

# The Great Diversification and its Undoing\*

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## Abstract

We investigate the hypothesis that macroeconomic fluctuations are primitively the results of many microeconomic shocks, and show that it has significant explanatory power for the evolution of macroeconomic volatility. We define “fundamental” volatility as the volatility that would arise from an economy made entirely of idiosyncratic microeconomic shocks, occurring primitively at the level of sectors or firms. In its empirical construction, motivated by a simple model, the sales share of different sectors vary over time (in a way we directly measure), while the volatility of those sectors remains constant. We find that fundamental volatility accounts for the swings in macroeconomic volatility in the US and the other major world economies in the past half century. It accounts for the “great moderation” and its undoing. Controlling for our measure of fundamental volatility, there is no break in output volatility. The initial great moderation is due to a decreasing share of manufacturing between 1975 and 1985. The recent rise of macroeconomic volatility is due to the increase of the size of the financial sector. We provide a model to think quantitatively about the large comovement generated by idiosyncratic shocks. As the origin of aggregate shocks can be traced to identifiable microeconomic shocks, we may better understand the origins of aggregate fluctuations. (JEL: E32, E37)

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# 1 Introduction

This paper explores the hypothesis that changes in the microeconomic composition of the economy during the post-war period can account for the “great moderation” and its unraveling, both in the US and in the other major world economies. We call “fundamental volatility” the volatility that would be derived only from microeconomic shocks. If aggregate shocks come in large part from microeconomic shocks (augmented by amplification mechanisms), then aggregate volatility should track fundamental volatility. To operationalize this idea, the key quantity we consider (which constitutes one departure from other studies) is the following definition of “fundamental volatility”:

$$\sigma_{Ft} = \sqrt{\sum_{i=1}^n \left(\frac{S_{it}}{\text{GDP}_t}\right)^2 \sigma_i^2}, \quad (1)$$

where  $S_{it}$  is the gross output (not just value added) of sector  $i$ , and  $\sigma_i$  is the standard deviation of the total factor productivity (TFP) in the sector. Note that the evolution of  $\sigma_{Ft}$  will only reflect the changing weights of different sectors in the economy, as micro-level TFP volatility is held constant through time. Notice also that in this measure the weights do not add up to one. These are the “Domar weights” that research in productivity studies (Domar 1961, Hulten 1978) has identified as the proper weights to study the impact of microeconomic shocks.

Figure 1 plots  $\sigma_{Ft}$  for the US. We see a local peak around 1975, then a fall (due to the decline of a handful of manufacturing sectors), followed by a new rise (which we will relate to the rise of finance). This looks tantalizingly like the evolution of the volatility of US GDP growth. Indeed, we show statistically that the volatility of the innovations to GDP is well explained by the fundamental volatility  $\sigma_{Ft}$ . In particular, our measure explains the great moderation: the existence of a break in the volatility of US GDP growth around 1984. After controlling for fundamental volatility, there is no break in GDP volatility. Our measure also accounts for the rise in GDP volatility, as finance became large from the mid 1990s onward, creating an increase in fundamental volatility.

In Figure 2 we present a similar analysis for the major economies for which we could get disaggregated data about shares and TFP movements: Japan, Germany, France, and the United Kingdom. The results also indicate that fundamental volatility tracks GDP volatility.

Our conclusion is that fundamental volatility appears to be a quite useful explanatory construct. It provides an operational way to understand the evolution of volatility, and sheds more light on the origins of the latter.

Hence, our paper may bring us closer to a concrete understanding of the origin of macro-

economic shocks. What causes aggregate fluctuations? It has proven convenient to think about aggregate productivity shocks, but their origin is mysterious: what is the common high-frequency productivity shock that affects Wal-Mart and Boeing? This is why various economists have progressively developed the hypothesis that macroeconomic fluctuations can be traced back to microeconomic fluctuations. This literature includes Long and Plosser (1983), who proposed a baseline multi-sector model. Its implementation is relatively complex, as it requires sector sizes that are constant over time (unlike the evidence we rely on), and the use of input-output matrices. Horvath (1998, 2000) perhaps made the greatest strides toward developing these ideas empirically, in the context of a rich model with dynamic linkages. The richness of the model might make it difficult to see what drives its empirical features, and certainly prevents the use of a simple concept like the concept of fundamental volatility. Dupor (1999) disputes that the origins of shocks can be microeconomic, on the grounds of the law of large numbers: if there is a large number of sectors, aggregate volatility should vanish proportionally to the square root of the number of sectors. Hence, Horvath’s result would stem from poorly disaggregated data. Carvalho (2009), taking a network perspective on sectoral linkages, shows that the presence of hub-like, general-purpose inputs, can undo the law of large numbers argument and enable microeconomic shocks to affect aggregate volatility<sup>1</sup>. Acemoglu, Ozdaglar, and Tahbaz-Salehi (2010) show how this perspective leads to volatility cascades.

Gabaix (2011) points out that “sectors” may be arbitrary constructs, and the fat-tailed Zipf distribution of firms (or perhaps very disaggregated sectors) necessarily leads to a high amount of aggregate volatility coming from microeconomic shocks, something he dubs the “granular” hypothesis. In that view, microeconomic fluctuations create a fat-tailed distribution of firms or sectors (Simon 1955, Gabaix 1999, Luttmer 2007). In turn, the large firms or sectors coming from that fat-tailed distribution create GDP fluctuations. Gabaix also highlights the conceptual usefulness of the notion of fundamental volatility. Di Giovanni and Levchenko (2009) show how that perspective helps explain the volatility of trade fluctuations across countries.

Against this backdrop, we use a simple way to cut through the complexity of the situation, and rely on a simple, transparent microeconomic construct, fundamental volatility, to predict an important macroeconomic quantity, GDP volatility.

By bringing fundamental volatility into the picture, we contribute to the literature on the

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<sup>1</sup>Interesting other conceptual contributions for the micro origins of macro shocks include Bak *et al.* (1993), Jovanovic (1987), Durlauf (1993), and Nirei (2006).

origins of the “great moderation,” a term coined by Stock and Watson (2002): the decline in the volatility of US output growth around 1984, up until about 2007 and the financial crisis. The initial contributions (McConnell and Perez-Quiros 2000, Blanchard and Simon 2001) diagnosed the decline in volatility, and conjectured that some basic explanations (including sectoral shifts – to which we will come back) did not seem promising. Perhaps better inventory management (Irvine and Schuh 2005) or better monetary policy (Clarida, Gali and Gertler 2000) were prime candidates. However, given the difficulty of relating those notions to data, much of the discussion was conjectural. Later, more full-fledged theories of the great moderation have been advanced. Arias, Hansen, and Ohanian (2007) attribute the changes in volatility to changes in TFP volatility within a one-sector model. Our work sheds light on the observable microeconomic origins of this change in TFP volatility. Justiniano and Primiceri (2008) demonstrate that much of the great moderation could be traced back to a change in the volatility of the investment demand function. Gali and Gambetti (2009) document a change in both the volatility of initial impulses and the impulse-response mechanism. Compared to those studies, we use much more disaggregated data, which allows us to calculate the fundamental volatility of the economy. Because we use richer disaggregated data, we can obviate some of the more heavy artillery of dynamic stochastic general equilibrium (DSGE) models, and have a parsimonious toolkit to think about volatility. We defer the discussion of Jaimovich and Siu (2009) to Section 6.2. We view our proposal as complementary to those other mechanisms presented in the literature.

Finally, we relate to the literature on technological diversification and its effects on aggregate volatility. Imbs and Wacziarg (2003) and Koren and Tenreyro (2007) have shown that cross-country variation in the degree of technological diversification helps explain cross-country variation in GDP volatility and its relation with the level of development of an economy. Here we concentrate on the time series dimension of this mechanism and show how it can generate the observed long swings in volatility for a given economy. We also differ in that we build our measures of diversification and sectoral level volatility from microeconomic TFP accounting rather than from unspecified sectoral shocks<sup>2</sup>.

The methodological principle of this paper is to use as simple and transparent an approach

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<sup>2</sup>The recent contributions of Koren and Tenreyro (2009) and Moro (2009) provide quantitative models - of, respectively, technological diversification and structural change - that generate a decline of volatility that is consistent with empirical evidence. However, they do not contemplate the possibility of a reversal of these patterns. We instead show, through our fundamental volatility construct, that such a reversal is present and is key to explaining the recent period of higher volatility. See also Caselli et al (2010) for an analysis of how increased trade openness contributed to diversification of cost shocks.

as possible. In particular, we find a way to avoid the use of the input-output matrix, which has no claim to be stable over time, and is not necessary in our framework. We examine the economics through a very simple two-period model, rather than an infinite-horizon DSGE model. Useful as they are for a host of macroeconomic questions, those models have many free parameters, and we find it instructive to focus our attention on a zero-free parameter construct, the fundamental volatility of the economy. This said, a potentially fruitful next step is to build a DSGE model on the many sectors in the economy.

Section 2 presents a very simple framework that motivates our concept of fundamental volatility and its implementation. Section 3 summarizes the basic empirical results. Section 4 presents a brief history of fundamental volatility. Section 5 discusses how the role of correlation in the business cycle, and why previous analyses were pessimistic about the role of microeconomic shocks. Section 6 expands on a variety of points, including the use of value added vs. sales, firms vs. sectors, and other robustness checks. Section 7 concludes on the role of policy and the use of fundamental volatility as an early warning system. The Appendices provide an account of the data and procedures we employ, as well as the proofs.

## 2 Framework and Motivation

In this section, we present a simple multisector model that exposes the basic ideas and motivates our empirical work. There are  $n$  sectors that produce intermediate and final goods, and two primitive factors, capital and labor. Sector  $i$  uses inputs capital and labor inputs  $K_i$  and  $L_i$ , and a vector of intermediary inputs  $\mathbf{X}_i = (X_{ij})_{j=1\dots n} \in \mathbb{R}^n$ : it uses a quantity  $X_{ij}$  from sector  $j$ . It produces a gross output  $Y_i = A_i F_i(K_i^{1-\alpha} L_i^\alpha, \mathbf{X}_i)$ , where  $F_i$  is homogenous of degree 1. We call  $\mathbf{C} = (C_1, \dots, C_n)$  and  $\mathbf{A} = (A_1, \dots, A_n)$  the vectors of consumption and productivity.

The utility function is  $\mathcal{C}(\mathbf{C}) - L^{1+1/\phi}$ , where  $\mathcal{C}$  is homogenous of degree 1, so that  $\mathcal{C}$  is like an aggregate good. Hence, given inputs  $K$  and  $L$ , the aggregate production function is defined as:

$$F(K, L; \mathbf{A}) = \max_{\substack{C_i, L_i, K_i, X_{ij} \\ 1 \leq i, j \leq n}} \mathcal{C}(\mathbf{C}) \text{ subject to} \\ \sum_i K_i \leq K; \quad \sum_i L_i \leq L; \quad \forall i, C_i + \sum_j X_{ji} \leq A_i F_i(K_i^{1-\alpha} L_i^\alpha, \mathbf{X}_i).$$

Note that this economic structure admits quite general linkages between sectors via the production functions  $F_i$  and the factor markets. The following lemma, whose proof is in the Appendix, describes some aggregation results in this economy.

**Lemma 1** *The aggregate production function can be written:*

$$F(K, L; \mathbf{A}) = \Lambda(\mathbf{A}) K^{1-\alpha} L^\alpha$$

for an aggregate TFP  $\Lambda(\mathbf{A})$ . After sectoral-level TFP shocks  $dA_i$ , the shock to aggregate TFP  $\Lambda$  is:

$$\frac{d\Lambda}{\Lambda} = \sum_{i=1}^N \frac{S_i}{Y} \frac{dA_i}{A_i} \quad (2)$$

where  $S_i$  is the dollar value of the gross output of sector  $i$ .

Formula (2) is Hulten (1978)'s result. We rewrite it slightly, using the “hat” notation for fractional changes,  $\widehat{Z} \equiv dZ/Z$ ,<sup>3</sup> and time subscripts. Shocks  $\widehat{A}_{it}$  to the productivity in sector  $i$  result in an aggregate TFP growth  $\widehat{\Lambda}_t$  expressed as:

$$\widehat{\Lambda}_t = \sum_{i=1}^N \frac{S_{it}}{Y_t} \widehat{A}_{it} \quad (3)$$

where  $S_{it}$  is the dollar value of sales (gross output) in sector  $i$ , and  $Y_t$  is GDP.  $S_{it}/Y_t$  is called the “Domar” weight.

Note that the sum of the weights  $\sum_{i=1}^N S_{it}/Y_t$  can be greater than 1. This is a well-known and important feature of models of complementarity. If on average the sales to value added ratio is 2, and each sector has a TFP increase of 1%, the aggregate TFP increase is 2%. This effect comes from the fact that a Hicks-neutral productivity shock increases gross output (sales), not just value added, and has been analyzed by Domar (1961), Hulten (1978), and Jones (2009).<sup>4</sup>

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<sup>3</sup>The rules are well known and come from taking the logarithm and differentiating. For instance,  $X^\alpha Y^\beta Z^\gamma = \alpha \widehat{X} + \beta \widehat{Y} + \gamma \widehat{Z}$ .

<sup>4</sup>The intuition for (3) is the following. Suppose there are just two sectors, say cars (a final good, sector 1) and plastics (both a final and an intermediary good, sector 2). Cars use plastics as an intermediary input. Suppose furthermore there is productivity growth of  $\widehat{A}_{1t} = 1\%$  in cars and  $\widehat{A}_{2t} = 3\%$  in plastics. Suppose that, after the shock, there is no reallocation of factors. We then have 1% more cars in the economy and 3% more plastics. Those goods have not yet been reallocated to production, but still, they have a “social value,” captured by their price. Hence, if the economy uses the same quantity of factors, GDP has increased by  $dY_t = 1\% \times \text{initial value of cars} + 3\% \times \text{initial value of plastic}$ , i.e.,  $dY_t = S_{1t} \times \widehat{A}_{1t} + S_{2t} \times \widehat{A}_{2t}$ . Dividing by  $Y_t$ , we get  $\frac{dY_t}{Y_t} = \sum_{i=1}^2 \frac{S_{it}}{Y_t} \cdot \widehat{A}_{it}$ . However, what has increased is the productive capacity of the economy. So, it is really TFP that has increased by  $\widehat{\Lambda}_t = \sum_{i=1}^2 \frac{S_{it}}{Y_t} \widehat{A}_{it}$ . GDP might increase more or less once we take into account the response of labor supply, something we shall consider very soon.

Consider the baseline case where productivity shocks  $\widehat{A}_{it}$  are uncorrelated across  $i$ 's, and unit  $i$  has a variance of shocks  $\sigma_i^2 = \text{var}(\widehat{A}_{it})$ . Then, we have  $\sigma_\Lambda = \sigma_F$ , where we define:

$$\sigma_{Ft} = \sqrt{\sum_{i=1}^N \left(\frac{S_{it}}{Y_t}\right)^2 \sigma_i^2}. \quad (4)$$

This defines the ‘‘fundamental’’ volatility, which comes from microeconomic shocks. Gabaix (2011) calls this the ‘‘granular’’ volatility.

To see the changes in GDP, we assume that capital can be rented at a price  $r$ . The agent’s consumption is  $Y - rK$  with  $Y = \Lambda K^{1-\alpha} L^\alpha$ . The competitive equilibrium implements the planner’s problem, which is to maximize the agent’s utility subject to the resource constraint:

$$\max_{K,L} C - L^{1+1/\varphi} \text{ subject to } C = \Lambda K^{1-\alpha} L^\alpha - rK.$$

The solution is obtained by standard methods detailed in the proof of Proposition 1.

$$Y = k\Lambda^{\frac{1+\varphi}{\alpha}},$$

for an unimportant constant  $k$ . Taking logs,  $\ln Y = \frac{1+\varphi}{\alpha} \ln \Lambda + \ln k$ , and a change in TFP  $\widehat{\Lambda}$  creates a change in GDP equal to

$$\widehat{Y} = \frac{1+\varphi}{\alpha} \widehat{\Lambda}.$$

Given that the volatility of TFP is the fundamental volatility  $\sigma_{Ft}$ , the volatility of GDP is  $\sigma_{GDPt} = \frac{1+\varphi}{\alpha} \sigma_{Ft}$ . We summarize the situation in the next proposition.

**Proposition 1** *The volatility of GDP growth is*

$$\sigma_{Yt} = \mu \cdot \sigma_{Ft}, \quad (5)$$

where the fundamental volatility  $\sigma_{Ft}$  is given by (4), and the productivity multiplier  $\mu$  is equal to

$$\mu = \frac{1+\varphi}{\alpha}. \quad (6)$$

Here  $\alpha$  is the labor share and  $\varphi$  is the Frisch elasticity of labor supply.

Our hypothesis is that, indeed,  $\sigma_{Ft}$  explains a substantial part of GDP volatility, as motivated by (5). In our baseline specification, we construct  $\sigma_{Ft}$  as in equation (4), taking the sales to value added weights directly from detailed sectoral data provided by Dale Jorgenson and associates. We use the same data source to compute sectoral TFP growth - by standard TFP accounting (with intermediate inputs) methods; see for example Jorgenson et al (1987) -

and then take its standard deviation to obtain  $\sigma_i$ <sup>5</sup>. Notice that we keep  $\sigma_i$  time-independent. We do this for two reasons. First, Section 6.1 shows that the volatility of TFP does not exhibit any marked trend at the micro level, and that our results are robust to time-varying volatility. Second, by using a constant  $\sigma_i$ , we highlight that the changes in fundamental volatility come only from changes in the shares of the largest sectors in the economies, rather than from their volatilities (which would make the explanation run the risk of being circular). We thus explain time-varying GDP volatility solely with time-varying shares of economic activity within the economy.

To interpret the results, it is useful to comment on the calibration. We interpret the elasticity of labor supply broadly, including not only changes in hours worked per employed worker, but also changes in employment, and changes in effort. Using this notion, recent research (e.g., summarized in Hall 2009a,b) is consistent with a value of  $\varphi = 2$ , in part because of the large reaction of employment and effort (as opposed to simply hours worked per employed worker) to business cycle conditions. Using these values and the labor share of  $\alpha = 2/3$ , we obtain a multiplier of  $\mu = 4.5$ .

Figure 1 shows the fundamental volatility graphs from 1960 to 2008. We see that fundamental volatility and GDP volatility track each other rather well. By 2008 we are already at mid 1980s levels. This suggests a good correlation between fundamental volatility and GDP volatility. The next section studies this systematically.

## 3 Fundamental Volatility and Low-Frequency Movements in GDP Volatility

### 3.1 US Evidence

#### 3.1.1 Basic Facts

As a baseline measure of cyclical volatility, we first obtain deviations from the HP trend of log quarterly real GDP (smoothing parameter 1600; sample 1947:Q1 to 2009:Q4; source FRED database). We then compute the standard deviation at quarter  $t$  using a rolling window of 10 years (41 quarters, centered around quarter  $t$ ) In order to extend the period to the latest

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<sup>5</sup>The original data is annual and provides a breakdown of the entire US economy into 88 sectors. Following much of the sectoral productivity literature we focus on private sector output and drop government sectors. See Appendix A for further details on the data sources and on the construction of our fundamental volatility measure.



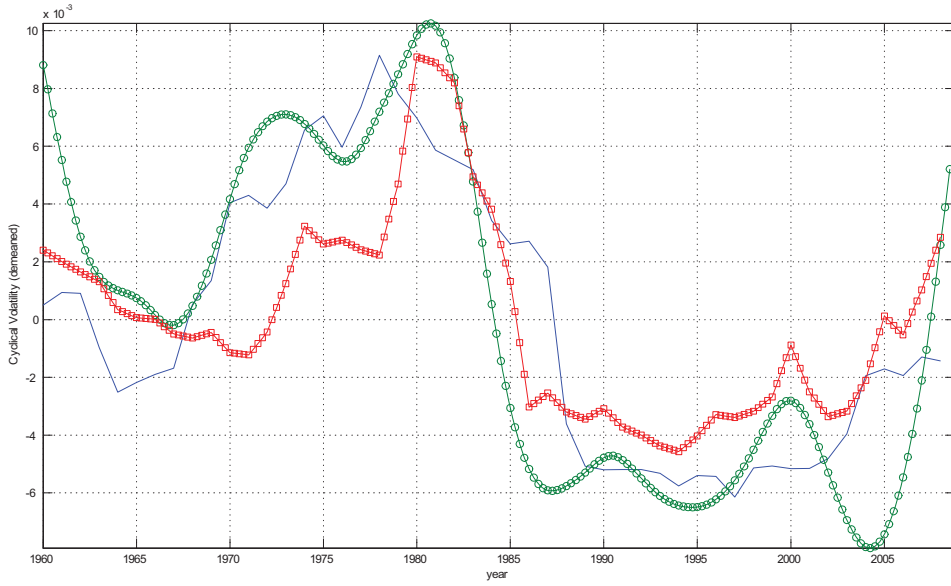


Figure 1: Fundamental Volatility and GDP Volatility. The squared line gives the fundamental volatility ( $4.5\sigma_{Ft}$ , demeaned). The solid and circle lines are annualized (and demeaned) estimates of GDP volatility, using respectively a rolling window estimate and an HP trend of instantaneous volatility.

recession, for 2005:Q1 until 2009:Q4 we use uncentered (i.e., progressively more one-sided) windows. We refer to this measure as  $\sigma_{Yt}^{\text{Roll}}$ .

As a robustness check, we also consider a different measure of cyclical volatility, namely the instantaneous quarterly standard deviation as computed by McConnell and Quiros (2000). For this measure, we start by fitting an AR(1) model to real GDP growth rates (1960:Q1 until 2008:Q4):

$$\Delta y_t = \psi + \phi \Delta y_{t-1} + \epsilon_t \quad (7)$$

where  $y_t$  is log GDP. We obtain as estimates  $\psi = 0.006$  ( $t = 6.78$ ) and  $\phi = 0.292$  ( $t = 4.20$ ). As is well known, an unbiased estimator of the annualized standard deviation is given by  $2\sqrt{\frac{\pi}{2}}|\hat{\epsilon}_t|$ , where the factor 2 converts quarterly volatility into annualized volatility, and the  $\sqrt{\frac{\pi}{2}}$  comes from the fact that, if  $\epsilon \sim N(0, \sigma^2)$ , then  $\sigma = E[\sqrt{\frac{\pi}{2}}|\epsilon|]$ . We refer to

$$\sigma_{Yt}^{\text{Inst}} \equiv 2\sqrt{\frac{\pi}{2}}|\hat{\epsilon}_t| \quad (8)$$

as the “instantaneous” measure of GDP volatility. We shall also use  $\sigma_{Yt}^{\text{HP}}$ , the Hodrick-Prescott smoothing of the instantaneous volatility  $\sigma_{Yt}^{\text{Inst}}$ .

Figure 1 plots the familiar great moderation graphs depicting the halving of volatility in the mid 1980s. Interestingly, both measures also point to a significant increase in volatility from the early 2000s on, mostly as a result of the recent crisis. It also depicts the sample fit of our fundamental volatility measure,  $\sigma_{Ft}$ , for the annual case given a baseline value of  $\mu = 4.5$ . In particular, it shows  $\sigma_{Yt}^{\text{Roll}}$  and  $\sigma_{Yt}^{\text{HP}}$  (annualized and demeaned), together with  $4.5\sigma_{Ft}$  (demeaned).

We run least squares regressions of the type:

$$\sigma_{Yt} = a + b\sigma_{Ft} + \eta_t \quad (9)$$

where  $\sigma_{Ft}$  is our measure of fundamental volatility, and  $\sigma_{Yt}$  is one of the measures of volatility described above:  $\sigma_{Yt}^{\text{Roll}}$  for the rolling window estimate,  $\sigma_{Yt}^{\text{Inst}}$  for the instantaneous standard deviation measure. Note that  $\sigma_{Ft}$  is only available on an annual basis. As such we pursue two different strategies: i) annualizing the standard deviation measure by averaging the left-hand side over four quarters (“annual” below), or ii) linearly interpolating our measures of fundamental volatility in order to obtain quarterly frequency data.

Table 1: GDP Volatility and Fundamental Volatility

|           | Annual Data- $\sigma_{Yt}^{\text{Roll}}$ | Annual Data- $\sigma_{Yt}^{\text{Inst}}$ | Quarterly Data- $\sigma_{Yt}^{\text{Roll}}$ | Quarterly Data- $\sigma_{Yt}^{\text{Inst}}$ |
|-----------|--|--|---|---|
| $\hat{a}$ | -0.029<br>(-5.53;0.005)                  | -0.0483<br>(-4.47;0.019)                 | -0.031<br>(-12.26;0.003)                    | -0.0478<br>(-4.10;0.012)                    |
| $\hat{b}$ | 4.815<br>(8.39;0.574)                    | 7.015<br>(5.89;1.190)                    | 5.056<br>(18.18;0.281)                      | 6.955<br>(5.40;1.288)                       |
| $R^2$     | 0.60                                     | 0.43                                     | 0.63  | 0.13  |

*Notes:* Regression of GDP volatility on fundamental volatility:  $\sigma_{Yt} = a + b\sigma_{Ft} + \eta_t$ . In parentheses are  $t$ -statistics and standard errors.

Table 1 summarizes the results, at both annual and quarterly frequency. We find good statistical and economic significance of  $\sigma_F$ .<sup>6</sup> It is the sole regressor, and its  $R^2$  is around 60% for the rolling estimate of volatility.<sup>7</sup> This shows that  $\sigma_{Ft}$  explains a good fraction of the

<sup>6</sup>Our model predicts an intercept  $a = 0$ . Simple variants could predict a positive  $a$ , or a negative  $a$ , as we find here empirically. A positive  $a$  is generated by adding other shocks to GDP. A negative  $a$  is generated if the multiplier  $\mu$  is increasing in  $\sigma_F$  rather than constant, i.e., if the economy’s technologies are more flexible when the environment is more volatile.

<sup>7</sup>As a two-step OLS can be inefficient econometrically, we have also performed an ARCH-type maximum-likelihood estimation, based on the joint system (7) and  $\sigma_t = \alpha + \beta\sigma_{Ft} + \eta_t$ . Its results are very similar to those in Table 1.

historical evolution on GDP volatility. Of course, the  $R^2$  of  $\sigma_{Y_t}^{\text{Inst}}$  is lower than for  $\sigma_{Y_t}^{\text{Roll}}$ , as  $\sigma_{Y_t}^{\text{Inst}}$  is a much more volatile measure of GDP volatility.

Note that, in our regressions, all the movements come from the sizes of sectors: their volatilities are fixed in our construction of  $\sigma_F$ . We do this for parsimony's sake, and also because it is warranted by the evidence: the average volatility of sectoral-level microeconomic volatility did not have noticeable trends in the sample. Indeed, the average sectoral-level volatility is 3.4% in the 1960-2005 period, 3.5% in the 1960-1984 period, and 3.2% in the 1984-2005 period. We cannot reject the null of equal mean volatility across the two sample periods (the  $p$ -value is 0.18). Hence, our construction of  $\sigma_F$  allows to isolate the impact of the changes in microeconomic composition of the economy.

### 3.1.2 Accounting for the Break in US GDP Volatility

A common way of quantifying the great moderation is to test the null hypothesis of a constant level in GDP volatility

$$\sigma_{Y_t} = a + \eta_t$$

against an alternative representation featuring a break in the level

$$\sigma_{Y_t} = a + cD_t + \eta_t$$

where  $D_t$  is a dummy variable assuming a value of 1 for periods  $t \geq T$  given an estimated break date  $T$ . Following common practice in the literature (see McConnell and Quiros (2000), Stock and Watson (2002) and Sensier and van Dijk (2004), we take  $\sigma_{Y_t}$  to be given by the instantaneous volatility measure  $\sigma_{Y_t}^{\text{Inst}}$ , and test for the presence of a break in level using Bai and Perron's (1998) SupLR test statistic<sup>8</sup>. In what follows, we look for a single break date  $T$ , where we assume  $T$  lies in a range  $[T_1, T_2]$  with  $T_1 = 0.2n$  and  $T_2 = 0.8n$ , and  $n$  is the total number of observations (i.e., the trimming percentage is set at 20% of the sample).<sup>9</sup>

To assure comparability and since our sample period does differ, we start by reconfirming the findings first reported in McConnell and Quiros (2000). We do find strong support for a level break with a SupLR statistic of 32.33 (the 5% critical value is 8.75). The estimated break date  $T$  is 1984:1 and is estimated with a 90% confidence interval given by 1981:2–1986:4.

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<sup>8</sup>McConnell and Quiros (2000) and Stock and Watson (2002) show that, for U.S. quarterly GDP, one cannot reject the null of no break in the autoregressive coefficients in the equation for GDP growth, thus enabling us the residuals in (7) to test for a break in the variance.

<sup>9</sup>Bai and Perron (2006) find that serial correlation can induce significant size distortions when low values of the trimming percentage are used, and recommend values of 15% or higher. We use code made available by Qu and Perron (2007) to compute the test statistics and obtain critical values.

The estimated value of  $c$  is  $-0.0104$  ( $t = -5.50$ ), implying a permanent decrease in aggregate volatility after this date.

Table 2: Break Tests with Fundamental Volatility

|                  | Break Test With or Without Fundamental Volatility on the Right-Hand Side |                                  |                                  |                                     |
|------------------|--|----------------------------------|----------------------------------|-------------------------------------|
|                  | Without  | With                             |                                  |                                     |
|                  | H <sub>0</sub> : No break in $a$   | H <sub>0</sub> : No Break in $a$ | H <sub>0</sub> : No Break in $b$ | H <sub>0</sub> : No Break in $a, b$ |
|                  | (i)  | (ii)                             | (iii)                            | (iv)                                |
| SupLR stat.      | 32.33  | 8.60                             | 8.96                             | 9.33                                |
| Null of no break | Reject   | Accept                           | Accept                           | Accept                              |
| Est. break date  | 1984:1   | None                             | None                             | None                                |

*Notes:* We perform a break test for equation  $\sigma_{Y_t}^{Inst} = a + \eta_t$  (column i) and  $\sigma_{Y_t}^{Inst} = a + b\sigma_{F_t} + \eta_t$ , the regression of instantaneous GDP volatility on fundamental volatility (columns ii-iv). Column (i) confirms that, without conditioning on fundamental volatility, there is a break in GDP volatility (the great moderation). Next, column (ii) performs a test on the  $a$  coefficient. We cannot reject the null hypothesis of no break. This means that, once we control for fundamental volatility, there is no break in GDP volatility. The subsequent tests for breaks in  $b$  and  $(a, b)$  are extra robustness checks (columns iii-iv); they confirm the conclusion that, after controlling for fundamental volatility, there is no break in GDP volatility. From Qu and Perron (2007), the 5% asymptotic critical values reported for the *SupLR* statistic are 13.34 for (ii) and (iii), and 11.17 for (iv).

We next test the hypothesis that, once our fundamental volatility measure is accounted for in the dynamics of  $\sigma_{Y_t}^{Inst}$ , there is no such level break in aggregate volatility. That is, we test for the null of no break in the intercept of the equation:<sup>10</sup>  $\sigma_{Y_t}^{Inst} = a + b\sigma_{F_t} + \eta_t$ . To rule out the additional possibility that the break in aggregate volatility is the result of a break in its link with fundamental volatility, we also test the null of no break on the slope parameter  $b$  and the joint null of no break in both  $a$  and  $b$ .<sup>11</sup> To maximize the total number of observations,

<sup>10</sup>The resulting SupLR test statistic reported in the Table is computed under the assumptions of no serial correlation in  $\eta_t$  and the same distribution of  $\eta_t$  across segments. The key conclusion (failure to reject the null of no break in  $a$  when we account for fundamental volatility) is unchanged when we relax either or both of these assumptions.

<sup>11</sup>When testing for the null of no break in only one of the parameters (either the intercept or the slope), we are imposing the restriction that there is no break in the other parameter.

we opt to use a quarterly interpolation of our fundamental volatility measure.

The results are in Table 2. We cannot reject the null of no break in any of these settings. We conclude that, after controlling for the time series behavior of fundamental volatility, there is no break in GDP volatility. This is the sense in which fundamental volatility explains the great moderation (and its undoing): after controlling for the changes in fundamental volatility, there is no statistical evidence of a residual great moderation.

### 3.2 International Evidence

We now extend the previous analysis to the four other major economies: France, Japan, Germany, and the UK. As is well known (see Stock and Watson 2005), these countries have exhibited quite different low-frequency dynamics of GDP volatility throughout the last half-century. Under the hypothesis of this paper, it should be the case that the evolution of our measure of fundamental volatility is also heterogenous across these economies.

Relative to the US, we face greater data limitations, along both the time series and cross-sectional dimensions. We are able to construct the Domar weight measures from 1970 to 2005 (from 1973 for Japan). Though we have considerable sectoral details for nominal measures, sector-specific price indexes are only available for half or less of the sectors in each country.<sup>12</sup> This renders impossible an accurate weighting by sectoral TFP volatility. Therefore, we choose to consider a special case of our fundamental volatility measure given by

$$\bar{\sigma}_{Ft} = \bar{\sigma} \sqrt{\sum_{i=1}^N \left(\frac{S_{it}}{Y_t}\right)^2} \quad (10)$$

where  $\bar{\sigma}$  is the average standard deviation of sectoral TFP over the entire sample period in the country under consideration. The values of  $\bar{\sigma}$  are 2.3% for France, 2.0% for Germany, 3.2% for Japan, 2.2% for the UK, and 3.4% for the US. Finally, motivated by our discussion above, we consider a multiplier  $\mu = 4.5$  to obtain the volatility of GDP implied by our fundamental volatility measure, i.e.,  $\sigma_{Yt} = 4.5\bar{\sigma}_{Ft}$ . As in the US case, we take as a baseline measure of cyclical volatility the 10-year rolling window standard deviation of HP-filtered quarterly real GDP. Figure 2 compares the evolution of these measures (where we, again, demean both measures).

As in the US case, our proposed measure seems to account well for the (different) low-frequency movements in GDP volatility in this set of countries. In the UK it captures the strong reduction in volatility in the late 1970s and its leveling off until the mid 1990s. We

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<sup>12</sup>See Appendix A for more details on the sources, description, and construction of these measures.

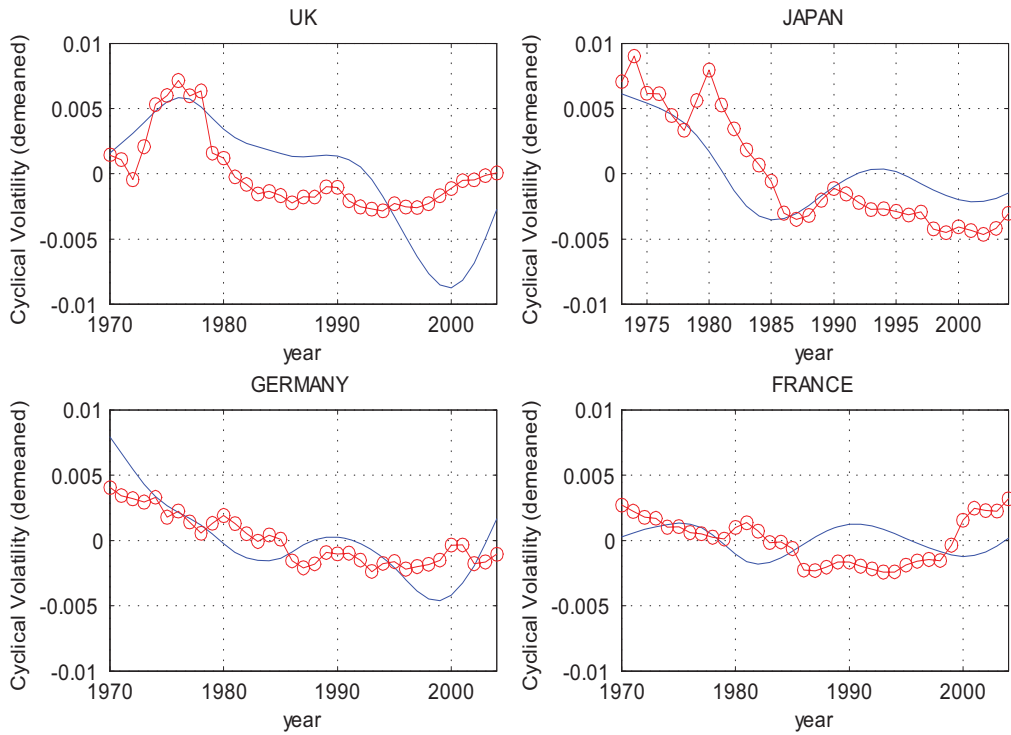


Figure 2: GDP Volatility and Fundamental Volatility in Four OECD countries. Solid line: smoothed rolling window standard deviation of deviations from HP trend of quarterly real GDP. Circle line: fundamental volatility measure,  $\sigma_{Y_t} = 4.5\bar{\sigma}_{F_t}$ . Both measures are demeaned. We report results for the four large countries for which we have enough disaggregated data.

cannot account for the short lived drop in UK volatility around 2000 (but notice that by 2005 the levels of our measure and the data are again very close). As Stock and Watson (2005) noticed, Germany provides a different picture, that of a large but gradual decline. Again, our measure does well, displaying a much smoother negative trend. For Japan, fundamental volatility tracks well the fall in GDP volatility in the late 1970s and early 1980s, as well as the often noted reversal occurring in the mid 1980s. For France, our measure displays no discernible trend, hovering around its mean throughout the sample period. This is in line with the muted low-frequency dynamics of French GDP volatility.

Table 3: GDP Volatility and Fundamental Volatility: International Evidence

|                   | $\sigma_{ct}$         | $\sigma_{ct}$         |
|-------------------|-----------------------|-----------------------|
| $\widehat{\beta}$ | 3.253<br>(7.91;0.411) | 1.941<br>(3.63;0.534) |
| $\chi t$          | No                    | Yes                   |
| Observations      | 172                   | 172                   |

*Notes:* We run the regression  $\sigma_{Yct} = \alpha_c + \chi t + \beta \bar{\sigma}_{Fct} + \varepsilon_{ct}$ , where  $\sigma_{Yct}$  is the country volatility using a rolling window measure,  $\bar{\sigma}_{Fct}$  is the fundamental volatility of the country defined in (10),  $\alpha_c$  a country fixed effect and  $\chi t$  a linear time trend.  $t$ -statistics and standard errors in parentheses.

To complement this, we consider running panel regressions

$$\sigma_{Yct} = \alpha_c + \chi t + \beta \bar{\sigma}_{Fct} + \varepsilon_{ct} \quad (11)$$

where  $\sigma_{Yct}$  is our rolling window measure of cyclical volatility for country  $c$  in year  $t$  and  $\bar{\sigma}_{Fct}$  are the country-specific fundamental volatility measures. We include the US along with the four other economies mentioned above. To preserve comparability, we construct and use  $\bar{\sigma}_{Fct}$  for the US (rather than  $\sigma_{Ft}$  described in the previous section). We use country fixed effects  $\alpha_c$ , and run the above panel with and without a common linear time trend  $\chi t$ . We view the specification without a time trend as the cross-country analog of the regressions run above for the US alone. The specification with a time trend allows us to control for potential common factors affecting volatility in all countries, and therefore identifies  $\beta$  through cross-country timing differences in the evolution of fundamental volatility. While this specification renders the value of  $\beta$  not comparable to the values obtained for the simple US regression,

it strengthens our results by minimizing possible spurious regression type problems in our baseline specification.<sup>13</sup>

Table 3 reports the results. All results are significant at the 1% level (they are also significant without the US). Again, we confirm the existence of a tight link between aggregate volatility and our fundamental volatility measure. Note that, for the specification without a linear trend, our measure is quantitatively similar in the cross-country case. Its significance survives when we allow for a linear time trend and when instrumented by its own lag.

## 4 A Brief History of Fundamental Volatility

The previous section has shown that fundamental volatility correlates well with GDP volatility. In this section, we present a brief account of the evolution of our fundamental volatility measure in the last half-century.<sup>14</sup> To make it quantitative, we define:<sup>15</sup>

$$H_i(t_1, t_2) = \frac{\left(\frac{S_{it_2}}{Y_{t_2}}\right)^2 \sigma_i^2 - \left(\frac{S_{it_1}}{Y_{t_1}}\right)^2 \sigma_i^2}{\sigma_{Ft_2}^2 - \sigma_{Ft_1}^2}. \quad (12)$$

That is,  $H_i(t_1, t_2)$  indicates how much of the change in squared fundamental volatility between  $t_2$  and  $t_1$  can be explained by the corresponding change in the squared Domar weight of industry  $i$ . By construction,  $\sum_i H_i(t_1, t_2) = 1$  for all  $t_1 \neq t_2$ .

### 4.1 United States

We find it useful to break our account of fundamental volatility into three questions: i) What accounts for the “long and large decline” of fundamental volatility from the 1960s to the early 1990s? ii) What accounts for the interruption of this trend from the mid 1970s to the early 1980s? iii) What is behind the reversal of fundamental volatility dynamics observed around the mid 1990s and its subsequent increase until 2008?

Our answers are the following: i) The long and large decline of fundamental volatility from the 1960s to the early 1990s is due to the smaller size of a handful of heavy manufacturing sectors. ii) The growth of the oil sector (which itself can be traced to the rise of the oil price) accounts for the burst of volatility in the mid 1970s. iii) The increase in the size of the financial

<sup>13</sup>We report heteroskedasticity-autocorrelation robust standard errors by using a Newey-West estimator with 2 lags.

<sup>14</sup>See Jorgenson and Timmer (forth.) for another analysis of structural change.

<sup>15</sup>For countries outside the US, in accordance to (10) we use  $H_i(t_1, t_2) = \frac{(S_{it_2}/Y_{t_2})^2 \bar{\sigma}^2 - (S_{it_1}/Y_{t_1})^2 \bar{\sigma}^2}{\sigma_{Ft_2}^2 - \sigma_{Ft_1}^2}$ .



sector is an important determinant of the increase in fundamental volatility. We now detail our answers.

The low-frequency decline in fundamental volatility observed from 1960 to 1990 can be accounted for almost entirely by the demise of a handful of heavy manufacturing sectors: Construction, Primary Metals, Fabricated Metal Products, Machinery (excluding Computers), and Motor Vehicles.<sup>16</sup> While only moderately large in a value-added sense in 1960 – accounting for 18% of total value added in 1960 –, these sectors are both relatively more intensive intermediate input users and relatively more volatile, thus accounting for a disproportionately large fraction of aggregate fundamental volatility in 1960 (30% of  $\sigma_F^2$ ).<sup>17</sup> In this sense, the relatively high aggregate volatility in the early 1960s was the result of an undiversified technological portfolio, loading heavily on a few heavy manufacturing industries. Their demise, starting around the early 1970s and accelerating around 1980, meant that by 1990 they accounted for only 10% of aggregate volatility.

Another way to see this is to compute a counterfactual fundamental volatility measure where we fix the Domar weights of these sectors to their sample average, while using the actual, time-varying Domar weights for the other sectors. This enables us to ask what would have happened to fundamental volatility had these sectors not declined during the period of analysis. We find (see Figure 3) that in this counterfactual economy the level of fundamental volatility would have barely changed from the early 1960s to the early 1990s. At the same time, it is also clear that the dynamics of these sectors do not account either for the spike in fundamental volatility around 1980 nor do they play a role in its continued rise from the mid 1990s onwards.

Instead, we find that the spectacular rise and precipitous decline in fundamental volatility from the early 1970s to the mid 1980s are largely accounted for by the dynamics of two energy-related sectors: Oil and Gas Extraction, and Petroleum and Coal Products: the  $H$  (1971, 1980) of these sectors is 0.64 and 0.30, respectively. By 1981, these two sectors accounted for 41% of fundamental volatility, a fourfold increase from the average over the remainder of the sample. The decline of these two sectors also accounts for the bulk of the fall in fundamental volatility during the 1980-1986 period ( $H$  (1980, 1986) = 0.55 and 0.24).

To analyze the rise in fundamental volatility since the mid 1990s, we build on Philippon’s (2008) analysis of the evolution of the GDP share of the financial sector, but revisit it

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<sup>16</sup>Between 1960 and 1989, their  $H$  is 0.58. The main drivers are Construction ( $H = 0.36$ ) and Primary Metals ( $H = 0.12$ ).

<sup>17</sup>The share of  $\sigma_F^2$  due to sector  $i$  is defined as  $\left(\frac{S_{it}}{Y_t}\right)^2 \sigma_i^2 / \sigma_F^2$ . Those shares add up to 1.

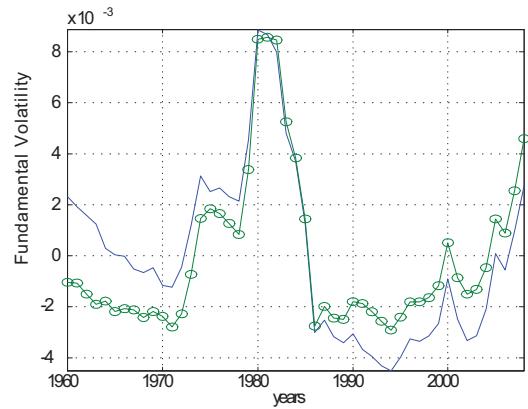
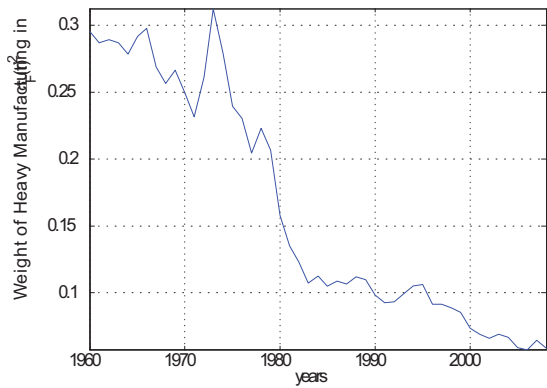


Figure 3: **Left:** Weight of heavy manufacturing sectors in  $\sigma_{F_t}^2$ . **Right:** The continuous line is the baseline fundamental volatility measure ( $4.5\sigma_{F_t}$  demeaned). The circled line gives a counterfactual volatility measure (also demeaned) where weights of heavy manufacturing sectors are fixed at their sample average.

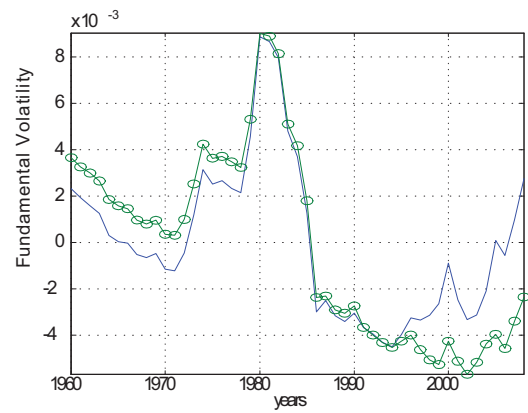
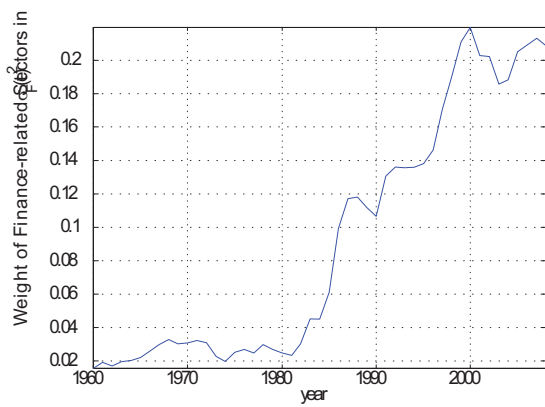


Figure 4: **Left:** Weight of finance-related sectors in  $\sigma_{F_t}^2$ . **Right:** The continuous line is the baseline fundamental volatility measure ( $4.5\sigma_{F_t}$  demeaned). The circled line gives a counterfactual volatility measure (also demeaned) where weights of finance-related sectors are fixed at their sample average.

through the metric of fundamental volatility. We find that the combined contribution of three finance-related sectors – Depository Institutions, Non-Depository Financial Institutions (including Brokerage Services and Investment Banks), and Insurance – to fundamental volatility increased tenfold from the early 1980s to the 2000s, with the latest of these sharp movements occurring in the mid 1990s and coinciding with the rise of our fundamental volatility measure ( $H(1990, 2007)$  is 0.44 for Non-Depository Financial Institutions and 0.19 for Depository Institutions). From the late 1990s onward, these three sectors have accounted for roughly 20% of fundamental volatility.

In a counterfactual economy where the weights of these sectors are held fixed, fundamental volatility would have prolonged its trend decline until the early 2000s (see Figure 4). While the renewed exposure to energy-related sectors from the mid-2000s onwards would have reversed this trend somewhat, the implied level of fundamental volatility at the end of the sample would have been lower, in line with that observed in the early 1990s (and not that of the early 1960s and 1970s as our baseline measure implies). The rise of finance is thus key to explaining the undoing of the great moderation from the mid 1990s onwards: as the US economy loaded more and more on these sectors, fundamental volatility rose, reflecting a return to a relatively undiversified portfolio of sectoral technologies.

We next turn to our four other major economies.

## 4.2 United Kingdom

The UK time series starts off with a short-term spike in fundamental volatility between 1972 and 1976 which is almost single-handedly explained by the high-frequency developments for Petroleum and Coal Products, yielding  $H(1972, 1976) = 0.59$ . Thereafter, we see a large decline from 1978 until the mid/late 1980s, again mostly explained by Petroleum and Coal Products ( $H(1978, 1988) = 0.61$ ). As can be seen in Figure 2, fundamental volatility stabilizes in 1985 and then gradually increases until 2005. A large portion of that surge in fundamental volatility can be attributed to Construction, whose share  $H(1985, 2005)$  is 0.41.

The rise of the finance sector also contributes to the modest but continuous rise in fundamental volatility since 2000: the combined  $H(2000, 2005)$  for Financial Intermediation and Insurance equals 0.46.

### 4.3 Japan

The most salient characteristic of the Japanese time series is the steep decline in fundamental volatility from 1970 to 1987. The decline of the steel industry and construction explains the development in fundamental volatility very well:  $H(1970, 1987) = 0.57$  and 0.33 for Basic Metals and Construction, respectively. Despite the decline from 1970 to 1987, fundamental volatility reaches very high levels in the 1970s, especially compared to Germany and France. The prominent role of the steel industry also explains this pattern of short-term spikes with  $H(1973, 1974) = 0.64$  and  $H(1978, 1980) = 0.23$  for Basic Metals, and  $H(1978, 1980) = 0.23$  for Construction. Finally, the sharp increase in fundamental volatility from 1987 to 1990 can be attributed to Construction, whose  $H(1987, 1990)$  is 1.02.

### 4.4 Germany

The major trend in the German time series of fundamental volatility is the latter's downturn from 1970 to 1987 which is very well explained by the drop in GDP shares for Basic Metals (mostly steel) and Construction: the respective values for  $H(1970, 1987)$  are 0.73 and 0.57. Furthermore, the downturn of fundamental volatility during the 1980s can in part be explained by Petroleum and Coal Products, with  $H(1981, 1988) = 0.19$ .

### 4.5 France

The French time series can be split as follows: a decline in fundamental volatility from 1970 to 1987/8, followed by a ten-year sequence of hardly any movement, and a steep increase from 1998 to 2005. Construction heavily contributes to this drop ( $H(1970, 1987) = 0.50$ ), and is accompanied by Petroleum and Coal Products in the 1980s ( $H(1981, 1988) = 0.23$ ). Lastly, the steep increase in fundamental volatility since 1998 is due to "other business activities," which contains the bulk of business services. This category comprises heterogenous activities, ranging from operative services such as security activities to services requiring highly qualified human capital.

## 5 Idiosyncratic Shocks and Comovement

### 5.1 Motivation and Summary

We next discuss how our results are consistent with comovement in the economy. We need some notations. Calling  $Y_i$  the value added of sector  $i$ , GDP growth is  $\widehat{Y} = \sum_i \frac{Y_i}{Y} \widehat{Y}_i$ , and its variance can be decomposed as:

$$\sigma_{Y_t}^2 = D_t + N_t \tag{13}$$

$$D_t = \sum_{i=1}^n \left( \frac{Y_{it}}{Y_t} \right)^2 \text{var} \left( \widehat{Y}_{it} \right), \quad N_t = 2 \sum_{1 \leq i < j \leq n} \frac{Y_{it}}{Y_t} \frac{Y_{jt}}{Y_t} \text{cov} \left( \widehat{Y}_{it}, \widehat{Y}_{jt} \right)$$

The term  $D_t$  represents the diagonal terms in GDP growth, while the term  $N_t$  represents the non-diagonal terms, i.e., in an accounting sense, the terms that come from common shocks or from linkages in the economy.

In this section, we address why previous research was pessimistic about the importance of microeconomic shocks (Blanchard and Simon 2001, McConnell and Perez-Quiros 2000, Stock and Watson 2002), and answer the following questions. (i) Previous research showed that comovement (the  $N_t$  term) accounts for the bulk of GDP volatility, so why focus on the diagonal terms? (ii) Previous literature showed that the off-diagonal  $N_t$  terms fell in volatility, doesn't that mean that the common shock they reflect is the main story (see, for instance, Stiroh 2009)? (iii) Don't we also detect comovement of TFP across sectors, which must mean that there is an extra common factor to take into account?

Before laying out the model that helps think about those questions, we present a summary of our answers. (i) We present a model with comovement in which all the primitive shocks are idiosyncratic (at the sector level). However, because of linkages, there is comovement. Indeed, to a good approximation, in this model

$$D_t \simeq c_D \sigma_{F_t}^2, \quad N_t \simeq c_N \sigma_{F_t}^2 \tag{14}$$

for two coefficients  $c_D$  and  $c_N$ . In our calibration, like in data, about 90% of the variance of output is indeed due to comovement ( $N_t/\sigma_Y^2 \simeq 0.9$ ). That comovement itself comes entirely from the primitive diagonal shocks whose variance is measured by  $\sigma_{F_t}^2$ . Hence, the off-diagonal terms are just the shadow of the primitive shocks, which are the diagonal terms captured by  $\sigma_{F_t}^2$ .

(ii) In terms of time-series evolution, in our hypothesis the prime mover is the change in  $\sigma_{F_t}^2$ , which comes from sectoral-level shares. By linkages, the off-diagonal terms  $N_t$  will reflect

those changes. The fact that the magnitude of off-diagonal terms changes simply reflects the fact that the primitive shocks captured by  $\sigma_{F_t}^2$  change.<sup>18</sup>

(iii) In our model, all primitive shocks are idiosyncratic. Hence, TFP movements across sectors are uncorrelated. However, even small measurement errors will create a comovement in measured TFP. Suppose that, when times are good (i.e., when the average idiosyncratic shock is positive), people work more, but this partly comes as higher effort, so that measured employment underestimates the true increase in labor supply. Each firm will look more productive than it really is. There will be a measured common productivity increase, but it is only due to mismeasurement. Quantitatively, our model shows that a small measurement error can account for the observed comovement in measured TFP.

We next proceed to the model that allows us to reach the above conclusions.

## 5.2 Model Setup

To think quantitatively about comovement, we specify the model of Section 2. We use a CES production function for one final good.<sup>19</sup> Thus, we can solve the structure of this economy in closed form, and quantify its comovement. Our model differs in a series of small ways from Long and Plosser (1983), Horvath (1998, 2000), Shea (2002), Foerster, Sarte, and Watson (2008), and Carvalho (2009).<sup>20</sup> Its main virtue is that it is solvable in closed form, so that the mechanisms are fairly transparent.

There is an aggregate good and  $n$  intermediary goods. Unit  $i$  uses  $L_i, K_i, X_i$  of labor, capital, and the aggregate good to produce  $Q_i$  goods  $i$ :

$$Q_i = \kappa A_i (L_i^\alpha K_i^{1-\alpha})^b X_i^{1-b} \quad (15)$$

with  $\kappa = 1 / (b^b (1-b)^{1-b})$ , and  $b$  is the share of intermediate inputs and will also be the ratio

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<sup>18</sup>However, in the model of Section 5, the fall in fundamental volatility will make all sectors' volatilities fall, and will indeed make a simulated shock  $\widehat{Y}_t^* = \sum_{i=1}^N (Y_i/Y)^* \cdot \widehat{Y}_{it}$  fall in variance, keeping constant output shares  $(Y_i/Y)^*$  but using the observed shocks  $\widehat{Y}_{it}$ . (A more analytical note on this point is available upon request.)

<sup>19</sup>In the CES world that we parameterize (with positive elasticity of substitution), a positive TFP shock increases a sector's size. This is not necessarily a good thing. When a sector is very new (say electronic gadgets), the size of that sector grows as the sector becomes more productive (that is, as more products are invented). However, perhaps in a long run sense, some sectors shrink as they become very productive (e.g., agriculture). Following the macro tradition, we eschew here a calibrated modeling of this heterogeneity in the link between productivity and size.

<sup>20</sup>Long and Plosser (1983) impose a Cobb-Douglas structure, with zero idiosyncratic movement in the sales per employee and dollar sales.

of value added to sales, both at the level of the unit and of the economy. GDP is production net of the intermediate inputs, the  $X_i$ 's:

$$Y = \left( \sum_i Q_i^{1/\psi} \right)^\psi - \sum_i X_i \quad (16)$$

with  $\psi > 1$ . The transformation from the goods  $Q_i$  to the final good  $\left( \sum_i Q_i^{1/\psi} \right)^\psi$  and the intermediary inputs is made by a competitive fringe of firms.

The representative agent's utility function is  $U = Y - L^{1+1/\varphi}$ . Capital can be rented at a rate  $r$ . Thus, the social planner's problem is:  $\max_{\{K_i, L_i, X_i\}} Y - L^{1+1/\varphi} - rK$  subject to  $\sum K_i = K; \sum L_i = L$ . We assume that the prices equal marginal cost. This could be caused by competition or by an input subsidy equal to  $\psi$  for the intermediary firms.

The model gives:

$$\text{GDP : } Y = \Lambda L^\alpha K^{1-\alpha} \quad (17)$$

$$\text{TFP : } \Lambda = \left( \sum_i A_i^{1/(\psi-1)} \right)^{(\psi-1)/b} \quad (18)$$

$$\frac{\text{Sales}_i}{\text{GDP}} : \frac{p_i Q_i}{Y} = \frac{1}{b} \left( \frac{A_i}{\Lambda^b} \right)^{1/(\psi-1)} \quad (19)$$

### 5.3 Comovement in Output

To study comovement, we assume that we start from a steady state equilibrium, and study the one-shot response of our economy to shocks.

Models such as (15) always deliver a Sales/Employees ratio that is independent of the unit's productivity. The reason for this almost surely counterfactual prediction is that labor is assumed to be costlessly adjustable. To capture the realistic case of labor adjustment costs, we assume that a fraction  $1 - \nu$  of labor is a quasi-fixed factor, in the sense of Oi (1962). Technically, we represent  $L_i = L_{V,i}^\nu L_{F,i}^{1-\nu}$ , where  $L_{V,i}$  and  $L_{F,i}$  are respectively the variable part of labor and the quasi-fixed part of labor. After a small shock, only  $L_{V,i}$  adjusts. The disutility of labor remains  $L^{1+1/\varphi}$ , where  $L = L_V^\nu L_F^{1-\nu}$  is aggregate labor. We assume that capital and intermediary inputs are flexible. The Online Appendix relaxes that assumption.

One can now study the effect of a productivity shock  $\hat{A}_i$  to each unit  $i$ . We call  $S_i = p_i Q_i$  the dollar sales of unit  $i$ . The next proposition, whose proof is in the Online Appendix (which contains a generalization to the case where all factors have finite elasticity), describes how the economy reacts to microeconomic shocks.

**Proposition 2** (*Aggregate factor emerging from microeconomic shocks*) Suppose that each unit  $i$  receives a productivity shock  $\hat{A}_i$ . Macroeconomic variables change according to

$$\text{GDP and TFP: } \hat{Y} = \frac{1 + \varphi}{\alpha} \hat{\Lambda}, \quad \hat{\Lambda} = \sum \frac{S_i}{Y} \hat{A}_i = \sum \frac{\text{Sales}_i}{\text{GDP}} \hat{A}_i \quad (20)$$

$$\text{Employment and Wage: } \hat{L} = \frac{\varphi}{1 + \varphi} \hat{Y}, \quad \hat{w} = \frac{1}{1 + \varphi} \hat{Y} \quad (21)$$

and microeconomic-level variables change according to

$$\text{Dollar sales and dollar value added: } \hat{S}_i = \hat{Y}_i = \beta \hat{A}_i + \bar{\Phi} \hat{Y} \quad (22)$$

$$\text{Production: } \hat{Q}_i = \psi \beta \hat{A}_i + (1 - \psi \Phi) \hat{Y} \quad (23)$$

$$\text{Price: } \hat{p}_i = -(\psi - 1) \beta \hat{A}_i + (\psi - 1) \beta \Phi \hat{Y} \quad (24)$$

$$\text{Employment: } \hat{L}_i = \nu \beta \hat{A}_i + \left( \frac{\varphi}{1 + \varphi} - \nu \Phi \right) \hat{Y} \quad (25)$$

$$\text{Use of intermediary inputs and capital: } \hat{X}_i = \hat{K}_i = \hat{S}_i \quad (26)$$

$$\beta = \frac{1}{\psi - 1 + b\alpha(1 - \nu)}, \quad \Phi = \frac{\beta b \alpha}{1 + \varphi}, \quad \bar{\Phi} = 1 - \Phi \quad (27)$$

In other terms, this economy exhibits a common factor  $\hat{Y}$  (equations 22-26) which is itself nothing but a sum of idiosyncratic shocks (equation 20).

The new results are the sectoral-level changes, in equations (22)-(26). The economy behaves like a one-factor model with an “aggregate shock,” the GDP factor  $\hat{Y}$ . Again, this factor stems from a multitude of idiosyncratic shocks (equation 20).<sup>21</sup> It causes all microeconomic-level quantities to comove. Economically, when sector  $i$  has a positive shock, it makes the aggregate economy more productive and affects the other sectors in three different ways. First, other sectors can use more intermediary inputs produced by sector  $i$ , thereby increasing their production. Second, sector  $i$  demands more inputs from the other firms (equation 22), which leads their production to increase. Third, given that sector  $i$  commands a large share of output, it will use more of the inputs of the economy, which tends to reduce the other sectors’ output. The net effect depends on the magnitudes of the elasticities.

We calibrate the model using conventional parameters to the extent possible, with parameters summarized in Table 4. Using the decomposition (13), the ratio  $f^{GDP} \equiv N/\sigma_Y^2$  captures how much of GDP variance is due to comovement.

<sup>21</sup>In this economy, firms are affected by this GDP factor, but this GDP factor is entirely made of firm-level shocks. That makes identification very difficult. It’s an instance of the “reflection problem” identified by Manski (1993). It has no general solution, but some of the partial techniques generated by Manski’s impulse might be useful to identify causality in the type of macroeconomic equilibrium described by Proposition 2.



Table 4: Model Calibration

| Calibrated Values  |                    |
|--|--------------------|
| Labor share  | $\alpha = 2/3$     |
| One minus share of intermediate inputs                     | $b = 1/2$          |
| Elasticity of labor supply                                 | $\varphi = 2$      |
| Product differentiation parameter                          | $\psi = 1.2$       |
| Share of labor that is variable in the short run           | $\nu = 1/2$        |
| Fraction of mismeasured temporary labor utilization        | $\theta = 0.43$    |
| Resulting Values   |                    |
| Micro-level productivity multiplier                        | $\beta = 2.7$      |
| Aggregate productivity multiplier                          | $\mu = 4.5$        |
| Elasticity $\Phi$  | $\Phi = 0.3$       |
| Fraction of GDP variance attributed to comovement          | $f^{GDP} = 0.90$   |
| Fraction of employment variance attributed to comovement   | $f^{Labor} = 0.95$ |
| Fraction of measured TFP variance attributed to comovement | $f^{TFP,m} = 0.59$ |

*Notes:* The first part of the table shows the postulated values. The second part shows the resulting values for a few quantities. Because of linkages, the fraction of variances attributed to comovement is non-zero although all primitive shocks are assumed to be idiosyncratic.

**Proposition 3** (*Magnitude of the comovement in output and employment*) Call  $f^{GDP}$  (resp.  $f^{Labor}$ ) the fraction of GDP (resp. employment) variance attributed to comovement in a variance-accounting sense. An exact value is given in equation (36) of the Appendix. If most shocks are idiosyncratic at the micro level ( $\beta^2 \sigma_A^2 \gg \bar{\Phi}^2 \sigma_Y^2$ ), we have

$$f^{GDP} = 1 - \left(\frac{b\beta}{\mu}\right)^2, \quad f^{Labor} = 1 - \left(\nu \left(1 + \frac{1}{\phi}\right) \frac{b\beta}{\mu}\right)^2 \quad (28)$$

and we have (14) with  $c_D \simeq b^2 \beta^2$  and  $c_N \simeq \mu^2 - b^2 \beta^2$ . However, economically, all the shocks are primitively idiosyncratic.

Of course, as  $\mu$  (defined in equation 6) increases, so does the fraction attributed to comovement. Using our calibration, we find  $f^{GDP} = 90\%$  and  $f^{Labor} = 95\%$ . This is to say that, even though primitive shocks are purely idiosyncratic in our model, linkages create such a large comovement that, in a volatility-accounting sense, 93% are mechanically attributed to comovement. This measure is congruent with the empirical findings of Shea (2002), who

finds estimates of  $f^{GDP}$  in the range 80%-85% and  $f^{Labor} = 95\%$ . It might also explain why Foerster, Sarte, and Watson (2008) find that a large part of fluctuations come from the non-diagonal part,  $N_t$ .

## 5.4 Comovement in Measured TFP

The data show a positive correlation (average pairwise correlation is 2.3%) in measured TFP innovations across sectors. The previous model generates positive comovement from independent TFP shocks. Hence, if there is perfect measurement of TFP, it will generate no comovement of TFP. Against this background, we interpret the data in the following way. We say that a fraction  $\theta$  of the change in the effective number of hours is not measured. For instance, a secretary will work harder when there is much work to do, and less intensely when there is less work. Still, the total number of hours that are counted is the same, say 40 hours per week. In that case,  $\theta = 1$ . If she does some overtime, so that some of her extra efforts appear in the labor supply statistics, then  $\theta < 1$ . For simplicity, we assume that only labor is mismeasured (the same argument would go through if more factors were mismeasured). The measured number of hours is:

$$\widehat{L}_i^m = (1 - \theta) \widehat{L}_i,$$

where the superscript  $m$  denotes the *measured* quantities. Measured TFP growth in sector  $i$  is:

$$\begin{aligned} \widehat{A}_i^m &= \widehat{Q}_i - b\alpha \widehat{L}_i^m - b(1 - \alpha) \widehat{K}_i - (1 - b) \widehat{X}_i \\ &= \left[ \widehat{Q}_i - b\alpha \widehat{L}_i - b(1 - \alpha) \widehat{K}_i - (1 - b) \widehat{X} \right] + \theta b\alpha \left( \widehat{L}_i - \widehat{L}_i^m \right) \end{aligned}$$

Hence:

$$\widehat{A}_i^m = \widehat{A}_i + \theta b\alpha \widehat{L}_i. \quad (29)$$

In other terms, the measured TFP is the true TFP plus the increase in effective labor  $\widehat{L}_i$  times labor share in output-cum-intermediary-inputs  $b\alpha$  times the mismeasurement factor  $\theta$ .

In this benchmark economy, the comovement in true productivity growth  $\widehat{A}_i$  is 0. However, there will be some comovement in measured productivity growth, as all sectors tend to increase factor utilization (in a partially unmeasured way) during booms. The following proposition quantifies this.

**Proposition 4** (*Magnitude of the comovement in measured TFP*) *Measured TFP follows the factor structure:*

$$\widehat{A}_i^m = c_A \widehat{A}_i + c_Y \widehat{Y}, \quad (30)$$

where  $c_A \equiv 1 + \theta b \alpha \nu \beta$  and  $c_Y \equiv \theta b \alpha \left( \frac{\varphi}{1+\varphi} - \nu \Phi \right)$ . Hence, if there is mismeasurement, measured TFP covaries. Call  $f^{TFP,m}$  the fraction of measured TFP variance attributed to comovement in a variance-accounting sense. If most shocks are idiosyncratic at the micro level, we have

$$f^{TFP,m} = 1 - \left( 1 + \frac{c_Y \mu}{c_A b} \right)^{-2}. \quad (31)$$

Note that, if there is no mismeasurement ( $\theta = 0$ ),  $c_Y = 0$  and  $f^{TFP,m} = 0$ : there is no comovement in TFP.

Empirically, we measure  $f^{TFP,m} = 0.59$  in US data. Solving for  $\theta$  in equation (31), this corresponds to  $\theta = 43\%$  of the variable labor input being undermeasured. It says that, if from trough to boom measured hours go up by 5.7%, effort goes up 4.3%.<sup>22</sup> The corresponding value of  $\sigma_F^m / \sigma_F$  is simply  $c_A = 1.19$ . So, mismeasurement of inputs affects a lot the apparent comovement between sectors (as it is the cause of comovement, and the productivity multiplier is large), but only relatively little the measurement of sectoral-level productivity (idiosyncratic factors generally dominate aggregate factors at the microeconomic level). In our model, all primitive shocks come from idiosyncratic microeconomic shocks, but there is comovement in output because of production linkages. In other terms, there is positive comovement in measured TFP because statistical agencies do not control well for unmeasured increases in labor inputs, i.e., “effort” or “utilization.”

We wish to conclude that our model simply illustrates important quantitative features of an economy with comovement. We suspect that the highlighted features will survive with other sources of comovement, e.g., a financial accelerator or expectations.

## 6 Robustness Checks and Extensions

### 6.1 Variants in the Empirical Constructs

#### 6.1.1 Sectors vs. Firms

In this paper, we primarily use sectors, because we have measures of gross output and value added for sectors in several countries. The firm-level data are spottier, yet encouraging as we shall see. We start with firms in the US, using the Compustat data set. We define the firm-based fundamental volatility as we did for sectors, cf. (1). To implement this formula, Compustat has several limitations which are rather important when studying long-run trends.

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<sup>22</sup> Adding the possibility of mismeasurement in capital utilization would decrease  $\theta$ .

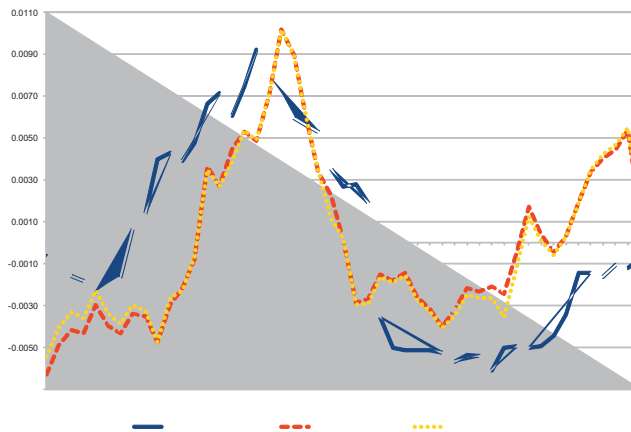


Figure 5: Firm-based Fundamental Volatility and GDP Volatility. The solid line gives the GDP volatility,  $\sigma_{Y_t}^{\text{Roll}}$  (demeaned). The dotted and dashed lines give the fundamental volatility based on firms (rather than sectors),  $4.5\sigma_F^{\text{Firms}}$  (demeaned). The dotted line is based on the largest 100 firms by sales in each year. The dashed line is based on all firms in Compustat.

It is not quite consistent over time, as it covers more and more firms. Additionally, the data are on worldwide sales, rather than domestic output. There are some data on domestic sales, which are unfortunately too spotty to be used.

For firm-level volatility, a firm-by-firm estimation gives quite volatile numbers, so we proceed as in the international section of this paper, using constant values of  $\sigma_i$  across firms. We use  $\sigma_i = 12\%$ , the typical volatility of sales/employees, sales and employee growth found in Gabaix (2011).

Figure 5 plots the detrended firm-level fundamental volatility for the top 100 firms and for all firms in the data set. As the top 100 firms are very large, most of the variation in  $\sigma_F^{\text{Firms}}$  is driven by them. We see that they track GDP volatility quite well. We also see that firm-based fundamental volatility and GDP volatility have a similar evolution.

To be more quantitative, we proceed as in Table 1 and regress  $\sigma_{Y_t} = a + b\sigma_{F_t}^{\text{Firms}}$ . We find a coefficient  $b = 4.8$  (s.e. 0.8) for  $\sigma_{Y_t}^{\text{Roll}}$  and yearly data, with an  $R^2$  equal to 0.44. We also replicate the break test of Table 2. Controlling for  $\sigma_{F_t}^{\text{Firms}}$ , there is no break in GDP volatility at the conventional confidence level. We conclude that the firm-based fundamental volatility does account for the great moderation and its undoing, just like the sector-based fundamental volatility we use in the rest of the paper.

The chief difficulty is for non-US data. For European countries the main data set is Amadeus. Unfortunately, it starts only in 1996, which is too short a time period to detect

the long-run trends that are the key object of this paper. We can only hope that a researcher will compile a historical database of the actions of the top firms in the major economies. We conjecture that going to more disaggregated data would enrich the economic understanding of microeconomic developments (e.g., the big productivity growth of the retail sector was due to Wal-Mart, rather than a mysterious shock affecting a whole sector), but data availability prevents us from pursuing that idea in this paper.

### 6.1.2 Correlations in TFP Innovations

We proceed as if TFP innovations are uncorrelated. This is a good benchmark, as the average correlation in TFP innovations in different sectors is only 2.3% in the US. Even that small correlation could be due to measurement error and factor hoarding, as seen in Section 5.4. We also considered the variant  $\sigma_{Ft}^{\text{Full}} = \sqrt{\sum_{i,j=1..N} \frac{S_{it}}{Y_t} \frac{S_{jt}}{Y_t} \sigma_i \sigma_j \rho_{ij}}$ . The results are quite similar, as detailed in the Online Appendix. We prefer our  $\sigma_{Ft}$  with only diagonal terms, as it is more parsimonious conceptually and empirically.

### 6.1.3 Time-Varying Sectoral Volatility

Our  $\sigma_{Ft}$  uses a constant sectoral volatility – largely for the sake of parsimony. We examine that benchmark here. First, we find that micro TFP volatility is not significantly different pre and post 1984: its average is 3.49% for 1960-1983 and 3.16% for 1984-2005, and the difference is not statistically significant. This warrants the benchmark of constant micro volatility.

Another way to explore whether our results depend on time-varying sectoral volatility is to construct  $\sigma'_{Ft} = \sqrt{(S_i/Y)^2 \sigma_{it}^2}$ , i.e.,  $\sigma_{Ft}$  with time-varying sectoral volatility while keeping sectoral shares constant at their time-series average, and  $\sigma''_{Ft} = \sqrt{(S_{it}/Y_t)^2 \sigma_{it}^2}$ , which has time-varying shares and volatility. We estimate  $\sigma_{it}$  by running a GARCH(1,1) for each sector  $i$ . We re-run the regression (9) with  $\sigma_Y^{\text{Inst}}$  and  $\sigma_Y^{\text{Roll}}$  on the right-hand side:  $\sigma'_{Ft}$  has insignificant explanatory power and a very low  $R^2$  (about 0.05). On the other hand,  $\sigma''_{Ft}$  has significant explanatory power and a good  $R^2$  (about 0.38). We conclude that the crucial explanatory factor is indeed the time-varying shares in the economy, not a potential change in sectoral-level volatility.

### 6.1.4 Gross vs. Net output

In this paper, we use the concept of gross output, rather than net output (i.e., value added). In that we follow the common best practice of the productivity literature (e.g., Basu, Fernald

and Kimball 2006). Part of the reason is data availability, part is conceptual: most models use inputs such as labor, capital, and other goods (the intermediary inputs), and for productivity there is no good reason to subtract the intermediary inputs.<sup>23</sup> Nonetheless, we did examine our results with a value-added productivity notion. We found them to be quite similar.

## 6.2 Time-Varying Elasticity of Labor Supply

Jaimovich and Siu (JS, 2009) find that changes in the composition of the labor force account for part of the movement in GDP volatility across the G7 countries. As the young have a more elastic labor supply, the aggregate labor supply elasticity should be increasing in the fraction of the labor force who is between 15 and 29 years old – which is the  $JS_t$  variable.

Table 5: Fundamental Volatility and Jaimovic-Siu Variable

|                    | (1)                   | (2)                    | (3)                   | (4)                   |
|--------------------|-----------------------|------------------------|-----------------------|-----------------------|
| $\widehat{\beta}$  | 1.580<br>(3.38;0.468) | 2.179<br>(3.75;0.580)  | 1.508<br>(3.16;0.477) | 2.123<br>(3.38;0.627) |
| $\widehat{\gamma}$ | 0.041<br>(3.76;0.011) | 0.0569<br>(4.99;0.011) | 0.044<br>(3.64;0.012) | 0.058<br>(4.96;0.012) |
| $\widehat{\delta}$ | -                     | -                      | 2.788<br>(0.91;3.05)  | 1.168<br>(0.71;3.164) |
| $\chi t$           | No                    | Yes                    | No                    | Yes                   |
| Observations       | 134                   | 134                    | 134                   | 134                   |

*Notes:* We run the regression  $\sigma_{Yct} = \alpha_c + \beta \bar{\sigma}_{Fct} + \gamma JS_{ct} + \delta (\bar{\sigma}_{Fct} - \bar{\sigma}_F) \times (JS_{ct} - \overline{JS}) + \chi t + \varepsilon_{ct}$ , where  $\sigma_{Yct}$  is the country volatility using a rolling window measure,  $\bar{\sigma}_{Fct}$  is the fundamental volatility of the country defined in (10),  $\bar{\sigma}_F$  is the mean of the  $\bar{\sigma}_{Fct}$ 's,  $JS_{ct}$  is the Jaimovich-Siu (2009) measure of the labor force share of the “volatile age group,”  $\overline{JS}$  is the mean of the  $JS_{ct}$ 's,  $\alpha_c$  a country fixed effect and  $\chi t$  a linear time trend.  $t$ -statistics and Newey-West standard errors (2 lags) in parentheses.

In terms of our model in (5)-(6), this corresponds to having a time-varying Frisch elasticity

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<sup>23</sup>To think about the problem, take a gross output production function  $F(L, X)$ : the inputs are labor and an intermediary input  $X$ , the net output is  $V(L, X) = F(L, X)$ . A Hicks-neutral increase in gross output productivity by a ratio of  $A$  means that the production function becomes  $AF(L, X)$ , so that the net output changes by  $AF(L, X) - X$ . In contrast, a neutral increase in productivity of net output would change the production function to  $AF(L, X) - AX$ . The interpretation of the latter is rather odd: with  $A = 1.1$ , the firm can produce 10% more gross output, but has also become 10% less efficient at handling inputs. This is one reason why it is conceptually easier to think about productivity in the gross output function.

of labor supply  $\varphi_t$ , i.e.,

$$\sigma_{Yt} = \frac{1 + \varphi_t}{\alpha} \sigma_{Ft}. \quad (32)$$

The JS composition of the workforce effect is in the term  $\varphi(t) = A + B \cdot JS_t$ , while the effect we focus on in this paper is the  $\sigma_{Ft}$  term. Put differently, the JS variable is about the amplification of primitive shocks, while our variable is about the primitive shocks themselves. To investigate (32), we run the panel regression:

$$\sigma_{Yct} = \alpha_c + \beta \bar{\sigma}_{Fct} + \gamma JS_{ct} + \delta (\bar{\sigma}_{Fct} - \bar{\sigma}_F) \times (JS_{ct} - \overline{JS}) + \chi t + \varepsilon_{ct} \quad (33)$$

That is, we return to the cross-country exercise above, this time including the JS measure of the labor force share of the “volatile age group” in each of our economies,  $JS_{ct}$ . The inclusion of this measure shortens our sample somewhat as it extends only to 1999 (and only begins in 1979 for the case of the UK). We also include an interaction term; to lessen orthogonality, we interact the demeaned versions of the variables,  $(\bar{\sigma}_{Fct} - \bar{\sigma}_F) \times (JS_{ct} - \overline{JS})$ . Finally, we run this regression with and without a linear time trend  $\chi t$ . Table 5 reports the results.

Both coefficients  $\beta$  and  $\gamma$  are positive and significant in all specifications. The interaction term has the expected sign but is insignificant, perhaps due to the lack of power to detect second-order terms. We conclude that the JS labor supply elasticity and the fundamental volatility are both relevant to explain the cross-country evolution of business cycle volatility.

### 6.3 Time-Varying Tail Risk in the Aggregate Economy

Recent events have forced economists and policy makers to reassess the mid-2000s belief of an ever more stable economy, and to update the probability of large fluctuations, or tail events, in aggregate GDP growth. For example, Blanchard, Dell’Ariccia, and Mauro (2010) state that “the great moderation led too many (including policy makers and regulators) to understate macroeconomic risk, ignore, in particular, tail risk and take positions [...] which turned out to be much riskier after the fact.” In this section, we ask whether, from the vantage point of our fundamental volatility construct, this understatement of tail risk was warranted.

We use our equation  $\widehat{Y} = \mu \sum_{i=1}^N \frac{S_{it}}{Y_t} \cdot \widehat{A}_{it}$ , with  $\mu = 4.5$ . For each time period, we feed the corresponding Domar weight vector and perform 10,000 draws of independent sectoral TFP shocks, normally distributed with mean zero and where the variance of each sector’s TFP growth is fixed at its full-sample empirical estimate. We then compute the probability of having that year’s GDP growth below  $-1.64\sigma_{GDP}$ , where  $\sigma_{GDP}$  is the model-implied average volatility of aggregate GDP for the full-sample period. The value 1.64 is chosen so that, if the volatility is constant, the tail probability is 5% ( $P(X \leq -1.64) = 0.05$  if  $X$  is a standard

normal). We then repeat this for all time periods and obtain “tail risk probabilities” from 1960 to 2008.

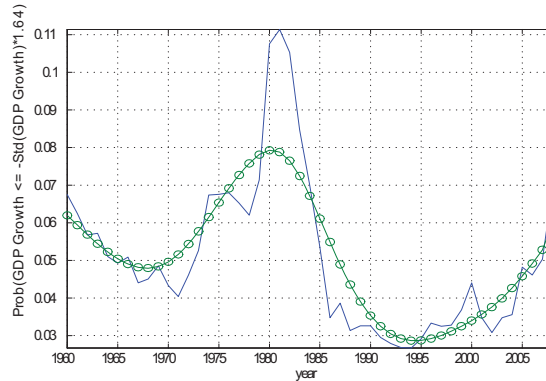


Figure 6: Tail Risk Probability. This figure plots for each year the probability of negative GDP growth in excess of 1.64 standard deviations, taking as given the level of fundamental volatility.

Figure 6 reports the result. The overall sample mean of tail risk is 0.051, very close to a prior of 0.05. Not surprisingly, tail risk follows the dynamics of our fundamental volatility measure.

Tail risk was reduced fourfold during the great moderation, from the high watermark of 0.11 in 1980 (implying a large fluctuation every 9 years) to a low of 0.028 in 1994 (a large fluctuation every 36 years). Tail risk has been on the rise since the mid 1990s, reaching 0.07 in 2008 (a large fluctuation every 14 years), roughly the same probability observed in the late 1970s. We conclude that, while the great moderation did imply a marked decrease of tail risk in GDP growth, larger (negative and positive) events were to be expected by the mid 2000s.

Note that this tail-risk measure does not inform us of the fragility of aggregate growth to *tail events in particular sectors* of the economy. To explore this alternative notion of fragility, or *conditional* tail risk, we consider what would have happened if a given sector had suffered a two-standard-deviation shock to its TFP growth rate.<sup>24</sup> Given the recent focus on risks posed by the “shadow banking system,” we focus on a particular finance sector in our sample: non-depository financial institutions (including security and commodity brokerage services

<sup>24</sup>To be precise, we implement the following procedure. Define  $\Sigma$  to be a diagonal matrix with non-zero entries given by each sector’s (full-sample) variance of TFP growth. Let  $B = \Sigma^{1/2}$ , and define  $G_t = BX_t$ , i.e.,  $G_{it} = \sum_j B_{ij}X_{jt}$ . Let  $j^*$  be the index of the particular sector we are interested in exploring. Then, the procedure is: simulate  $X_{jt}$  i.i.d. with zero mean and unit standard deviation for  $j \neq j^*$ , and set  $X_{j^*t} = -2$  (for a shock equal to two standard deviations). Then, compute  $G_{it} = BX_t$ , and finally TFP. We repeat this for every  $t$  from 1960 to 2008, feeding in the actual Domar weights.



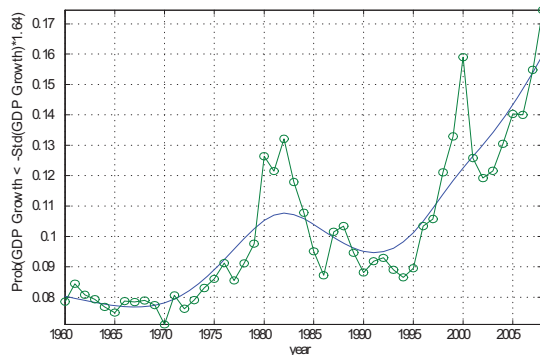


Figure 7: Tail Risk Probability and Fragility to the Non-Depository Financial Sector. This figure plots for each year the probability of negative GDP growth in excess of 1.64 standard deviations, given two-standard-deviation shocks to the financial sector.

and investment banks). Figure 7 shows a dramatic rise in the exposure of the aggregate economy to tail events in this sector. By 2008, the probability of a large negative event in GDP conditional on a negative tail event in this sector reaches a peak of 17.3% (or one every six years). While, by definition, the conditioning event is a rare one, we note that the fragility of the aggregate economy to any such event in this sector had been unprecedentedly high since the late 1990s: the 1998-2008 average is 14%.

## 7 Conclusion

We have investigated the explanatory power of “fundamental volatility” to understand the swings in macroeconomic volatility, and found it to be quite good. Fundamental volatility explains the great moderation and its undoing. It has a clear economic foundation and has the advantage of being easy to measure.

Our findings do support the view that the key to macroeconomic volatility might be found in microeconomic shocks. This is a meaningful, nontrivial empirical result: many other factors (e.g., policy, taxes, globalization) change the structure of the economy, so it is not clear a priori that the microeconomic composition of the economy would have such a large explanatory power.

Of course, microeconomic shocks need to be enriched by some propagation mechanisms. Their identification might be simplified if we think that microeconomic shocks are the primary factors that are propagated. For instance, we do not deny that monetary shocks may be important. However, they may largely be part of the response to other shocks (e.g., real

shocks caused by oil or finance).

Our findings pose the welfare consequences of the microeconomic composition of an economy. In models with financial frictions, a rise in volatility is typically welfare-reducing. Perhaps finance was too big and created too much volatility in the 2000s? Perhaps the oil-dependent industries were too big and created too much volatility in the 1970s?

In addition, fundamental volatility can serve as an “early warning system” to measure future volatility. In retrospect, the surge in the size of finance in the 2000s could have been used to detect a great source of new macroeconomic volatility.

In any case, we think that fundamental microeconomic volatility is a useful theoretical and empirical concept to consider when thinking about the causes and consequences of aggregate fluctuations.

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## A Data Appendix

*US data.* The main data source for this paper was constructed by Dale Jorgenson and associates,<sup>25</sup> and provides a detailed breakdown of the entire US economy into 88 sectors. The data is annual and covers the period between 1960 and 2005. The original sources are input-output tables and industry gross output data compiled by the Bureau of Labor Statistics and the

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<sup>25</sup>The data set is available for download at Dale Jorgenson’s Dataverse website, <http://dvn.iq.harvard.edu/dvn/dv/jorgenson>.

Bureau of Economic Analysis. The data are organized according to the KLEM methodology reviewed in Jorgenson, Gallop, and Fraumeni (1987) and Jorgenson, Ho, and Stiroh (2005). In particular, the input data incorporate adjustments for quality and composition of capital and labor. To the best of our knowledge, this is the most detailed (balanced) panel coverage of US sectors available, offering a unified data set for the study of sectoral productivity.<sup>26</sup>

In this data set, for each year-industry pair, we observe nominal values of sectoral gross output, capital, labor, and material inputs supplied by the 88 sectors (plus non-competing imports) as well as the corresponding price deflators. Following Jorgenson, Ho, and Stiroh (2008) and Basu et al. (2009), we concentrate on private sector output, thus excluding services produced by the government, but including purchases of private-sector goods and services by the government (a robustness check shows that this does not materially affect our result). We also exclude from the analysis the imputed service flow from owner-occupied housing.<sup>27</sup> This yields a panel of 77 sectors which forms the basis of all computations.

Finally, to construct aggregate output volatility measures, we obtained quarterly real GDP data from the Federal Reserve Economic Data (FRED).

*International data.* For UK, Japan, Germany, and France we resort to the EU KLEMS database (see Timmer et al. 2007 for a full account of the data set). As the name indicates, this database is again organized according the KLEM methodology proposed by Jorgenson and associates. To preserve comparability, we focus on private sector accounts by excluding publicly provided goods and services. We thus exclude sectors under the heading "non-market services" in the data set. These also include real estate services as the database does not make a distinction between real estate market services and the service flow from owner-occupied residential buildings (see Timmer et al. 2007, Appendix Table 3 for definitions and discussion).

For each country we obtain a panel of nominal sectoral gross output and value added at the highest level of disaggregation possible. For the UK we end up with 66 sectors, Japan 58, Germany 50, and France 46. For roughly half of these sectors we can compute TFP growth from the gross output perspective. We then compute the average (across sectors) standard deviation of TFP growth during the entire sample period. For UK, Germany, and France the resulting panel runs from 1970-2005. For Japan it starts in 1973 due to the unavailability of earlier data.

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<sup>26</sup>The NBER-CES manufacturing database provides further detail but only includes manufacturing industries. As made clear in the paper, it is crucial to account for the growth of service sectors when looking at cross-sectoral diversification, the great moderation and its undoing.

<sup>27</sup>Finally we drop two sectors for which there are no data in the original data set, "Uranium Ore" and "Renting of Machinery."

To obtain aggregate output volatility for these four countries we extended the quarterly GDP data in Stock and Watson (2005) till 2009:Q4 with data from the OECD Economic Outlook.

## B Proof Appendix

**Proof of Lemma 1:** The lemma makes two claims: the Cobb-Douglas form and the Hulten formula. We start with the Cobb-Douglas form. The Lagrangean is

$$\mathcal{L} = \mathcal{C}(\mathbf{C}) + \sum_i p_i \left( A_i F_i(K_i^{1-\alpha} L_i^\alpha, \mathbf{X}_i) - C_i + \sum_j X_{ji} \right) + r \left( K - \sum_i K_i \right) + w \left( \sum_i L_i - L \right)$$

Defining  $I_i = K_i^{1-\alpha} L_i^\alpha$ , the aggregate factor input in sector  $i$ , and  $I = \sum_i I_i$ , we have:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial K_i} &= \frac{1-\alpha}{K_i} I_i p_i A_i \partial_1 F_i - r = 0, & \frac{\partial \mathcal{L}}{\partial L_i} &= \frac{\alpha}{L_i} I_i p_i A_i \partial_2 F_i - w = 0 \\ \therefore \frac{K_i}{L_i} &= \frac{1-\alpha}{\alpha} \frac{w}{r} = \frac{\sum_j K_j}{\sum_j L_j} = \frac{K}{L} \\ \therefore \frac{K_i}{K} &= \frac{L_i}{L} = \left( \frac{K_i}{K} \right)^{1-\alpha} \left( \frac{L_i}{L} \right)^\alpha = \frac{I_i}{K^{1-\alpha} L^\alpha} \\ \therefore 1 &= \sum_i \frac{K_i}{K} = \sum_i \frac{I_i}{K^{1-\alpha} L^\alpha} = \frac{I}{K^{1-\alpha} L^\alpha} \end{aligned}$$

so that  $I = K^{1-\alpha} L^\alpha$ . The production function is homogenous of degree 1 in  $(I_i, X_{ij})$ : if a plan  $(\mathbf{C}, I_i, X_{ij})$  is doable, so is a plan  $(\lambda \mathbf{C}, \lambda I_i, \lambda X_{ij})$ . Hence, the production function has a form  $Y = I \cdot \Lambda(\mathbf{A}) = K^{1-\alpha} L^\alpha \Lambda(\mathbf{A})$ .

Second, we turn to the Hulten formula. Shocks  $dA_i$  create a change in welfare:

$$d\mathcal{L} = \sum_i p_i F_i(K_i^{1-\alpha} L_i^\alpha, \mathbf{X}_i) dA_i = \sum_i A_i p_i F_i \frac{dA_i}{A_i} = \sum_i S_i \frac{dA_i}{A_i}$$

i.e.,  $\frac{dY}{Y} = \sum_i \frac{S_i}{Y} \frac{dA_i}{A_i}$ , or  $\frac{d\Lambda}{\Lambda} = \sum_i \frac{S_i}{Y} \frac{dA_i}{A_i}$ .

**Proof of Proposition 1** The planner's problem is  $\max_{K,L} \Lambda K^{1-\alpha} L^\alpha - L^{1+1/\varphi} - rK$ . The first order conditions with respect to  $K$  and  $L$  give:  $(1-\alpha) \frac{Y}{K} = r$ ,  $\alpha \frac{Y}{L} = (1+1/\varphi) L^{1/\varphi}$ , so that  $K = (1-\alpha) Y/r$ ,  $L = \left( \frac{\alpha}{1+1/\varphi} Y \right)^{\frac{\varphi}{1+\varphi}}$  and

$$Y = \Lambda K^{1-\alpha} L^\alpha = \Lambda \left( (1-\alpha) \frac{Y}{r} \right)^{1-\alpha} \left( \frac{\alpha}{1+1/\varphi} Y \right)^{\frac{\alpha\varphi}{1+\varphi}} = \left( \frac{1-\alpha}{r} \right)^{1-\alpha} \left( \frac{\alpha}{1+1/\varphi} \right)^{\frac{\alpha\varphi}{1+\varphi}} \Lambda Y^{1-\frac{\alpha}{1+\varphi}}$$

Finally,  $Y = k \Lambda^{\frac{1+\varphi}{\alpha}}$  with  $k = \left[ \left( \frac{1-\alpha}{r} \right)^{1-\alpha} \left( \frac{\alpha}{1+1/\varphi} \right)^{\frac{\alpha\varphi}{1+\varphi}} \right]^{\frac{1+\varphi}{\alpha}}$ .

**Proof of Proposition 3** In this model, value added is proportional to sales,  $Y_i = bS_i$  (this comes from the first order condition with respect to  $X_i$ ). So

$$\begin{aligned} D &\equiv \sum_i \left(\frac{Y_i}{Y}\right)^2 \text{var}(\widehat{Y}_i) = \sum_i b^2 \left(\frac{S_i}{Y}\right)^2 \text{var}(\widehat{S}_i) \\ &= b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 \text{var}(\beta\widehat{A}_i + \bar{\Phi}\widehat{Y}) \end{aligned} \quad (34)$$

and  $f = N/\sigma_Y^2 = 1 - D/\sigma_Y^2$ . Consider first the case where most shocks at the microeconomic level are idiosyncratic, i.e.,  $\beta^2\sigma_A^2 \gg \bar{\Phi}^2\sigma_Y^2$ . Then,

$$f^{GDP} \simeq 1 - \frac{b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 \beta^2 \sigma_A^2}{\sigma_Y^2} = 1 - \frac{b^2 \beta^2 \sigma_F^2}{\mu^2 \sigma_F^2} = 1 - \frac{b^2 \beta^2}{\mu^2}. \quad (35)$$

and  $c_D \simeq b^2 \beta^2$  and  $c_N \simeq \mu^2 - b^2 \beta^2$ .

In the general case, (20) implies  $\text{cov}(\widehat{A}_i, \widehat{Y}) = \frac{S_i}{Y} \mu \sigma_A^2$ , so (34) gives:

$$\begin{aligned} f^{GDP} &= 1 - \frac{b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 \left[ \beta^2 \sigma_A^2 + \bar{\Phi}^2 \mu^2 \sigma_F^2 + 2\beta\bar{\Phi} \left(\frac{S_i}{Y}\right) \mu \sigma_A^2 \right]}{\mu^2 \sigma_F^2} \\ &= 1 - \frac{b^2 \beta^2}{\mu^2} - \bar{\Phi}^2 b^2 \left( \sum_i \left(\frac{S_i}{Y}\right)^2 \right) - \frac{2b^2 \beta \bar{\Phi} \sigma_A^2}{\mu \sigma_F^2} \sum_i \left(\frac{S_i}{Y}\right)^3 \\ \therefore f^{GDP} &= 1 - \frac{b^2 \beta^2}{\mu^2} - \bar{\Phi}^2 b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 - \frac{2b^2 \beta \bar{\Phi} \sum_i \left(\frac{S_i}{Y}\right)^3}{\mu \sum_i \left(\frac{S_i}{Y}\right)^2}. \end{aligned} \quad (36)$$

We verify numerically that the approximation (35) is quite good. Likewise, the comovement in labor follows, using (21), (25), and  $L_i/L = bS_i/Y$

$$\begin{aligned} 1 - f^{Labor} &= \frac{\sum_i \text{var}(\widehat{L}_i) (L_i/L)^2}{\text{var}(\widehat{L})} \simeq \frac{\sum_i (\nu\beta)^2 \sigma_A^2 (bS_i/Y)^2}{\left(\frac{\varphi}{1+\varphi}\right)^2 \sigma_Y^2} \\ &= \left(\frac{\nu\beta(1+\phi)}{\phi}\right)^2 \frac{b^2 \sigma_F^2}{\mu^2 \sigma_F^2} = \left(\frac{\nu(1+\phi)b\beta}{\phi \mu}\right)^2. \end{aligned}$$



## C Additional Derivations

**Proof of Proposition 2** We found it useful to state a general proposition with an arbitrary number of fixed and variable factors. We call  $\mathcal{F}$  the primitive factors (e.g., labor and capital) and  $\mathcal{F} \cup X$  the set of all factors – the primitive factor and the intermediary inputs. Consider the microeconomic production function:

$$Q_i = \kappa A_i \left( \prod_f F_{if}^{\alpha_f} \right)^b X_i^{1-b}$$

with  $\kappa = 1 / (b^b (1-b)^{1-b})$ , where sector  $i$  produces a quantity  $Q_i$  using  $F_{if}$  of factor  $f \in \mathcal{F}$  and  $X_i$  intermediary inputs. This can be rewritten

$$Q_i = \kappa A_i \prod_{f \in \mathcal{F} \cup X} F_{if}^{\gamma_f} \quad (37)$$

where we define  $\gamma_f = b\alpha_f$  for  $f$  a primitive factor, and  $\gamma_f = (1-b)$  for  $f$  the intermediary input. Using this notation, the intermediary inputs in sector  $i$  are  $X_i = F_{iX}$ .

For instance, in the economy studied in Section 5 there are three factors:

$$(F_f)_{f=1\dots 3} = (\text{Labor, Capital, Intermediary inputs}) = (L, K, X)$$

and their weights are:  $(\gamma_f)_{f=1\dots 3} = (\alpha b, (1-\alpha)b, 1-b)$ . We will call it the “3-factor economy.”

Each factor  $F_f$  has a cost  $C_f F_f^{1/\xi_f}$  for a constant  $f$ . In the the 3-factor economy,  $1/\xi_1 = 1 + 1/\varphi$ , i.e.,  $\xi_1 = \varphi / (1 + \varphi)$ . On the other hand, as the cost of the intermediary good  $X$  is linear in  $X$ ,  $\xi_3 = C_3 = 1$ . If capital is elastic in the short run,  $\xi_2 = 1$  and  $C_2 = r$ ; if it is completely inelastic,  $\xi_2 = 0$  in all results below.

GDP is output net of intermediary inputs:  $Y = H - X = H - \sum_i F_{iX}$ , with

$$H = \left( \sum_i Q_i^{1/\psi} \right)^\psi \quad (38)$$

GDP is the solution of the planner’s problem:

$$\max_{F_{if}} H - \sum_f C_f F_f^{1/\xi_f} \text{ s.t. for all } f, \sum_i F_{if} \leq F_f$$

Finally, as in the body of the paper, a fraction  $\nu_f$  of a factor  $f$  is flexible in the short run. We start with a general proposition.

**Proposition 5** (*General case*) *The static equilibrium is described by  $Y = \Lambda \prod_{f \in \mathcal{F}} F_f^{\alpha_f}$  with (18)-(19). Furthermore, suppose that each unit  $i$  receives a productivity shock  $\widehat{A}_i$ . Macroeconomic variables change according to:*

$$TFP: \widehat{\Lambda} = \sum_i \frac{S_i}{Y} \widehat{A}_i = \sum_i \frac{Sales_i}{GDP} \widehat{A}_i \quad (39)$$

$$GDP: \widehat{Y} = \frac{1}{1 - \sum_f \alpha_f \xi_f} \widehat{\Lambda} \quad (40)$$

$$\text{Employment of factor } f: \widehat{F}_f = \xi_f \widehat{Y} \quad (41)$$

$$\text{Wage of factor } f: \widehat{w}_f = (1 - \xi_f) \widehat{Y} \quad (42)$$

*Microeconomic-level variables change according to:*

$$\text{Dollar sales: } \widehat{S}_i = \beta \widehat{A}_i + \Phi \widehat{Y} \quad (43)$$

$$\text{Production: } \widehat{Q}_i = \psi \beta \widehat{A}_i + (1 - \psi \Phi) \widehat{Y} \quad (44)$$

$$\text{Price: } \widehat{p}_i = -(\psi - 1) \beta \widehat{A}_i + (\psi - 1) \beta \Phi \widehat{Y} \quad (45)$$

$$\text{Employment of factor } f: \widehat{F}_{if} = \nu_f \beta \widehat{A}_i + (\xi_f - \nu_f \Phi) \widehat{Y} \quad (46)$$

$$\text{Use of intermediary input: } \widehat{X}_i = \widehat{S}_i \quad (47)$$

where

$$\begin{aligned} \beta &= \frac{1}{\psi - \sum_f \gamma_f \nu_f} = \frac{1}{\psi - (1 - b) - b \sum_f \alpha_f \nu_f} \\ &= \frac{1}{\psi - \sum_{f \in \mathcal{F} \cup X} \text{Share of factor } f \times \text{Flexibility ratio of factor } f} \end{aligned}$$

and

$$\begin{aligned} \Phi &= \beta \left( 1 - \sum_f \gamma_f \xi_f \right) = \beta b \left( 1 - \sum_f \alpha_f \xi_f \right) \\ &= \beta \left( 1 - \sum_{f \in \mathcal{F} \cup X} \text{Share of factor } f \times \text{Adjusted supply elasticity of factor } f \right) \end{aligned}$$

where  $f \in \mathcal{F} \cup X$  denotes the primitive factors (labor, capital) and also the intermediary input  $X$ .

**Proof of Proposition 5** *Step 1. Frictionless equilibrium.* The price of unit  $i$  is  $p_i = \frac{\partial H}{\partial Q_i}$ , hence by (38):  $S_i/H = p_i Q_i/H = Q_i \frac{\partial H}{\partial Q_i}/H$  and

$$\frac{S_i}{H} = \left( \frac{Q_i}{H} \right)^{1/\psi} \quad (48)$$

Because  $H$  is homogenous of degree 1,  $H = \sum \frac{\partial H}{\partial Q_i} Q_i = \sum S_i$ .  $H$  is the sum of sales in the economy.

Unit  $i$  solves:  $\max_{F_{if}} p_i Q_i - \sum_f w_f F_{if}$ , which gives:  $F_{if} = S_i \gamma_f / w_f \propto S_i$ . We use  $\propto$  to mean that the variables are proportional, up to a factor that does not depend on  $i$ . So,  $S_i^\psi \propto Q_i \propto A_i S_i$  by (15), so  $S_i \propto A_i^{1/(\psi-1)}$ . Calling  $B = \sum_i A_i^{1/(\psi-1)}$ , and using the adding up constraint  $\sum F_{if} = F_f$ , we find the constant of proportionality:  $F_{if} = F_f A_i^{1/(\psi-1)} / B$ . Plugging this in (38), we obtain  $H = \kappa B^{\psi-1} \prod_{f \in \mathcal{F} \cup X} F_f^{\gamma_f}$ . Now, we solve for  $X$ :  $\max_X Y = H - X$ , i.e.,  $\max_X \kappa B^{\psi-1} \left( \prod_{f \in \mathcal{F}} F_f^{\alpha_f} \right)^b X^{1-b} - X$ . The solution yields  $X = (1-b)H$ ,  $Y = H - X = B^{(\psi-1)/b} \prod_{f \in \mathcal{F}} F_f^{\alpha_f}$ , i.e.,

$$Y = \Lambda \prod_{f \in \mathcal{F}} F_f^{\alpha_f}, \quad \Lambda = B^{(\psi-1)/b} \quad (49)$$

as announced in the statement of Proposition 5. In the 3-factor economy we obtain (17). Also,  $Y = H - X = bH$ .

*Step 2. Changes, assuming  $\nu_f = 1$ .* To keep the proof streamlined, we first consider the case  $\nu_f = 1$ , i.e., the case with no frictions in the adjustment of labor, and with the possibility that  $\sum_f \gamma_f$  is not 1. TFP growth comes from (18), and is also Hulten's formula.  $Y = bH$  gives  $\widehat{Y} = \widehat{H}$ . The optimal use of factor  $f$  maximizes  $Y - \sum C_f F_f^{1/\xi}$ , for constants  $C_f$ . Hence  $\gamma_f Y / F_f = C_f F_f^{1/\xi_f - 1}$ , and  $F_f = Y^{\xi_f}$  times a constant, and

$$\widehat{F}_f = \xi_f \widehat{Y} \quad (50)$$

Equation (49) implies that:

$$\widehat{Y} = \widehat{\Lambda} + \sum_f \alpha_f \widehat{F}_f = \widehat{\Lambda} + \left( \sum_f \alpha_f \xi_f \right) \widehat{Y}$$

and  $\widehat{Y} = \frac{\widehat{\Lambda}}{1 - \sum_f \alpha_f \xi_f}$ . The wage is  $w_f = \frac{1}{\xi_f} L^{1/\xi_f - 1}$ , so:  $\widehat{w}_f = \left( \frac{1}{\xi_f} - 1 \right) \widehat{F}_f$ , hence

$$\widehat{w}_f = (1 - \xi_f) \widehat{Y}. \quad (51)$$

It is convenient that one can solve for changes in the macroeconomic variables without revisiting the sectors' decision problems.

We now turn to the unit-level changes. Optimization of the demand for labor gives  $w_f F_{if} = \gamma_f S_i$ , so

$$\widehat{F}_{if} = \widehat{S}_i - \widehat{w}_f = \widehat{S}_i - (1 - \xi_f) \widehat{Y}$$

We have, from (15),

$$\begin{aligned} \widehat{Q}_i &= \widehat{A}_i + \sum_f \gamma_f \widehat{F}_{if} = \widehat{A}_i + \sum_f \gamma_f \left( \widehat{S}_i - (1 - \xi_f) \widehat{Y} \right) \\ &= \widehat{A}_i + \left( \sum_f \gamma_f \right) \widehat{S}_i - \sum_f \gamma_f (1 - \xi_f) \widehat{Y} \end{aligned}$$

Eq. (48) gives  $\widehat{Q}_i = \psi \widehat{S}_i + (1 - \psi) \widehat{H}$ , and using  $\widehat{Y} = \widehat{H}$ ,

$$\psi \widehat{S}_i + (1 - \psi) \widehat{Y} = \widehat{Q}_i = \widehat{A}_i + \left( \sum_f \gamma_f \right) \widehat{S}_i - \sum_f \gamma_f (1 - \xi_f) \widehat{Y}$$

which gives

$$\begin{aligned} \widehat{S}_i &= \frac{\widehat{A}_i + \left[ \psi - 1 - \sum_f \gamma_f (1 - \xi_f) \right] \widehat{Y}}{\psi - \sum_f \gamma_f} = \frac{\widehat{A}_i}{\psi - \sum_f \gamma_f} + \left( 1 - \frac{1 - \sum_f \gamma_f \xi_f}{\psi - \sum_f \gamma_f} \right) \widehat{Y} \\ &= \beta \widehat{A}_i + \overline{\Phi} \widehat{Y} \end{aligned}$$

where

$$\beta = \frac{1}{\psi - \sum_f \gamma_f}, \quad \Phi = \beta \left( 1 - \sum_f \gamma_f \xi_f \right)$$

and, as  $\widehat{Q}_i = \psi \widehat{S}_i + (1 - \psi) \widehat{Y}$ , we obtain the announced expressions for  $\widehat{S}_i$  and  $\widehat{Q}_i$ .  $\widehat{F}_{if}$  comes from  $\widehat{F}_{if} = \widehat{S}_i - \widehat{w}_f$ .  $S_i$  was defined as  $S_i = p_i Q_i$ , which gives  $\widehat{p}_i = \widehat{S}_i - \widehat{Q}_i$ .

*Step 3.* With general  $\nu_f \in [0, 1]$ . After the changes  $\widehat{A}_i$ , only  $L_{V,i}$  can adjust. The planner optimizes the variable part of labor supply:  $\max_{L_{V,f}} A \prod_f \left( F_{Vf}^{\nu_f} F_{Ff}^{1-\nu_f} \right)^{\gamma_f} - \left( F_{Vf}^{\nu_f} F_{Ff}^{1-\nu_f} \right)^{1/\xi_f}$ .

Note that this is isomorphic to optimizing the total labor supply, defining  $F_f = F_{Vf}^{\nu_f} F_{Ff}^{1-\nu_f}$ . Hence, we have (50) and (51).

For the unit-level variables, one replaces  $(\alpha_f, \gamma_f, \xi_f)$  by  $(\alpha'_f, \gamma'_f, \xi'_f) = (\alpha_f \nu_f, \gamma_f \nu_f, \xi_f / \nu_f)$ , which delivers

$$\beta = \frac{1}{\psi - \sum_f \gamma_f \nu_f}, \quad \Phi = \beta \left( 1 - \sum_f \gamma_f \xi_f \right).$$

Remember that (22). Then, the expression for employment stemming from the optimization of labor demand becomes:

$$\begin{aligned}\widehat{F}_{V,i} &= \nu_f \left[ \widehat{S}_i - \left( 1 - \frac{\xi_f}{\nu_f} \right) \widehat{Y} \right] = \nu_f \left[ \beta \widehat{A}_i + (1 - \Phi) \widehat{Y} - \left( 1 - \frac{\xi_f}{\nu_f} \right) \widehat{Y} \right] \\ &= \nu_f \left[ \beta \widehat{A}_i + \left( \frac{\xi_f}{\nu_f} - \Phi \right) \widehat{Y} \right] = \nu_f \beta \widehat{A}_i + (\xi_f - \nu_f \Phi) \widehat{Y}\end{aligned}$$

This concludes the proof of Proposition 5.

**Proof of Proposition 2** We apply the results from Proposition 5. We particularize them to the case of flexible capital ( $\xi_2 = 1, \nu_2 = 1$ ) and flexible intermediary inputs ( $\xi_3 = 1, \nu_3 = 1$ ), whereas labor is less flexible ( $\xi_1 = \xi, \nu_1 = \nu$  can be less than 1). Then, using  $\xi = \varphi / (1 + \varphi)$ ,

$$\begin{aligned}\widehat{Y} &= \frac{1}{1 - (1 - \alpha) - \alpha \xi_1} \widehat{\Lambda} = \frac{1}{\alpha(1 - \xi)} \widehat{\Lambda} = \frac{1 + \varphi}{\alpha} \widehat{\Lambda} \\ \widehat{L} = \xi \widehat{Y} &= \frac{\varphi}{1 + \varphi} \widehat{Y}, \quad \widehat{w} = (1 - \xi) \widehat{Y} = \frac{\widehat{Y}}{1 + \varphi}.\end{aligned}$$

Also,

$$\begin{aligned}\beta^{-1} &= \psi - \sum_f \gamma_f \nu_f = \psi - b\alpha\nu - b(1 - \alpha) - (1 - b) = \psi - 1 + b\alpha(1 - \nu) \\ \Phi &= \beta \left( 1 - \sum_f \gamma_f \xi_f \right) = \beta (1 - b\alpha\xi - b(1 - \alpha) - (1 - b)) = \beta b\alpha(1 - \xi) = \beta b\alpha \frac{1}{1 + \varphi}.\end{aligned}$$

**Proof of Proposition 4** Equation (30) comes from (25) and (29). The measured change in productivity is (using  $\sum_i S_i/Y = 1/b$ )

$$\widehat{\Lambda}^m = \sum_i \frac{S_i}{Y} \widehat{A}_i^m = \sum_i \frac{S_i}{Y} \left( c_A \widehat{A}_i + c_Y \widehat{Y} \right) = c_A \widehat{\Lambda} + \frac{c_Y}{b} \widehat{Y} = \left( c_A + \frac{c_Y}{b} \mu \right) \widehat{\Lambda}$$

so the volatility of measured TFP,  $\sigma_F^{Full,m} = Var \left( \widehat{\Lambda}^m \right)^{1/2}$ , is  $\sigma_F^{Full,m} = \left( c_A + \frac{c_Y}{b} \mu \right) \sigma_F$ . On the other hand, the measured productivity using only the diagonal terms is  $\sigma_F^m = c_A \sigma_F$  by (30). Hence, we obtain

$$\frac{\sigma_F^{Full,m}}{\sigma_F^m} = 1 + \frac{c_Y \mu}{c_A b} \quad (52)$$

Finally, we yield  $f^{TFP,m} = 1 - \left( \frac{\sigma_F^{Full,m}}{\sigma_F^m} \right)^{-2} = 1 - \left( 1 + \frac{c_Y \mu}{c_A b} \right)^{-2}$ .

## D Some Additional Empirical Results

### D.1 Including the Covariance Terms

Here we expand a bit on  $\sigma_{Ft}^{\text{Full}}$  as defined in Section 6.1. Call  $\rho_{ij} = \text{corr}(\widehat{A}_i, \widehat{A}_j)$  the cross-correlation in TFP innovation between sectors  $i$  and  $j$ . We can also define:

$$\sigma_{Ft}^{\text{Full}} = \sqrt{\sum_{i,j=1\dots N} \left(\frac{S_{it}}{Y_t}\right) \left(\frac{S_{jt}}{Y_t}\right) \rho_{ij} \sigma_i \sigma_j} \quad (53)$$

Note that  $\sigma_{Ft}^{\text{Full}}$  should be, essentially by construction, the volatility of TFP. The advantage of this construct, though, will be to do the following thought experiment. Suppose that the shares  $S_{it}/Y_t$  change and the variance-covariance matrix  $(\rho_{ij} \sigma_i \sigma_j)$  does not change, how much should GDP volatility change? Figure 8 shows the “fundamental volatility” graph including the full covariance matrix, i.e., accounting for cross terms.

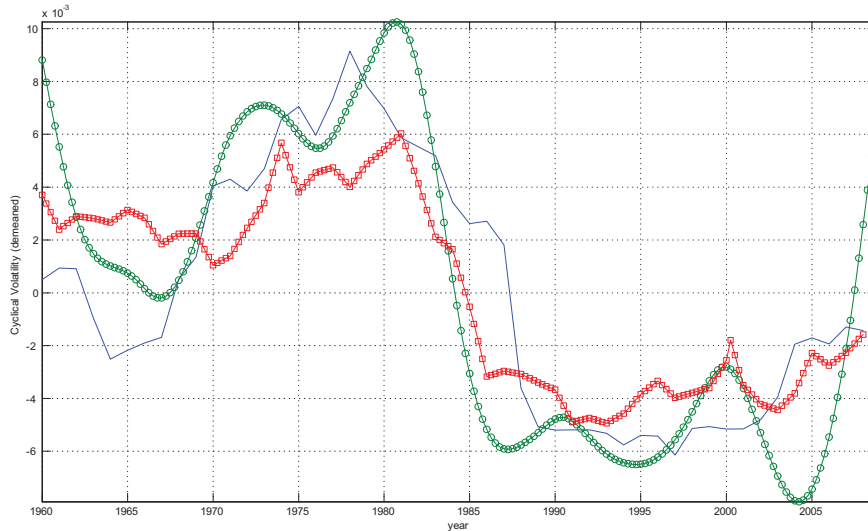


Figure 8: Fundamental Volatility (Full Matrix) and GDP Volatility. The squared line gives the fundamental volatility drawn from the full variance-covariance matrix of TFP ( $4.5\sigma_{Ft}^{\text{Full}}$ , also demeaned). The solid and circle lines are annualized (and demeaned) estimations of GDP volatility. The solid line depicts rolling window estimates of the standard deviation of GDP volatility. The circle line depicts the HP trend of the instantaneous standard deviation.

Table 6 shows that the results of Table 1 hold also with  $\sigma_{Ft}^{\text{Full}}$ , the  $R^2$ 's are actually a bit higher. The advantage of  $\sigma_F$ , with only the diagonal terms, compared to  $\sigma_F^{\text{Full}}$ , which has

Table 6: GDP Volatility and Fundamental Volatility

|           | Annual Data- $\sigma_{Y_t}^{\text{Roll}}$ | Annual Data- $\sigma_{Y_t}^{\text{Inst}}$ | Quarterly Data- $\sigma_{Y_t}^{\text{Roll}}$ | Quarterly Data- $\sigma_{Y_t}^{\text{Inst}}$ |
|-----------|---|---|--|--|
| $\hat{a}$ | -0.041<br>(-7.13;0.006)                   | -0.0459<br>(-4.27;0.013)                  | -0.041<br>(-14.46;0.003)                     | -0.067<br>(-4.88;0.014)                      |
| $\hat{b}$ | 3.981<br>(9.74;0.409)                     | 5.727<br>(4.70;1.085)                     | 4.046<br>(19.66;0.206)                       | 5.919<br>(5.98;0.989)                        |
| $R^2$     | 0.67                                      | 0.38                                      | 0.67   | 0.16   |

*Notes:* Regression of GDP volatility on fundamental volatility. We regress  $\sigma_{Y_t} = a + b\sigma_{F_t}^{\text{Full}} + \eta_t$  on fundamental volatility. In parentheses are  $t$ -statistics and standard errors.

both diagonal and non-diagonal terms, is that  $\sigma_F$  is easier to interpret (volatility coming from independent shocks) and requires less data, so it is easier to apply to non-US countries (where the data are sparser, and we replace all variances by a constant and all covariances by 0).

## D.2 Technological Diversification Patterns

Recall that, in the construction of our fundamental volatility measure, the only time-varying element that we allow for is the sum of squared Domar weights,  $H_t^D = \sum_{i=1}^N (S_{it}/Y_t)^2$ , where  $S_{it}$  is sector  $i$  nominal gross output in year  $t$  and  $Y_t$  gives the total (nominal) value added for the private sector economy in year  $t$ . While Domar weights do not sum to one – as gross output at the sector level exceeds sectoral value added by the amount of intermediate input consumed by that sector – this measure is akin to the more usual Herfindahl indexes of concentration. In particular, looking at the cross-sectional (uncentered) second moment to characterize dispersion/concentration in technology loadings is still valid. The graph below shows the evolution of this measure for the US.

From the peak in 1960 to the trough in 1997, there is a 33% drop in the  $H_t^D$  measure. These dynamics are key to explaining the evolution of our fundamental volatility measure. As such, it is important to perform a number of robustness checks and confirm that the same pattern obtains: i) in more disaggregated data and across different classification systems, ii) for different dispersion/concentration measures, and iii) for value-added shares.

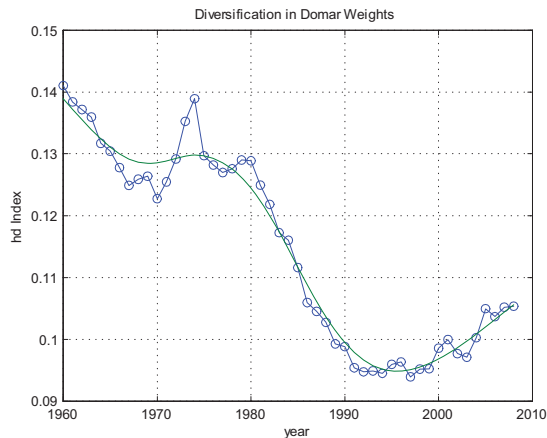


Figure 9: Evolution of  $H_t^D$ , 1960-2008

### D.2.1 More Disaggregated Data

First, we look at the raw BLS data underlying much of the construction of Jorgenson’s data set. These data are both defined at a more disaggregated level and according to different classification systems. Namely, we source two different vintages of BLS “interindustry relationships” data (i.e., Input-Output data). The first is based on SIC classifications (a mixture of two and three digit SIC sectors) running from 1977 to 1995 for a total of 185 sectors. The second is the latest vintage produced by the BLS, based on the newer NAICS classification system, and runs from 1993-2008 (for a total of 200 sectors).

Based on these data, we calculate the implied Herfindahl-like measure. Note that, given the difference in underlying classification systems, the levels are not comparable, neither among themselves nor with the ones reported above. Nevertheless, the dynamics seem to be in broad agreement with those above: a fall in technological concentration in the late 1970s and early 1980s and, if anything, a stronger reversal of this pattern from the late 1990s onwards.

### D.2.2 Checking Against Other Dispersion Measures

Though directly relevant to the story of this paper, Herfindahl indexes of concentration are not the only dispersion measure. However, looking at alternative measures of concentration, such as the Gini Index, the broad story is unchanged: there is a decline in concentration for both measures up until the early 1990s, followed by re-concentration from the mid to late 1990s where the latter now shows up much more strongly.

The patterns above are robust to other measures of dispersion: cross-sectional standard deviation; coefficient of variation; max-min spread and max-median spread.



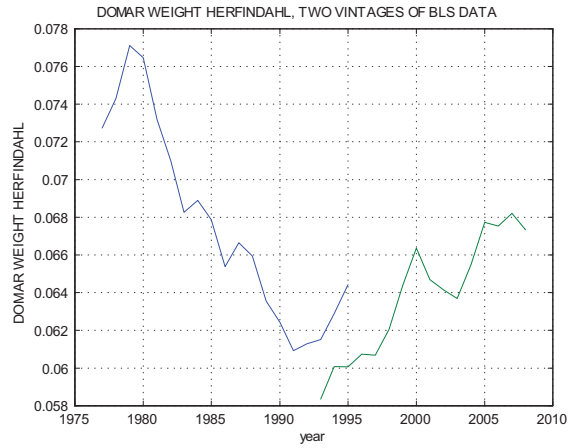


Figure 10: Herfindahl Measure for Domar Weights Computed from Two Vintages of Raw BLS Data (1977-1995 SIC Data; 1993-2008 NAICS Data)

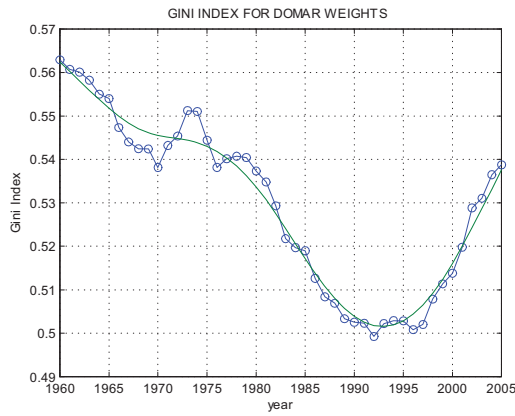


Figure 11: Gini Index for Sectoral Domar Weights

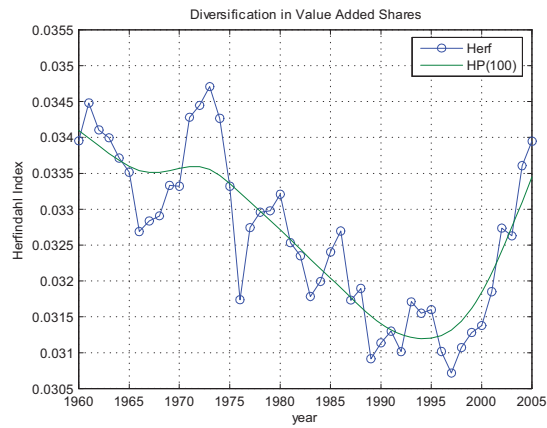


Figure 12: Herfindahl Index of Sectoral Concentration from Value-added Shares Data from 1960-2005.

An alternative is to look into diversification patterns in sectoral value-added shares where the corresponding Herfindahl in year  $t$ ,  $H_t^{VA}$ , is defined as  $H_t^{VA} = \sum_{i=1}^N (Y_{it}/Y_t)^2$ , where  $Y_{it}$  is nominal value added in sector  $i$  in year  $t$ . Figure 12 depicts the evolution of this index and the corresponding HP-filtered series.

Again, looking at value-added shares leads to the same U-shaped pattern. We also see that, quantitatively, the peak to trough movement in the value-added Herfindahl is smaller. This is as it should be: manufacturing technologies are relatively more intermediate input intensive. As such, the gradual move away from manufacturing and into services implies that the Domar weights-based measure of concentration fell relatively more than the value added one: not only has economic activity relied more on what were initially small – in a value added sense – sectors but these latter sectors have relatively lower gross output to value added ratios.<sup>28</sup>

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<sup>28</sup>A way to confirm this is to regress the average growth rate of each sector's value-added share on the average weight of intermediate input purchases in sectoral gross output (average over the entire sample period 1960-2005). We obtain a negative and significant slope coefficient.