, and in generational terms, $\delta^n = 0.787$.

capita. Using a PPP of 3.333 Yuans/\$, we obtain 0.503 thousands of international dollars per capita.

Eaton and Kortum (1999) conclude, in a general equilibrium model which uses a knowledge production function similar to ours, that “relative to the adoption of their own potentially useful ideas, countries generally adopt from one half to three-fourths of those generated abroad” (page 539). We adopt a conservative position and take a generational technological diffusion value of $\lambda = 0.5$, which corresponds to an annual rate of 2.7%.

Finally, $\bar{w}^{nN} = \frac{i_t^n}{x_t^n} \approx \frac{i_t^n}{1/3 x_t \epsilon^n}$, where i_t^n is the average annual expenditure per capita in

knowledge, $1/3 x_t$ is the total efficient units of labor, and ϵ^n the share of labor devoted to the production of knowledge. We take $\epsilon^n = 0.05$ (5% of total labor) and use the average values of expenditures and total labor for the last generation (1976-2000) to obtain

$$\bar{w}^{nN} = \frac{i_{1976-2000}^n}{1/3 \times x_{1976-2000} \times 0.05} = 49.5 \text{ thousands of 2000 dollars.}$$

Using the depreciation rate $\hat{\delta}^n = 0.06$ and the estimations obtained for $\lambda = 0.5$ and $\bar{w}^{nN} = 49.5$, we compute $k_3 = 649.349$, $k_{3N} = 133.267$, $k_{3S} = 516.082$, in accordance with (A2) and (A3).

Initial stock of knowledge for North

The time series of the stock of knowledge in North is constructed by the perpetual inventory method (PIM), using US data for 1960-2000 and taking 1960 as initial value. For the stock of knowledge in 1960 we take $S_{1960}^n = i_{1960}^n / (\hat{\delta}^n + g^n) = 4.21$ thousands of constant 2000 dollars per capita, where i^n represents total expenditure per capita in R&D plus public and private investment in software, and g^n represents the average yearly growth rate between 1960-1970.¹² The value for the stock of knowledge in the year 2000 is $\bar{S}_{2000}^n = 15.64$ (in thousands of 2000 dollars per capita).

Initial stock of knowledge for South

We assign to South the stock per capita in China. The time series of the stock of knowledge in China is constructed by PIM, using the annual knowledge equation with technology diffusion represented in (A1) We take 1/3 of the GDP per capita in 1980 (i.e. 278 international dollars per capita) as the initial value of the stock of knowledge in China.¹³ The date is unusually recent for applying PIM, but it can be justified by the particular circumstances of China.¹⁴ This year roughly coincides with the new development path set by Deng Xiaoping after the failure of the “Great Leap” experiment.¹⁵ As Song (2008) argues, “for the first time in China’s history, science and technology were viewed as driving force behind economic development” (p. 236). The reform also

¹² See Footnote 5 above.

¹³ Currency is always in constant 2000 PPP international dollars.

¹⁴ The choice of the initial value has a moderate effect for the stock in 2000. Choosing as initial value R&D investment in 1980 would decrease the year 2000 stock in less than \$10 per capita. But this figure most likely underestimates the real value (see notes in the OECD statistics). On the other hand, choosing total GDP would increase year 2000 stock in less than \$10 per capita.

¹⁵ Deng Xiaoping reforms started in 1978. We choose 1980 instead since this is the first year for which we have a PPP conversion factor.

initiated the flow of many students to the West for further scientific study, which also justifies the use of a rate of diffusion starting in 1980.¹⁶ For the time series of investment in knowledge, we take the data on R&D investment 1980-2000 from Gao and Jefferson (2007) and the China Science and Technology Statistical Data Yearbook (MOST 1998-2000).¹⁷ Table B.2 presents the values.

Year	R&D expenditures (% of GDP)	GDP per capita (constant 2000 PPP international dollars)	Stock of knowledge per capita ('000 of constant 2000 PPP international dollars)
1980	0.50	832.990952	0.278
1981	0.50	925.201251	0.327
1982	0.50	1057.864047	0.372
1983	0.50	1190.165201	0.415
1984	0.50	1337.966701	0.456
1985	0.50	1401.073683	0.495
1986	0.51	1465.743412	0.532
1987	0.50	1573.055288	0.569
1988	0.50	1589.853498	0.605
1989	0.50	1559.017183	0.641
1990	0.70	1564.995961	0.678
1991	0.70	1632.907862	0.718
1992	0.80	1741.064853	0.761
1993	0.70	1743.559496	0.801
1994	0.70	1650.748636	0.839
1995	0.60	1624.658512	0.875
1996	0.60	1693.194783	0.913
1997	0.64	1834.659152	0.952
1998	0.69	1997.791384	0.996
1999	0.83	2187.609702	1.045
2000	1.00	2357.338331	1.103

Table B.2: Time series of the stock of knowledge in China using the perpetual inventory method with a 6% annual depreciation rate and an 5% annual diffusion rate. R&D expenditure and GDP from Gao and Jefferson (2007) and S&T Statistics Data Yearbook.

B.9. The calibration of the education production function

We assume both regions have access to the same production function of education (the one in the North). The parameter k_4 , capturing the productivity of education, plays an important role in the model. By definition, $k_4 = N_t^J x_t^J / N_{t-1}^J x_{t-1}^e$, where both the numerator and the denominator are measured in efficiency units. We can transform efficiency units into hours by the equality

$$\frac{N_t^J x_t^J}{N_{t-1}^J x_{t-1}^e} = \frac{(1+s)^T \hat{x}_t}{(1+s)^{T-1} \hat{x}_{t-1}} = (1+s) \frac{\hat{x}_t}{\hat{x}_{t-1}} \quad (\text{for some } T),$$

¹⁶ By 2006, 1.67 million Chinese students had enrolled in universities in more than 108 countries. "This confirms that the policy of free access to overseas education is and will continue to be instrumental in China's drive toward modernization." (Song, 2008, p. 236).

¹⁷ Since there is only data available from 1986, we take investment in R&D constant at 0.5% of GDP for the decade of the 80s (the value for the years where we have data).

where $(1 + s)$ is the growth factor of human capital per generation, and where the “hats” represent the data in total annual hours. Hence, the calibration of k_4 is based on two rates: s and the share $\hat{x}_{t-1}^e/\hat{x}_t$ of time devoted to education out of total time. Note that k_4 is increasing in s and decreasing in the share $\hat{x}_{t-1}^e/\hat{x}_t$.

We take the value $\hat{s} = 0.67\%$ for the average yearly growth rate of the human capital stock, which yields the per-generation factor $(1 + s) = (1 + \hat{s})^{25} = 1.0067^{25}$. This figure, supported by the 1960-85 average provided by de la Fuente and Domènech (2001), is lower than the 0.93% average for 1960-2000 found in Barro and Lee (2001).

The rate $\hat{x}_{t-1}^e/\hat{x}_t$ is the product of the rate of education in labor and the rate of labor in total time. Assuming the ratios to remain constant over time, we infer from our time series that about 10% of total labor is devoted to education, and that labor accounts for 1/3 of total time. It follows that

$$k_4 = (1.0067)^{25} \frac{1}{0.0334} = 35.45. \quad (\text{A4})$$

This figure is conservative in the sense that higher growth rates of human capital, lower labor rates, and population growth would yield a larger value for k_4 .

B.10. Initial values for total labor and labor allocation

We construct the USA human capital stock (in efficiency units) by normalizing year 1950 equal to 1 and taking the average yearly growth rate of human capital stock equal to 0.67% (de la Fuente and Domènech, 2001). Hence, $x_t^N = 1.0067^{t-1950}$ in 1950-USA efficiency units, and therefore $\bar{x}_0^N = 1.0067^{50} = 1.396$.

We take the standard assumption of 67% of time devoted to working hours, allocated between education, knowledge and production according to their average proportions in the US (Labor Bureau of Statistics): 10% in education, 5% in knowledge, and 85% in the production of output.

For the estimation of human capital in South, we use the ratio of years of education between China and USA. We obtain from Barro and Lee (2001) the average years of school of the total population aged 15 and over for USA and China: 12.05 and 6.35 years, respectively.¹⁸ Therefore, $\bar{x}_0^S = 1.396 \times (6.35 / 12.05) = 0.736$, in 1950-USA efficiency units.

Based on the study by Li and Zax (2003), we take Chinese workers to devote 65% of their time to leisure. We set 8% of working time devoted to education, and use the data of the National Bureau of Statistics of China (2008) for the allocation of the remaining working time: 1.3% allocated in knowledge, and 90.7% in the production of output.

C. THE DIFFUSION RATE OF KNOWLEDGE AND THE KNOWLEDGE EQUATION FOR SOUTH

As specified in Section 2.5 above, the production of new knowledge in the North requires only efficiency labor, while knowledge depreciates at a positive rate, i. e.,

¹⁸ More sophisticated analyses, like those of Wang and Yao (2001) and Perkins and Rawski (2008), find similar values for China

$$S_t^n \leq (1 - \delta^n) S_{t-1}^{nN} \frac{N_{t-1}^N}{N_t^N} + k_3 x_t^{nN}, \quad t \geq 1,$$

where x_t^n is the average annual efficiency units of labor per capita devoted to the production of knowledge by Generation t in North.

The law of motion in South captures the presence of international technological diffusion. We assume that, insofar the stock of knowledge per capita in North is larger than in South, a fraction $\hat{\lambda}$ of that difference spills over from North to South. South can in addition invest in its own knowledge stock. Let the *annual* knowledge equation for South be as follows:¹⁹

$$\hat{N}_t^S \hat{S}_t^{nS} = (1 - \hat{\delta}^n) \hat{N}_{t-1}^S \hat{S}_{t-1}^{nS} + \hat{N}_t^S \hat{i}_t^{nS} + (1 - \hat{\delta}^n) \hat{\lambda} (\hat{N}_{t-1}^N \hat{S}_{t-1}^{nN} - \hat{N}_{t-1}^S \hat{S}_{t-1}^{nS}).$$

In per capita terms,

$$\hat{S}_t^{nS} = (1 - \hat{\delta}^n) \frac{\hat{N}_{t-1}^S}{\hat{N}_t^S} \hat{S}_{t-1}^{nS} + \hat{i}_t^{nS} + (1 - \hat{\delta}^n) \hat{\lambda} \left(\frac{\hat{N}_{t-1}^N}{\hat{N}_t^S} \hat{S}_{t-1}^{nN} - \frac{\hat{N}_{t-1}^S}{\hat{N}_t^S} \hat{S}_{t-1}^{nS} \right).$$

Here, t is measured in years and $\hat{\lambda}$ is the diffusion rate of knowledge from North to South *per year*. South learns a fraction $\hat{\lambda}$ of North's knowledge differential from the previous year. This can be written as:

$$\hat{S}_t^{nS} = (1 - \hat{\delta}^n) \left((1 - \hat{\lambda}) \frac{\hat{N}_{t-1}^S}{\hat{N}_t^S} \hat{S}_{t-1}^{nS} + \hat{\lambda} \frac{\hat{N}_{t-1}^N}{\hat{N}_t^S} \hat{S}_{t-1}^{nN} \right) + \hat{i}_t^{nS}. \quad (\text{B1})$$

Next, we convert the knowledge equation to generational time units. Let us write out (B1) for the first two years:

$$\begin{aligned} \hat{S}_1^{nS} &= (1 - \hat{\delta}^n) (1 - \hat{\lambda}) \frac{\hat{N}_0^S}{\hat{N}_1^S} \hat{S}_0^{nS} + (1 - \hat{\delta}^n) \hat{\lambda} \frac{\hat{N}_0^N}{\hat{N}_1^S} \hat{S}_0^{nN} + \hat{i}_1^{nS}, \\ \hat{S}_2^{nS} &= \left((1 - \hat{\delta}^n) (1 - \hat{\lambda}) \right)^2 \frac{\hat{N}_0^S}{\hat{N}_2^S} \hat{S}_0^{nS} + (1 - \hat{\delta}^n) (1 - \hat{\lambda}) (1 - \hat{\delta}^n) \hat{\lambda} \frac{\hat{N}_0^N}{\hat{N}_2^S} \hat{S}_0^{nN} + \\ &\quad (1 - \hat{\delta}^n) (1 - \hat{\lambda}) \frac{\hat{N}_1^S}{\hat{N}_2^S} \hat{i}_1^{nS} + (1 - \hat{\delta}^n) \hat{\lambda} \frac{\hat{N}_1^N}{\hat{N}_2^S} \hat{S}_1^{nN} + \hat{i}_2^{nS}. \end{aligned}$$

We assume that the investment in knowledge and the population are the same in each of the 25 years of a generation's life. After 25 years, we have:

$$\begin{aligned} \hat{S}_{25}^{nS} &= \left((1 - \hat{\delta}^n) (1 - \hat{\lambda}) \right)^{25} \frac{\hat{N}_0^S}{\hat{N}_{25}^S} \hat{S}_0^{nS} + (1 - \hat{\delta}^n) \hat{\lambda} \sum_{t=0}^{24} \left((1 - \hat{\delta}^n) (1 - \hat{\lambda}) \right)^{24-t} \frac{\hat{N}_0^N}{\hat{N}_{25}^S} \hat{S}_t^{nN} + \\ &\quad \hat{i}_{25}^{nS} \left(\frac{1 - \left((1 - \hat{\delta}^n) (1 - \hat{\lambda}) \right)^{25}}{1 - (1 - \hat{\delta}^n) (1 - \hat{\lambda})} \right). \end{aligned} \quad (\text{B2})$$

¹⁹ Recall that we denote with a tilde variables in annual terms.

As for \hat{S}_t^{nN} , we assume that North makes a constant investment each year of i^{nN} and that population keeps constant within a generation. Capital recursion then yields:

$$\hat{S}_T^{nN} = (1 - \hat{\delta}^n)^T \frac{N_0^N}{N_T^N} \hat{S}_0^{nN} + \left(\frac{1 - (1 - \hat{\delta}^n)^T}{\hat{\delta}^n} \right) i^{nN}. \quad (\text{B3})$$

Substituting (B3) into (B2) gives:

$$\begin{aligned} \hat{S}_{25}^{nS} &= ((1 - \hat{\delta}^n)(1 - \hat{\lambda}))^{25} \frac{N_0^S}{N_{25}^S} \hat{S}_0^{nS} + \\ & (1 - \hat{\delta}^n) \hat{\lambda} \sum_{t=0}^{24} ((1 - \hat{\delta}^n)(1 - \hat{\lambda}))^{24-t} \frac{N_0^N}{N_{25}^S} \left((1 - \hat{\delta}^n)^t \frac{N_0^N}{N_{25}^N} \hat{S}_0^{nN} + \left(\frac{1 - (1 - \hat{\delta}^n)^t}{\hat{\delta}^n} \right) i^{nN} \right) + \\ & i^{nS} \left(\frac{1 - ((1 - \hat{\delta}^n)(1 - \hat{\lambda}))^{25}}{1 - (1 - \hat{\delta}^n)(1 - \hat{\lambda})} \right), \end{aligned}$$

which can be written as:

$$\begin{aligned} \hat{S}_{25}^{nS} &= \left((1 - \hat{\delta}^n)(1 - \hat{\lambda}) \right)^{25} \frac{N_0^S}{N_{25}^S} \hat{S}_0^{nS} + (1 - \hat{\delta}^n)^{25} \hat{\lambda} \left(\frac{1 - (1 - \hat{\lambda})^{25}}{\hat{\lambda}} \right) \frac{N_0^N}{N_{25}^S} \frac{N_0^N}{N_{25}^N} \hat{S}_0^{nN} + \\ & (1 - \hat{\delta}^n) \frac{\hat{\lambda}}{\hat{\delta}^n} \frac{N_0^N}{N_{25}^S} i^{nN} \sum_{t=0}^{24} \left((1 - \hat{\delta}^n)(1 - \hat{\lambda}) \right)^{24-t} (1 - (1 - \hat{\delta}^n))^t + \\ & i^{nS} \left(\frac{1 - ((1 - \hat{\delta}^n)(1 - \hat{\lambda}))^{25}}{1 - (1 - \hat{\delta}^n)(1 - \hat{\lambda})} \right). \end{aligned}$$

The \sum_0^{24} term equals

$$\frac{1 - ((1 - \hat{\delta}^n)(1 - \hat{\lambda}))^{25}}{1 - (1 - \hat{\delta}^n)(1 - \hat{\lambda})} - (1 - \hat{\delta}^n)^{24} \left(\frac{1 - (1 - \hat{\lambda})^{25}}{\hat{\lambda}} \right),$$

and therefore

$$\begin{aligned}
\hat{S}_{25}^{nS} &= \left((1-\hat{\delta}^n)(1-\hat{\lambda}) \right)^{25} \frac{N_0^S}{N_{25}^S} \hat{S}_0^{nS} + (1-\hat{\delta}^n)^{25} \left(1 - (1-\hat{\lambda})^{25} \right) \frac{N_0^N}{N_{25}^S} \frac{N_0^N}{N_{25}^N} \hat{S}_0^{nN} + \\
&\frac{N_0^N}{N_{25}^S} \frac{i^{nN}}{\hat{\delta}^n} \left((1-\hat{\delta}^n) \hat{\lambda} \frac{1 - \left((1-\hat{\delta}^n)(1-\hat{\lambda}) \right)^{25}}{1 - (1-\hat{\delta}^n)(1-\hat{\lambda})} - (1-\hat{\delta}^n)^{25} \left(1 - (1-\hat{\lambda})^{25} \right) \right) + \quad (\text{B4}) \\
&i^{nS} \left(\frac{1 - \left((1-\hat{\delta}^n)(1-\hat{\lambda}) \right)^{25}}{1 - (1-\hat{\delta}^n)(1-\hat{\lambda})} \right).
\end{aligned}$$

(As a check, notice that if $\hat{\lambda} = 0$, this does reduce to the standard knowledge equation without diffusion.)

We can write (B4) in *generational* terms as

$$\begin{aligned}
S_t^{nS} &= \left((1-\hat{\delta}^n)(1-\hat{\lambda}) \right)^{25} \frac{N_{t-1}^S}{N_t^S} S_{t-1}^{nS} + (1-\hat{\delta}^n)^{25} \left(1 - (1-\hat{\lambda})^{25} \right) \frac{N_{t-1}^N}{N_t^S} \frac{N_{t-1}^N}{N_t^N} S_{t-1}^{nN} + \\
&\frac{N_{t-1}^N}{N_t^S} \frac{i^{nN}}{\hat{\delta}^n} \left((1-\hat{\delta}^n) \hat{\lambda} \frac{1 - \left((1-\hat{\delta}^n)(1-\hat{\lambda}) \right)^{25}}{1 - (1-\hat{\delta}^n)(1-\hat{\lambda})} - (1-\hat{\delta}^n)^{25} \left(1 - (1-\hat{\lambda})^{25} \right) \right) + \\
&\hat{i}_t^{nS} \left(\frac{1 - \left((1-\hat{\delta}^n)(1-\hat{\lambda}) \right)^{25}}{1 - (1-\hat{\delta}^n)(1-\hat{\lambda})} \right).
\end{aligned}$$

Equivalently,

$$\begin{aligned}
S_t^{nS} &= (1-\delta^n)(1-\lambda) \frac{N_{t-1}^S}{N_t^S} S_{t-1}^{nS} + (1-\delta^n) \lambda \frac{N_{t-1}^N}{N_t^S} \frac{N_{t-1}^N}{N_t^N} S_{t-1}^{nN} + \\
&\frac{N_{t-1}^N}{N_t^S} \frac{i^{nN}}{\hat{\delta}^n} \left((1-\hat{\delta}^n) \hat{\lambda} \frac{1 - (1-\delta^n)(1-\lambda)}{1 - (1-\hat{\delta}^n)(1-\hat{\lambda})} - (1-\delta^n) \lambda \right) + \\
&\hat{i}_t^{nS} \left(\frac{1 - (1-\delta^n)(1-\lambda)}{1 - (1-\hat{\delta}^n)(1-\hat{\lambda})} \right),
\end{aligned}$$

where $\delta^n = 1 - (1-\hat{\delta}^n)^{25}$ and $\lambda = 1 - (1-\hat{\lambda})^{25}$ are the depreciation and diffusion rates per generation, respectively.

Finally, because investment in knowledge is written in efficiency units of labor per capita,

then

$$S_t^{nS} = (1 - \delta^n)(1 - \lambda) \frac{N_{t-1}^S}{N_t^S} S_{t-1}^{nS} + (1 - \delta^n) \lambda \frac{N_{t-1}^N}{N_t^N} \frac{N_{t-1}^N}{N_t^S} S_{t-1}^{nN} + k_{3N} \frac{N_{t-1}^N}{N_t^S} x_t^{nN} + k_{3S} x_t^{nS} \quad ,$$

where

$$k_{3N} = \frac{1}{\hat{\delta}^n} \left((1 - \hat{\delta}^n) \hat{\lambda} \frac{1 - (1 - \delta^n)(1 - \lambda)}{1 - (1 - \hat{\delta}^n)(1 - \hat{\lambda})} - (1 - \delta^n) \lambda \right) \bar{w}^{nN} \quad , \text{ and}$$

$$k_{3S} = \left(\frac{1 - (1 - \delta^n)(1 - \lambda)}{1 - (1 - \hat{\delta}^n)(1 - \hat{\lambda})} \right) \bar{w}^{nN} \quad .$$

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