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LCA Solvability of Chain Covering Problem

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Abstract

The aim of this research is to study the question of solvability of chain covering problem given on n -vertex N -weighted graphs with multicriteria Vector Objective Function (*VOF*) by means of Linear Convolution Algorithms (*LCA*). In the paper we show that *LCA* do not guarantee finding the solution to the considered class of problems. We proved unsolvability of chain covering problem only for *VOFs* consisting of *MAXMIN* and *MAXSUM* criteria. However, following the same approach it is possible to research solvability of chain covering problem for other types of *VOFs*.

1 Introduction

The aim of this research is to study the question of solvability of chain covering problem given on n -vertex N -weighted graphs with multicriteria Vector Objective Function (VOF) by means of Linear Convolution Algorithms (LCA). In the paper we show that LCA do not guarantee finding the solution to the considered class of problems.

The solvability by means of LCA has been researched for many problems of mathematical programming. To prove that some instance problem is unsolvable by means of LCA it is sufficient to give a particular example of this instance problem that can not be solved by mean of LCA . Such examples have been already provided by [1], [2], [3], [4] for N -criteria statements of spanning trees problem, chains (paths) between a pair of vertexes problem, travelling-salesman problem, perfect matching problem. Unfortunately, this approach does not allow to study the question of solvability of chain covering problem in general. There is an infinite set of instance problems belonging to the class of chain covering problem. To deal with this complication in the paper we suggest a method that allows us to construct for each instance chain covering problem an example that has non-empty set of solutions and still is unsolvable by means of LCA .

In the paper we research solvability of chain covering problem for $VOFs$ consisting of $MAXMIN$ and $MAXSUM$ criteria. However, following the same approach it is possible to study solvability of chain covering problem for other types of $VOFs$.

2 Notations and statement of the problem

A multicriteria problem is said to be defined if for every solution x of Feasible Set of Solutions (FSS) $X = \{x\}$ the value of VOF is given

$$F(x) = (F_1(x), \dots, F_v(x), \dots, F_N(x)).$$

Criteria of VOF are to be optimized (minimized or maximized)

$$F_v(x) \rightarrow \min, \quad v = 1, \dots, N.$$

A feasible solution $\tilde{x} \in X$ is called Pareto Optimum (PO) if there is no element $x^* \in X$ such that $F(x^*) \geq F(\tilde{x})$ and $F(x^*) \neq F(\tilde{x})$. The set \tilde{X} of all PO is called Pareto Set (PS).

Subset $\hat{X} \subseteq \tilde{X}$ is called a Complete Set of Alternatives (CSA) if it's cardinality $|\hat{X}|$ is minimum and equality $F(X^*) = F(\hat{X})$ is satisfied, $F(X^*) = \{F(x) : x \in X^*\}, \forall X^* \in X$.

The Mass Multi-Objective Problem ($MMOP$) is an arbitrary problem where CSA is to be found in explicit form. The term "instance MOP " is used for denoting a particular MOP defined by a pair (X, F) , where $X = \{x\}$ is FSS and $F = F(x)$ is VOF .

For any vector

$$\lambda \in \Lambda_N \left\{ \lambda = (\lambda_1, \dots, \lambda_N) : \sum_{v=1}^N \lambda_v = 1, \lambda_v > 0 \right\}$$

an element x^* minimizing on the *FSS X* linear convolution

$$F^\lambda(x) = \sum_{v=1}^N \lambda_v F_v(x)$$

of *VOF* is *PO*.

This fact is laid in the basis of *LCA* approach used for solving multicriteria problems in mathematical programming. However, for some instance *MOPs PS* can not be found by means of *LCA*. Therefore such instance *MOPs* are called unsolvable.

More precisely *N*-criteria mass *MOP* is called unsolvable by means of *LCA* if there is an instance problem of this *MOP* such that

$$\exists \tilde{x} \in \tilde{X} : F^\lambda(\tilde{x}) > \min \{ F^\lambda(x) : x \in X \}, \forall \lambda \in \Lambda_N,$$

where $F^\lambda(x)$ is the linear convolution of the criteria.

Let us now precisely formulate the statement of the problem studied in the paper. Let $G = (V, E)$ be a graph given by the set of vertex $V = VG$ and the set of edges $E = EG$. For every edge $e \in E$ of the graph G the vector of weights $w(e) = (w_1(e), \dots, w_v(e), \dots, w_N(e))$, is given.

Let also the set H be such that $H = \{h\} \subseteq \{2, \dots, n\}$. A subgraph $x = (V, E_x)$ is called a covering of the graph G by chains of the set H if every connected component of subgraph x is a simple chain that covers h vertices of the graph G (the length of such chain is $h - 1$) and h -chain belongs to H .

In all problems studied in the paper, *FSS X* = $\{x\}$ consists of the coverings of the graph G by the chains of the set H and for each element x of the set X the value of *VOF* $F(x)$ is defined. The solution to the problem is to be given in the form of *CSA*.

Slightly abusing notations we denote instance *MOP* of the *MMOP* described above by $(X(G, H), F)$, where $X(G, H)$ is *FSS* of the problem of covering of the graph G by chains of the set H .

In addition we also found convenient to use the following notations:

G_n to denote an arbitrary n -vertices graph G ;

G_n^c to denote a complete n -vertices graph G , i.e. the graph $G = (V, E)$, with the cardinality of the set of edges is $|E| = \frac{n(n-1)}{2}$;

$x = \bigcup_{j=1}^J h_j$, to denote a covering x of the graph G by chains of the set H , $h_j \in H$;

$x = \emptyset$, to denote a covering x of an empty graph $G = (V, E)$, $V = \{\emptyset\}$, $E = \{\emptyset\}$.

3 General results

Lemma 3.1

If $X(G_n^c, H) = \{\emptyset\}$, then $(X(G_n, H), F)$ is unsolvable by means of LCA.

Proof:

Suppose that $X(G_n^c, H) = \{\emptyset\}$ and that using some LCA algorithm we found CSA \hat{X} for some problem $(X(G_n, H), F)$ such that $\hat{X} = \{\hat{x}\} \neq \{\emptyset\}$. By definition of CSA any element $\hat{x} \in \hat{X}$ can be written as $\hat{x} = \bigcup_{j=1}^m h_j$, $h_j \in H$.

Then, it directly follows from definition of covering that $\hat{x} \in X(G_n^c, H)$. But this implies that $X(G_n^c, H) \neq \{\emptyset\}$, a contradiction. \square

Consider FSS $X(G_n^c, H)$ of an arbitrary problem $(X(G_n^c, H), F)$, $n \geq 4$. If $X(G_n^c, H) \neq \{\emptyset\}$, then $X(G_n^c, H)$ can be written as $X(G_n^c, H) = \{x_i\}$, $i = 1, \dots, I$, where I is the number of the coverings in the set $X(G_n^c, H)$. Each solution $x_i \in X(G_n^c, H)$ is a combination of chains that covers n vertices of the graph G_n^c , i.e. $x_i = \bigcup_{j=1}^{m_i} h_{ij}$, $h_{ij} \in H$, $i = 1, \dots, I$, where m_i is the number of chains

in the solution x_i . Let $x_{\max}(G_n^c, H) = \bigcup_{j=1}^m h_j$, $h_j \in H$ be any of the solutions x_i for which $m = \max(m_i)$, $i = 1, \dots, I$. In subsequent part of the paper we extensively use the properties of the chosen covering x_{\max} .

Definition: Let the elements c_p , $p = 1, \dots, n$ be such that:

- $c_1 = 2 \cup 2$, i.e. c_1 is a pair of 2-chains.
- $c_2 = 2 \cup 3$, i.e. c_2 is 2-chain and 3-chain.
- $c_3 = 3 \cup 3$, i.e. c_3 is a pair of 3-chains.
- $c_p = p$, $\forall p \geq 4$, i.e. c_p is p -chain.

Lemma 3.2

For $\forall n \geq 4$, and $\forall H$ such that $X(G_n^c, H) \neq \{\emptyset\}$ the solution $x_{\max}(G_n^c, H)$ and the set H satisfy at least one of the following conditions:

- 1). $2 \in H$ and $c_1 \in x_{\max}(G_n^c, H)$;
- 2). $2, 3 \in H$ and $c_2 \in x_{\max}(G_n^c, H)$;
- 3). $3 \in H$ and $c_3 \in x_{\max}(G_n^c, H)$;
- 4). $p \in H$, $c_p = p \in x_{\max}(G_n^c, H)$, $p \geq 4$ and $X(G_p, H \setminus p) = \{\emptyset\}$.

Proof:

Suppose that $c_p = p \notin x_{\max}$, $p \geq 4$. Then the covering x_{\max} consists only of 2-chains and 3-chains. If also $2 \cup 2 \notin x_{\max}$, $2 \cup 3 \notin x_{\max}$, and $3 \cup 3 \notin x_{\max}$ then there exist only two nonempty sets x_{\max} , namely, $x_{\max} = 2$, and $x_{\max} = 3$. For $\forall n \geq 4$ neither of these sets satisfies the definition of the solution to the problem $(X(G_n^c, H), F)$ and thus $X(G_n^c, H) = \{\emptyset\}$. However, this result contradicts to the assumption stated in **Lemma 3.2** that $X(G_n^c, H) \neq \{\emptyset\}$. Consequently,

$x_{\max}(G_n^c, H)$ includes at least one of the combinations of chains 1)-4) listed in **Lemma 3.2**.

Note, that from the definition of covering and the definition of FSS X immediately follows that h -chain belongs to the covering $x_{\max}(G_n^c, H) \in X(G_n^c, H)$ only if the h -chain belongs to the set H . Consequently, the restrictions imposed on the set H in **Lemma 3.2** are precise.

Let us prove that if $c_p = p \in x_{\max}(G_n^c, H)$, then the set H is necessarily such that $X(G_p, H \setminus p) = \{\emptyset\}$. Suppose not, i.e. that $X(G_p, H \setminus p) \neq \{\emptyset\}$. Without a loss of generality let $x_{\max}(G_n^c, H)$ be such that $x_{\max} = p \cup \bigcup_{j=2}^m h_j$, $h_j \in H$. Take

any element $x_v = \bigcup_{k=1}^w h_k$, $h_k \in H$ from FSS $X(G_p, H \setminus p) = \{x_v\}$, $v = 1, \dots, V$.

If $w = 1$, then from the definition of covering directly follows that $h_k = p$ which contradicts to the assumption that the chain p was excluded from the set H . Consequently, we conclude that $w > 1$. Consider the element

$$\tilde{x} = x_v \cup \bigcup_{j=2}^m h_j = \bigcup_{k=1}^w h_k \cup \bigcup_{j=2}^m h_j.$$

Note, that from the fact that \tilde{x} is the combination of chains that covers n -vertices and both $h_k, h_j \in H$ follows that $\tilde{x} \in X(G_n^c, H)$. The number of chains in \tilde{x} is $\bar{m} = w + m - 1 > m$. However, the last result contradicts to the condition of choice of the solution x_{\max} , that was chosen such that $m = \max(m_i)$, $i = 1, \dots, I$. \square

Definition: Define the set $\aleph \equiv$

$\{X(G^1, \{2\}), X(G^1, \{2, 4\}), X(G^2, \{2, 3\}), X(G^2, \{2, 3, 5\}), X(G^3, \{3\}), X(G^p, \{p\})\}$ where G^1, G^2, G^3, G^p , ($p \geq 4$) are the graphs shown on figure 1.

Definition: Define the set \aleph be such that $Y \in \aleph$ iff $Y = \{z_k \cup y\}$, where $\{z_k\} = Y \in \aleph$ and $y = \bigcup_{i=1}^I h_i$, $h_i \in \{2, 3, \dots\}$.

Lemma 3.3

For $\forall H, n \geq 4$ s.t. $X(G_n^c, H) \neq \{\emptyset\} \exists (X(\widetilde{G}_n, H), F), X(\widetilde{G}_n, H) \in \aleph \cup \aleph$.

Proof:

Consider FSS $X(G_n^c, H)$ of the problem $(X(G_n^c, H), F)$, $n \geq 4$. If the set $X(G_n^c, H) \neq \{\emptyset\}$, then according to **Lemma 3.2** we can write the solution $x_{\max}(G_n^c, H)$ as $x_{\max} = \bigcup_{j=1}^m h_j = c_p \cup y$, where the element y is such that if

$x_{\max}(G_n^c, H) = c_p$, then $y = \emptyset$ and if $x_{\max}(G_n^c, H) \neq c_p$, then $y = \bigcup_{i=1}^I h_i$, $h_i \in$

H . On the combination of chains c_p that belongs to the solution $x_{\max}(G_n^c, H)$ construct the graph G^p , $p = 1, \dots, n$ as it is shown on figure 1.

Case 1). If $y = \emptyset$ let the graph $\widetilde{G}_n \equiv G^p$.

Consider the problem $(X(\widetilde{G}_n, H), F)$, where graph $\widetilde{G}_n = G^1$. Note, that if the set H is such that

a). $2 \in H, 4 \notin H$, then $X(\widetilde{G}_n, H) = X(G^1, \{2\})$;

b). $2 \in H, 4 \in H$, then $X(\widetilde{G}_n, H) = X(G^1, \{2, 4\})$;

c). $2 \notin H, 4 \in H$, then $X(\widetilde{G}_n, H) = X(G^1, \{4\})$;

d). $2 \notin H, 4 \notin H$, then $X(\widetilde{G}_n, H) = \{\emptyset\}$.

From the way we constructed the graph \widetilde{G}_n , $x_{\max}(G_n^c, H) \in X(\widetilde{G}_n, H)$. Thus $X(\widetilde{G}_n, H) \neq \{\emptyset\}$, so the set H does not satisfy requirement d).

Note, that we constructed the graph $\widetilde{G}_n = G^1$ on the combination of chains c_1 and thus from **Lemma 3.1** follows that if $c_1 \in x_{\max}(G_n^c, H)$, then the set H necessarily includes 2-chain, so it does not satisfy requirement c) either.

Consequently, for the constructed problem $(X(\widetilde{G}_n, H), F)$ FSS X is necessarily such that either $X(\widetilde{G}_n, H) = X(G^1, \{2\})$ or $X(\widetilde{G}_n, H) = X(G^1, \{2, 4\})$. Thus, $X(\widetilde{G}_n, H) \in \aleph$.

In the same way we can prove that for the problem $(X(\widetilde{G}_n, H), F)$, where $\widetilde{G}_n = G^2$ FSS $X(\widetilde{G}_n, H)$ is such that either $X(\widetilde{G}_n, H) = X(G^2, \{2, 3, 5\})$ or $X(\widetilde{G}_n, H) = X(G^2, \{2, 3\})$ and for the problem $(X(\widetilde{G}_n, H), F)$, where $\widetilde{G}_n = G^3$ FSS $X(\widetilde{G}_n, H)$ is such that $X(\widetilde{G}_n, H) = X(G^3, \{3\})$. Consequently, for these problems $X(\widetilde{G}_n, H) \in \aleph$.

Consider the problem $(X(\widetilde{G}_n, H), F)$, where $\widetilde{G}_n = G^p$, $p \geq 4$. It follows from **Lemma 3.2** that if $c_p = p \in x_{\max}(G_n^c, H)$, then the set H necessarily includes a p -chain and $X(\widetilde{G}_n, H \setminus p) = \{\emptyset\}$. Consequently, $X(\widetilde{G}_n, H) = X(G^p, \{p\})$ and thus $X(\widetilde{G}_n, H) \in \aleph$.

Case 2). If $y \neq \emptyset$, then define the graph G as $G \equiv y = \bigcup_{j=1}^I h_j$, $h_j \in H$, and

let the graph \widetilde{G}_n be such that $\widetilde{G}_n \equiv G^p \cup G$.

In the **case 1).** we had already shown that for each of the constructed graphs G^p $X(G^p, H) \in \aleph$. Consider the problem $(X(\widetilde{G}_n, H), F) = (X(G^p \cup G, H), F)$, $G^p = (VG^p, EG^p)$, $G = (VG, EG)$. From the fact that $VG^p \cap VG = \{\emptyset\}$ and $VG^p \cap VG = \{\emptyset\}$ and from the definition of covering follows that only the elements $\{z_k \cup y_l\}$ belong to FSS $X(\widetilde{G}_n, H)$, where $\{z_k\} = X(G^p, H) \in \aleph$ and $\{y_l\} = X(G, H)$. Note, that from the way we constructed the graph G follows that $y \in X(G, H)$, where $y \equiv G = \bigcup_{j=1}^I h_j$, $h_j \in H$ and thus FSS $X(G, H) \neq \{\emptyset\}$. Let us prove that the element y is the unique solution to the

problem $(X(G, H), F)$. Write the graph G as $G = \bigcup_{i=1}^I G_{h_i}$, where $G_{h_i} \equiv h_i$.

Assume that $\exists \tilde{y}$, such that $\tilde{y} \neq y$ and $\tilde{y} \in X(G, H)$. Then, the element \tilde{y} can be written as $\tilde{y} = \bigcup_{i=1}^I \bigcup_{v=1}^{w_i} h_{iv}$, $h_{iv} \in H$, i.e. each graph G_{h_i} is covered by the combination of chains $\bigcup_{v=1}^{w_i} h_{iv}$, $h_{iv} \in H$. From the assumption $\tilde{y} \neq y$ follows that

$\exists i: \bigcup_{v=1}^{w_i} h_{iv} \neq G_{h_i}$, and $\bigcup_{v=1}^{w_i} h_{iv} \neq h_i$. Consequently, $X(G_{h_i}, H \setminus h_i) \neq \{\emptyset\}$ that contradicts to the result obtained in **Lemma 3.2**. The contradiction allows us to conclude that the FSS $X(G, H) = y = \bigcup_{j=1}^I h_j$, $h_i \in H$. Thus $X(\widetilde{G}_n, H) \in \mathfrak{R}$.

According to the fact that $\mathfrak{N} \cap \mathfrak{R} = \{\emptyset\}$ **Lemma 3.3** is proved. \square

Theorem 3.1

If VOF $F(x)$ is such that N -criteria problem (X, F) is unsolvable by the mean of LCA for any $X \in \mathfrak{N} \cup \mathfrak{R}$, then for given VOF the N -criteria problem of covering $(X(G_n, H), F)$ is also unsolvable by mean of LCA for $\forall n$.

Proof:

The proof of **Theorem 3.1** directly follows from the results obtained in **Lemmas 3.1-3.3**. From **Lemmas 3.1** follows that for every n and H for which FSS $X(G_n^c, H) = \{\emptyset\}$, FSS and thus CSA of any problem $(X(G_n, H), F)$ is also empty $X(G_n, H) = \{\emptyset\}$. Consequently, by definition this problem is unsolvable by means of LCA.

From **Lemmas 3.2, 3.3** follows that for $\forall n, H$ for which $X(G_n^c, H) \neq \{\emptyset\}$ there exist the problem $(X(\widetilde{G}_n, H), F)$ such that $X(\widetilde{G}_n, H) \in \mathfrak{N} \cup \mathfrak{R}$. According to the assumption of **Theorem 3.1** the VOF $F(x)$ is such that for any $X \in \mathfrak{N} \cup \mathfrak{R}$ N -criteria problem (X, F) is unsolvable by mean of LCA. Consequently, **Theorem 3.1** is proved. \square

Thus, we can summarize the main result of this chapter in the following way:

To prove that for given VOF N -criteria chain covering problem is unsolvable by mean of LCA it is sufficient to show that the problem (X, F) is unsolvable for each $X \in \mathfrak{N} \cup \mathfrak{R}$.

In the next chapter this result is used for proving unsolvability of chain covering problem for VOFs consisting of *MAXMIN* and *MAXSUM* criteria.

4 *MAXMIN* and *MAXSUM* VOFs

In the following part of the paper we study solvability by means of LCA only for N -criteria chain covering problem $(X(G, H), F)$, $N \geq 2$ with VOF $F(x)$ consisting at least of one *MAXMIN* and at least of one *MAXSUM* criterion:

$$F_v(x) = \min_{e \in E_x} w_v(x) \rightarrow \max,$$

$$F_v(x) = \min \sum_{e \in E_x} w_v(x) \rightarrow \max.$$

By 2-criteria problem we call a chain covering problem (X, F) with VOF consisting of one *MAXMIN* and of one *MAXSUM* criteria.

Lemma 4.1

For 2-criteria problems (X, F) , $X \in \mathfrak{N} \cup \mathfrak{R}$ it is possible to define the weights w_1 and w_2 of the edges of the relevant graph such that the CSA \widehat{X} of this problem consists of the elements $\{x_1, x_2, x_3\}$ and $F(x_1) = (4, 1)$, $F(x_2) = (1, 4)$, $F(x_3) = (2, 2)$.

Proof:

The proof of **Lemma 4.1** is quite straightforward i.e. we show that for each problem (X, F) where $X \in \aleph \cup \aleph$ we can define the weights of the edges of the relevant graph in such a way that we obtain the values of the criteria stated in **Lemma 4.1**.

Note, that for every subsequently studied problem we define the weights of the relevant graph such that $CSA \hat{X}$ of the problem consists of the first 3 elements $\{x_1, x_2, x_3\}$.

Part 1. Let us study the case when $X \in \aleph$.

1.1). The problem $(X(G^1, \{2\}), F)$.

Define the weights of edges of the graph G^1 :

$$w_1(e_1) = w_1(e_3) = 4; w_2(e_1) = w_2(e_3) = \frac{1}{2};$$

$$w_1(e_2) = w_1(e_4) = 1; w_2(e_2) = w_2(e_4) = 2;$$

$$w_1(e_5) = w_1(e_6) = 2; w_2(e_5) = w_2(e_6) = 1.$$

$FSS X$ of this problem consists of the solutions $\{x_1, x_2, x_3\}$, on which VOF takes the following values:

$$F(x_1 = \{e_1, e_3\}) = (4, 1)$$

$$F(x_2 = \{e_2, e_4\}) = (1, 4)$$

$$F(x_3 = \{e_5, e_6\}) = (2, 2)$$

1.2). The problem $(X(G^1, \{2, 4\}), F)$.

$$w_1(e_1) = w_1(e_2) = w_1(e_3) = 4; w_2(e_1) = w_2(e_2) = w_2(e_3) = \frac{1}{3};$$

$$w_1(e_4) = 2; w_2(e_4) = \frac{4}{3};$$

$$w_1(e_5) = w_1(e_6) = 1; w_2(e_5) = \frac{7}{3}; w_2(e_6) = \frac{1}{3}.$$

FSS of the given problem consists of the solutions $\{x_1, \dots, x_{15}\}$, on which VOF takes the following values:

$$F(x_1 = \{e_1, e_2, e_3\}) = (4, 1)$$

$$F(x_2 = \{e_1, e_2, e_4\}) = (1, 4)$$

$$F(x_3 = \{e_2, e_3, e_4\}) = (2, 2)$$

$$F(x_4 = \{e_1, e_2, e_4\}) = (2, 2)$$

$$F(x_5 = \{e_1, e_5, e_6\}) = (1, 3)$$

$$F(x_6 = \{e_1, e_5, e_3\}) = (1, 3)$$

$$F(x_7 = \{e_1, e_6, e_3\}) = (1, 1)$$

$$F(x_8 = \{e_1, e_4, e_3\}) = (2, 2)$$

$$F(x_9 = \{e_2, e_5, e_6\}) = (1, 3)$$

$$F(x_{10} = \{e_2, e_6, e_4\}) = (1, 2)$$

$$F(x_{11} = \{e_3, e_5, e_6\}) = (1, 3)$$

$$F(x_{12} = \{e_4, e_5, e_6\}) = (1, 4)$$

$$F(x_{13} = \{e_2, e_4\}) = (2, \frac{4}{3})$$

$$F(x_{14} = \{e_1, e_3\}) = (4, \frac{2}{3})$$

$$F(x_{15} = \{e_5, e_6\}) = (1, \frac{8}{3})$$

1.3). The problem $(X(G^2, \{2, 3\}), F)$.

$$w_1(e_1) = w_1(e_2) = w_1(e_4) = 4; w_2(e_1) = w_2(e_2) = w_2(e_3) = \frac{1}{3};$$

$$w_1(e_4) = 2; w_2(e_4) = \frac{4}{3};$$

$$w_1(e_5) = w_1(e_6) = 1; w_2(e_5) = \frac{7}{3}; w_2(e_6) = \frac{1}{3}.$$

FSS of the given problem consists of the solutions $\{x_1, \dots, x_7\}$, on which *VOF* takes the following values:

$$\begin{aligned} F(x_1 = \{e_4, e_1, e_2\}) &= (4, 1) \\ F(x_2 = \{e_5, e_3, e_2\}) &= (1, 4) \\ F(x_3 = \{e_1, e_4, e_3\}) &= (2, 2) \\ F(x_4 = \{e_4, e_2, e_6\}) &= (1, 1) \\ F(x_5 = \{e_4, e_6, e_1\}) &= (1, 1) \\ F(x_6 = \{e_2, e_4, e_5\}) &= (1, 3) \\ F(x_7 = \{e_3, e_5, e_1\}) &= (1, 4) \end{aligned}$$

1.4). The problem $(X(G^2, \{2, 3, 5\}), F)$.

$$\begin{aligned} w_1(e_1) &= 2; w_2(e_1) = \frac{5}{4}; \\ w_1(e_2) &= w_1(e_3) = w_1(e_4) = w_1(e_5) = 4; \\ w_2(e_2) &= w_2(e_3) = w_2(e_4) = w_2(e_5) = \frac{1}{4}; \\ w_1(e_6) &= 1; w_2(e_6) = \frac{9}{4}; \end{aligned}$$

FSS of the given problem consists of the solutions $\{x_1, \dots, x_{14}\}$, on which *VOF* takes the following values:

$$\begin{aligned} F(x_1 = \{e_2, e_3, e_4, e_5\}) &= (4, 1) \\ F(x_2 = \{e_1, e_6, e_3, e_4\}) &= (1, 4) \\ F(x_3 = \{e_1, e_2, e_3, e_4\}) &= (2, 2) \\ F(x_4 = \{e_3, e_4, e_5, e_1\}) &= (2, 2) \\ F(x_5 = \{e_4, e_5, e_1, e_2\}) &= (2, 2) \\ F(x_6 = \{e_5, e_1, e_2, e_3\}) &= (2, 2) \\ F(x_7 = \{e_2, e_6, e_5, e_4\}) &= (1, 3) \\ F(x_8 = \{e_1, e_2, e_4\}) &= (2, \frac{7}{4}) \\ F(x_9 = \{e_2, e_3, e_5\}) &= (4, \frac{3}{4}) \\ F(x_{10} = \{e_3, e_4, e_1\}) &= (2, \frac{7}{4}) \\ F(x_{11} = \{e_4, e_5, e_2\}) &= (4, \frac{3}{4}) \\ F(x_{12} = \{e_5, e_1, e_3\}) &= (2, \frac{7}{4}) \\ F(x_{13} = \{e_1, e_6, e_4\}) &= (1, \frac{7}{4}) \\ F(x_{14} = \{e_2, e_6, e_4\}) &= (1, \frac{11}{4}) \end{aligned}$$

1.5). The problem $(X(G^3, \{3\}), F)$.

$$\begin{aligned} w_1(e_1) &= w_1(e_2) = w_1(e_4) = w_1(e_5) = 4; \\ w_2(e_1) &= w_2(e_2) = w_2(e_4) = w_2(e_5) = \frac{1}{4}; \\ w_1(e_3) &= 2; w_2(e_3) = \frac{5}{4}; \\ w_1(e_6) &= 1; w_2(e_6) = \frac{9}{4}; \end{aligned}$$

FSS of the given problem consists of the solutions $\{x_1, \dots, x_9\}$, on which *VOF* takes the following values:

$$\begin{aligned} F(x_1 = \{e_1, e_2, e_4, e_5\}) &= (4, 1) \\ F(x_2 = \{e_2, e_3, e_4, e_6\}) &= (1, 4) \\ F(x_3 = \{e_1, e_3, e_4, e_5\}) &= (2, 2) \\ F(x_4 = \{e_2, e_3, e_4, e_5\}) &= (2, 2) \\ F(x_5 = \{e_1, e_2, e_5, e_6\}) &= (1, 3) \\ F(x_6 = \{e_2, e_3, e_5, e_6\}) &= (1, 4) \\ F(x_7 = \{e_1, e_2, e_4, e_6\}) &= (1, 3) \end{aligned}$$

$$F(x_8 = \{e_1, e_3, e_4, e_6\}) = (1, 4)$$

$$F(x_9 = \{e_1, e_3, e_5, e_6\}) = (1, 4)$$

1.6). The problem $(X(G^p, \{p\}), F)$.

$$w_1(e_1) = 2; w_2(e_1) = 2 - \frac{p-3}{p-2};$$

$$w_1(e_2) = \dots = w_1(e_p) = 4; w_2(e_2) = \dots - w_2(e_p) = \frac{1}{p-2};$$

$$w_1(e_{p+1}) = 1; w_2(e_{p+1}) = 2 + \frac{1}{p-2};$$

FSS of the given problem consists of the solutions $\{x_1, \dots, x_{p+1}\}$, on which *VOF* takes the following values:

$$F(x_1 = \{e_2, e_3, \dots, e_p\}) = (4, 1)$$

$$F(x_2 = \{e_1, e_{p+1}, e_3, \dots, e_{p-1}\}) = (1, 4)$$

$$F(x_3 = \{e_3, e_4, \dots, e_p, e_1\}) = (2, 2)$$

$$F(x_4 = \{e_2, e_{p+1}, e_p, \dots, e_4\}) = (1, 3)$$

$$F(x_i = \{e_1, e_2, \dots, e_{i-3}, \dots, e_{i-1}, \dots, e_p\}) = (2, 2), i = 5, \dots, p+1$$

Thus, we proved **Lemma 4.1** for each problem (X, F) , where $X \in \mathfrak{R}$.

Part 2. We will obtain the same results for the set of problems (X, F) , where $X \in \mathfrak{R}$.

By the definition $Y \in \mathfrak{R}$ if $Y = \{z_k \cup y\}$, where $\{z_k\} = Y \in \mathfrak{N}$ and $y = \bigcup_{i=1}^l h_i$, $h_i \in \{2, \dots\}$. Thus, each set $Y \in \mathfrak{R}$ is *FSS* of the chain covering

problem given on the graph $\widetilde{G}_n \equiv G^p \cup G$ that was constructed in **Lemma 3.3**. Define the first weights w_1 of the edges of the graph G^p as we did when we proved **part 1** of **Lemma 4.1** and prescribe the first weight $w_1 = 4$ to each edge of the graph G . Let the second weights w_2 of the edges of the graph G^p be equal to $\frac{1}{2}$ of the weights that we prescribed to the edges of the graph G^p in **part 1** of **Lemma 4.1**. Prescribe the second weights $w_2 = \frac{1}{2n_x}$ to each edge of the graph G , where n_x is the total number of edges in the graph G . The CSA $\widehat{X}(\widetilde{G}_n, H)$ of constructed problem $(X(\widetilde{G}_n, H), F)$ consists of the elements $\{x_1, x_2, x_3\} = \{z_1 \cup y, z_2 \cup y, z_3 \cup y\}$ with the meanings of criteria that satisfy to the condition of **Lemma 4.1**. \square

Lemma 4.2

*If the values of criteria on the elements of CSA $\{x_1, x_2, x_3\}$ of 2-criteria problem are such that $F(x_1) = (1, 4)$, $F(x_2) = (4, 1)$, and $F(x_3) = (2, 2)$, then the N -criteria problem is unsolvable by mean of *LCA*.*

Proof:

Let *VOF* of N -criteria problem consists of k *MAXMIN* criteria and $N - k$ *MAXSUM* criteria. Define the weights of the edges w_i , $i = 3, \dots, N$ of N -criteria problem that corresponds to the 2-criteria problem such that $w_i = w_1$ if the criterion is *MAXMIN* and $w_i = w_2$ if the criterion is *MAXSUM* and consider the criteria convolutions. By the definition of *LCA* we obtain:

$$F^\lambda(x_1) = (\lambda_1 + \dots + \lambda_k) + 4(\lambda_{k+1} + \dots + \lambda_{N-k-1}) + 4(1 - \lambda_1 - \dots - \lambda_{N-k-1})$$

$$= 4 - 3 \sum_{i=1}^k \lambda_i;$$

$$F^\lambda(x_2) = 4(\lambda_1 + \dots + \lambda_k) + (\lambda_{k+1} + \dots + \lambda_{N-k-1}) + (1 - \lambda_1 - \dots - \lambda_{N-k-1})$$

$$= 1 + 3 \sum_{i=1}^k \lambda_i;$$

$$F^\lambda(x_3) = 2(\lambda_1 + \dots + \lambda_k) + 2(\lambda_{k+1} + \dots + \lambda_{N-k-1}) + 2(1 - \lambda_1 - \dots - \lambda_{N-k-1}) = 2$$

It is easy to see that

$$\max \{F^\lambda(x_1), F^\lambda(x_2)\} > F^\lambda(x_3) \text{ for } \forall \sum_{i=1}^k \lambda_i, 0 \leq \sum_{i=1}^k \lambda_i \leq 1.$$

Consequently, the solution x_3 that by the definition belongs to *CSA* can not be found by means of *LCA*. \square

On the basis of **Lemmas 4.1-4.2** we obtain the following result:

For any $n \geq 4$, N -criteria problem of covering with VOF consisting of MAXMIN and MAXSUM criteria is unsolvable by means of LCA.

In conclusion we should mention that taking into account the fact that if applied *LCA* algorithms do not guarantee finding the Pareto Set it would be also interesting to study statistical properties of *LCA* algorithms in terms of "almost all graphs" i.e. to obtain theoretical estimates characterizing expected efficiency of *LCA* algorithms.

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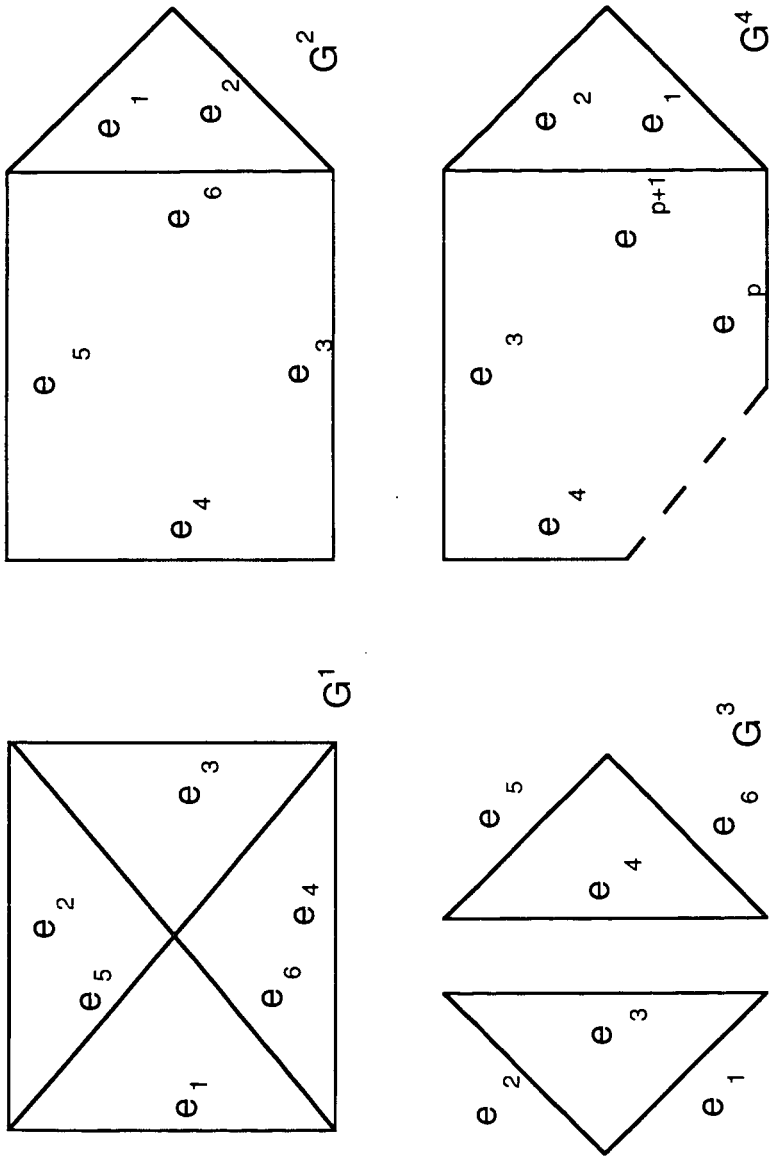


Figure 1.

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