

Adverse Selection, Credit, and Efficiency: the Case of the Missing Market^{*†}

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September 2009

Abstract

We analyze a standard environment of adverse selection in credit markets. In our environment, entrepreneurs who are privately informed about the quality of their projects need to borrow from banks. Conventional wisdom says that, in this class of economies, the competitive equilibrium is typically inefficient.

We show that this conventional wisdom rests on one implicit assumption: entrepreneurs can only borrow from banks. If an additional market is added to provide entrepreneurs with additional funds, efficiency can be attained in equilibrium. An important characteristic of this additional market is that it must be non-exclusive, in the sense that entrepreneurs must be able to simultaneously borrow from many different lenders operating in it. This makes it possible to attain efficiency by pooling all entrepreneurs in the new market while separating them in the market for bank loans.

Keywords: Adverse Selection, Credit Markets, Collateral, Exclusive Contracts, Screening

JEL Classification: D82, G20, D62

*I am grateful to Paolo Siconolfi, Filippo Taddei, Jaume Ventura, seminar participants at CREI and Universitat Pompeu Fabra, and to participants at the 2008 Annual Meeting of the SED for their comments. I acknowledge financial support from the Spanish Ministry of Education and Sciences (grants SEJ2005-00126 and CONSOLIDER-INGENIO 2010 CSD2006-00016, and Juan de la Cierva Program), the Generalitat de Catalunya (AGAUR, 2005SGR 00490), and from CREA-Barcelona Economics.

†An early draft of this paper, circulated in 2006, was titled “On the Efficiency of Financial Markets under Adverse Selection: the Role of Exclusivity”.

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1 Introduction

Imagine a setting in which entrepreneurs need to borrow money from banks in order to fund a project. Some entrepreneurs have good projects that succeed often, but others have bad projects that succeed only seldom. Imagine also that each entrepreneur knows the quality of his investment project, whereas banks do not. This is the typical situation that gives rise to adverse selection. If banks wish to identify the quality of potential borrowers, they need to design different types of loan contracts that are incentive compatible, so that each one of them attracts a different class of borrowers. In order to attract only entrepreneurs with good projects, banks need to design a contract that is appealing to this class of borrowers but is unappealing to entrepreneurs with bad projects: such a contract might, for example, entail small interest payments but require either a low loan size or a high level of collateral. Since bad projects tend to fail more often, entrepreneurs with these projects are less willing to provide collateral because they stand to lose it with a greater probability.

Conventional wisdom suggests that, in a situation like the one just described, the competitive equilibrium will typically be inefficient. The reason for this is quite intuitive. If loan contracts are incentive compatible, whatever is offered to borrowers with good projects cannot be so good as to attract borrowers with bad projects as well. This is similar to saying that the presence of bad borrowers is imposing an externality, since their choice is limiting – through the incentive compatibility constraints – what can be offered to borrowers with good projects.

Why can this externality not be internalized in a competitive equilibrium? Good borrowers could, for example, make the bad loans look more attractive by making side payments to those that apply to them: this would increase the profits that bad borrowers obtain from their loan contract, it would decrease their willingness to apply to the good contract, and it would therefore allow good borrowers to receive a better contract as well. If optimally tailored, these side payments would induce bad borrowers to internalize the externality that their choice imposes on good borrowers. Note that these side payments would lead to separation between borrowers – since at equilibrium good and bad borrowers would be choosing different contracts – while at the same time entailing cross-subsidization from good to bad borrowers.

An efficient arrangement like the one just mentioned, though, cannot be part of a competitive equilibrium. The reason is that, in such an allocation, different borrowers are choosing different contracts and those with good projects are making transfers to those with bad projects. This poses two related problems. First, when considered individually, banks make positive expected profits on the loan contracts of good borrowers while they make negative expected profits on the loan

contracts of bad borrowers. Hence, each bank has an incentive to offer only the good contracts while withdrawing the bad ones from the market: we will refer to this as the problem of *cross-subsidization*. Second, banks can profit by designing deviating contracts that – by implying a lower degree of cross-subsidization – manage to lure only good entrepreneurs away from the constrained optimal allocation: we will refer to this as the problem of *cream-skimming*. Although closely related, it is useful to consider these two problems separately in order to convey the intuition behind our main result.

In this paper, we argue that the conventional wisdom for this class of environments rests on one implicit assumption: entrepreneurs can only borrow from banks. If an additional market is added, in which entrepreneurs can obtain additional funds beyond those borrowed from banks, we show that the efficient allocation is an equilibrium of the economy. An important characteristic of this additional market is that – contrary to what is commonly assumed in environments of adverse selection – it must be non-exclusive, in the sense that entrepreneurs must be able to simultaneously borrow from many different lenders operating in it. In a sense, this requirement is similar to assuming that individual trades cannot be perfectly monitored in this additional market. We therefore refer to it as the “non-exclusive market”.

The intuition for our main result is the following: if good entrepreneurs can distinguish themselves in the eyes of banks by pledging more collateral, it might be beneficial for them to raise more of it through the non-exclusive market. Of course, by this market’s very non-exclusive nature, doing so is costly. If entrepreneurs with good projects borrow from it, bad entrepreneurs have an incentive to also do so in order to benefit from cross-subsidization. Good entrepreneurs, then, face a trade-off: borrowing in the non-exclusive market is directly costly because it entails cross-subsidization of bad borrowers, but it is indirectly beneficial because it allows them to raise collateral and relax the incentive compatibility constraint in the market for bank loans. We show that there is an equilibrium of our economy in which this trade-off is exploited optimally to attain the efficient levels of investment. This equilibrium is resilient to both of the problems discussed above. In the first place, such an equilibrium entails pooling of all borrowers in the non-exclusive market and separation of different types of borrowers in the bank-loans market: hence, no individual contract is expected to yield negative profits and the problem of cross-subsidization is thus overcome. In the second place, all cross-subsidization is undertaken through non-exclusive – and hence, unmonitored – trades. This drastically reduces the set of profitable deviations that banks can design to attract only good entrepreneurs, since their bad counterparts can also apply to such deviations without giving up any transfers that they receive through the non-exclusive market. In this manner, the problem of cream-skimming is overcome as well.

One interpretation of the welfare-enhancing role of the non-exclusive market in our equilibrium is that it allows good entrepreneurs to “buy” an efficient screening technology. In this model, good entrepreneurs can be screened by distorting their investment or by pledging wealth as collateral: of the two, the latter is costless whereas the former is not. If the initial problem is one of scarcity of the resource that allows for efficient screening, an additional market helps by allowing good entrepreneurs to purchase more of it. Of course, merely enabling entrepreneurs to undertake such a purchase is not enough: being costly for good entrepreneurs, it is also needed that this purchase is resilient to attempts at cream-skimming. In our model, this resilience is provided by the non-exclusive environment in which the purchase is carried out. In this sense, and contrary to common results in environments of asymmetric information, welfare is enhanced by enabling entrepreneurs to engage in unmonitored trades.

The literature most closely related to this paper is the one dealing with the efficiency properties of competitive equilibria under adverse selection. Bisin and Gottardi (2006) and Rustichini and Siconolfi (2003, 2004) most clearly pose the problem in terms of consumption externalities arising from the incentive compatibility constraint. Bisin and Gottardi also propose a particular mechanism to deal with the problem: if the problem is one of externalities, they say, it can be solved by introducing markets that allow agents to internalize them. Their mechanism requires the introduction of consumption rights for each type of agent. In the context of our environment, this means that if an entrepreneur wants to borrow a certain amount as a good borrower, he must also provide a certain amount of “good-borrower rights”. If these rights are initially distributed among the population in the appropriate manner, and if markets are created in which these rights can be traded, Bisin and Gottardi show that efficiency can be attained in equilibrium. The present paper differs from their work in two dimensions. On the one hand, our result is admittedly less general, since it applies to problems of adverse selection in credit markets in which collateral is a useful screening device. On the other hand, though, our result shows that efficiency in such a setting can be attained through the use of competitive markets alone, without the need of intervention by a central planner to setup and manage a particular mechanism.

In its modeling approach, this paper draws mostly from Martin (2009), which is in turn closely related to the strategic models of competition under adverse selection. The modeling of the market for bank loans, in which intermediaries compete by designing contracts, is in the tradition of the screening literature derived from Rothschild and Stiglitz (1976), Stiglitz and Weiss (1981), and Bester (1985, 1987). A key difference between our setting and most of those analyzed by previous papers is that we allow for a concave investment function so that the size of projects is determined

endogenously in equilibrium.¹ This feature is crucial since it allows entrepreneurs to be screened through the amounts of collateral that they provide and of investment that they undertake.

The paper is structured as follows. Sections 2 and 3 discuss the baseline model of the credit market and characterize its equilibrium. Section 4 shows that the equilibrium is inefficient and illustrates different interventions by a central planner that could achieve constrained efficiency. Finally, Section 5 shows how constrained efficiency can be attained as a competitive equilibrium when the economy is modified through the introduction of an additional market.

2 The Baseline Model

Assume an economy that is populated by a continuum of consumers and entrepreneurs. There are two periods indexed by $t \in \{0, 1\}$, that we refer to as Today and Tomorrow. All agents are risk-neutral and have monotonic preferences over the economy's only consumption good Tomorrow, although the economy's only endowment is in terms of the consumption good Today. The economic problem that we are considering, then, is that of transforming goods Today into consumption Tomorrow in the most efficient way.

To do so, agents in our economy have two options. They may use a storage technology that yields one unit of the consumption good Tomorrow for every unit stored Today. Alternatively, they may use a productive technology that produces Tomorrow's good by using Today's good as an input. We will make assumptions so that it is always in principle beneficial for the economy to simultaneously use the storage and production technologies. The latter, though, can be operated solely by entrepreneurs, and it may therefore be subject to informational frictions. Assumptions on technology are as follows:

Assumption 1 (Productive Technology). *Entrepreneurs, which are uniformly distributed in the interval $[0, 1]$, may be either of type B (“Bad”) or G (“Good”) depending on the productivity of their technology. Entrepreneurs of each type are distributed over intervals of length λ^j , $j \in \{B, G\}$, where $\lambda^G + \lambda^B = 1$. An entrepreneur of type j has a successful (unsuccessful) state tomorrow with probability p^j ($1 - p^j$), where $p^G > p^B$. If successful (unsuccessful), an entrepreneur of type j who invests I units of the consumption good Today obtains a gross return of $\alpha^j f(I)$ (zero) Tomorrow, where $\alpha^B > \alpha^G$ and $p^G \alpha^G > p^B \alpha^B$. It is assumed that $f(\cdot)$ is increasing, concave, and satisfies Inada conditions.*

Note that the technological assumptions are similar to those commonly used in the credit rationing literature, namely second-order stochastic dominance (SOSD). The only difference is that,

¹In this regard, our environment is closest to the one analyzed by Besanko and Thakor (1987).

in the present setup, the B technology is not just a mean-preserving spread of its G counterpart but actually has a lower expected return. This assumption allows the technologies to be unambiguously ranked.

Entrepreneurs are endowed with an amount W of the consumption good Today, which they cannot use to finance their production.² In order to do so they need to borrow funds from consumers, which they do indirectly through banks. Banks are finite in number and they act as intermediaries that collect deposits from consumers and entrepreneurs and offer loan contracts to the latter. Banks are assumed to be risk neutral and competitive: on the deposit side, they take the gross interest factor on deposits r as given and they compete on the loan market by designing contracts that take the following form:

Assumption 2 (Loan Contracts). *Entrepreneurs and banks sign a contract of the form (I, R, c) , where I is the amount borrowed and invested, R is the interest factor on the loan and c is the percentage of the loan that entrepreneurs must collateralize by using their own wealth. In the event of a successful state, entrepreneurs pay back the amount borrowed adjusted by the interest factor: otherwise, they default and the bank keeps the goods put up as collateral, the interest borne by them and the residual value of the project. Finally, and since they cannot invest it directly in the project, entrepreneurs deposit their endowment in the bank for a gross interest factor of r . This implies that the expected profit that a j -type entrepreneur obtains from loan contract (I, R, c) is given by*

$$\pi^j(I, R, c) = p^j \cdot [f(I) - R \cdot I] - (1 - p^j) \cdot (c \cdot I) \cdot r + r \cdot W. \quad (1)$$

Since competition among banks is crucial in determining the types of contracts that are offered in equilibrium, it is important to specify how we choose to model it. For the sake of simplicity, we will follow Rothschild and Stiglitz (1976) and model the credit market as a two-stage game of screening.³ In the first stage, banks design contracts: in the second stage, entrepreneurs apply for these contracts and all applications are accepted. It is assumed that each bank gets the same share of total deposits and, if they design the same contract, they get the same share and composition of loan applications. A bank's expected profits of accepting an application for a contract (I, R, c)

²This assumption is introduced to simplify the exposition, but it is not restrictive. The contracts that we study, which entail collateral, could be rewritten as contracts in which entrepreneurs invest in their projects and banks provide additional funds. We adopt the former characterization because it makes contracts easier to analyze, it delivers a more tractable framework and it encompasses the cases in which entrepreneurial wealth can or cannot be directly invested.

³None of our main results would change if we modeled perfectly competitive and atomistic credit markets in which contracts are traded, as in Gale (1992), Geanakoplos and Dubey (2002), and Martin (2007). We provide a brief discussion of this in Section 5.1.

from an entrepreneur of type j are given by

$$p^j \cdot (R \cdot I) + (1 - p^j) \cdot (c \cdot I) - r \cdot I. \quad (2)$$

3 Conventional Wisdom

Up to here, we have explained the main features of our benchmark economy.⁴ We now characterize what, according to conventional wisdom, would be the equilibria of this economy with and without asymmetric information. Before doing so, we provide the reader with a brief intuition for the analysis that follows.

In the adverse selection literature, two assumptions are crucial when modeling competition through screening. The first is a condition of no cross-subsidization (henceforth, NCS), by which banks are not allowed to offer contracts that lose money in expectation. This assumption is crucial for the existence of equilibrium because, if banks are allowed to engage in cross-subsidization, it will typically be possible for them to attract all entrepreneurs by designing an incentive compatible menu of contracts that entails transfers from some types of contracts to others. But this allocation, as our introductory discussion suggests, cannot be sustained in equilibrium. The second assumption that is crucial is that of exclusivity (E), by which entrepreneurs can apply to at most one of the contracts offered. This assumption, which implicitly guarantees that banks can monitor contract applications made by entrepreneurs, is what makes screening possible in the first place. Without knowing the total amount of credit that an entrepreneur obtains in equilibrium, it would be impossible for a bank to design incentive-compatible contracts.

Keeping this in mind, we characterize the equilibrium of our economy under these two assumptions. We then show the reader that this equilibrium is inefficient and that the NCS condition lies at the heart of this inefficiency. Indeed, if we allow the planner to cross-subsidize among different types of contracts, it is always able to outperform the competitive equilibrium. This is exactly the intuition behind the result of Bisin and Gottardi (2006): one could think of their mechanism as a way to relax the NCS condition in equilibrium. But what happens instead if we maintain the assumption of exclusivity among bank contracts, while allowing entrepreneurs to borrow from other sources as well? After all, it seems unrealistic to assume that entrepreneurs must obtain all of their funds from a single source. This is in essence the approach that we adopt in the current paper: although we preserve the existence of an exclusive market for bank loans, we introduce an additional non-exclusive source of funds for entrepreneurs. As we will show, this dimension suffices

⁴This setup is essentially identical to the one developed in Martin (2009). Hence, whenever possible, we omit formal proofs of some of the arguments and refer the interested reader to that paper.

to achieve efficiency in equilibrium.

3.1 Full-information equilibrium

In the absence of asymmetric information, the equilibrium of our economy is trivial. Letting $\{(I^{B^*}, R^{B^*}, c^{B^*}), (I^{G^*}, R^{G^*}, c^{G^*})\}$ denote the equilibrium contracts under full information, it is straightforward to verify that they satisfy

$$\left. \begin{aligned} f'(I^{j^*}) &= \frac{r}{\alpha^j p^j} \\ p^j \cdot R^{j^*} + (1 - p^j) \cdot c^{j^*} \cdot r &= r \end{aligned} \right\} \text{for } j \in \{G, B\}.$$

Hence, under full information, good entrepreneurs invest more than bad ones and banks break even in both contracts. It is important to note that, in this case, investment is independent of entrepreneurial wealth W . If entrepreneurs have no wealth, they will simply repay everything in the event of success by setting $R^{j^*} = (r/p^j)$ for $j \in \{G, B\}$.⁵ To simplify the exposition, we henceforth assume that the total amount of resources in the economy is large enough so that the gross risk-free interest rate r is always determined by the return to the storage technology and therefore equal to one.⁶

What changes once asymmetric information is introduced? In such a scenario, banks do not know whether a particular applicant is good or bad. They may then find it optimal to screen applicants by designing different contracts that attract different types of entrepreneurs. In our environment, there are two dimensions along which entrepreneurs can be screened, namely; (a) the size of the loan and; (b) the rate of collateralization.⁷ In order to differentiate good from bad entrepreneurs, banks might offer contracts that require good entrepreneurs to distort their investment. Likewise, they may offer contracts that require higher rates of collateralization: these contracts will be more appealing to good than to bad entrepreneurs, since the former have a higher probability of success and are therefore less likely to lose pledged collateral.

As is usually the case in models of adverse selection, any equilibrium must correspond to the Rothschild-Stiglitz separating allocation. If this allocation is dominated by a pooling contract, the model has no equilibrium. In the next section, we characterize the separating equilibrium of our economy and analyze the conditions under which it fails to exist.

⁵To be precise, the economy under full information displays many equilibria. Indeed, banks are indifferent between making entrepreneurs pay only in the event of success (i.e., setting $c^j = 0$) and making them pay partially in the event of success and partially in the event of failure (i.e., setting $c^j > 0$). All of these equilibria entail the same level of investment and are therefore equivalent in terms of the equilibrium allocation.

⁶This assumption requires that the amount of consumption goods Today exceed the highest possible amount of investment. None of our results depend on it.

⁷Once that rate of collateralization is chosen, the contractual rate of interest is immediately determined by banks' zero-profit condition.

3.2 Equilibrium under asymmetric information

Under assumptions (E) and (NCS), a separating equilibrium is defined as follows:

Definition 1. For a given level of entrepreneurial wealth W , a separating equilibrium is a set of contracts $C^{SEP}(W) = \{(I^B, R^B, c^B), (I^G, R^G, c^G)\}$ satisfying the following conditions:

1. *Feasibility: contracts must respect the collateralization constraint,*

$$c^j \in [0, \frac{W}{I^j}] \text{ for } j \in \{B, G\}. \quad (3)$$

2. *Incentive Compatibility: each entrepreneur applies to the contract designed for his type,*

$$\pi^j(I^j, R^j, c^j) \geq \pi^j(I^i, R^i, c^i) \text{ for } i \neq j, i, j \in \{B, G\}. \quad (4)$$

3. *Zero profit condition for banks: each contract must yield banks zero profits in expectation,*

$$1 = p^j \cdot R^j + (1 - p^j) \cdot c^j \text{ for } j \in \{B, G\}. \quad (5)$$

4. *No bank can profit by offering alternative contracts.*

Conditions (3)-(5) are standard: note simply that (5) stems from bank competition together with no cross-subsidization. Clearly, since banks compete to attract good entrepreneurs, a separating equilibrium requires that the profits of these entrepreneurs are maximized subject to conditions (3)-(5). The resulting contracts are characterized in the following proposition:

Proposition 1. For a given level of entrepreneurial wealth W , the separating equilibrium of our economy is a set of contracts $C^{SEP}(W) = \{(I^B, R^B, c^B), (I^G(W), R^G(W), c^G(W))\}$ satisfying,

$$(I^B, R^B, c^B) = (I^{B*}, \frac{1}{p^B}, 0), \quad (6)$$

$$\alpha^G p^G f'(I^G(W)) > 1 \Rightarrow c^G = \frac{W}{I^G}, \text{ and,} \quad (7)$$

$$c^G(W) = \frac{[\alpha^B p^B f(I^G(W)) - \frac{p^B}{p^G} I^G(W)] - [\alpha^B p^B f(I^B) - I^B]}{(1 - \frac{p^B}{p^G}) I^G(W)} \leq 1. \quad (8)$$

Proof. See Martin (2009). □

Condition (6) implies that – at equilibrium – contracts taken by bad entrepreneurs entail no distortions. Thus, they are lent the amount that is efficient given the interest rate and they have

no need to provide collateral. It is therefore on the contracts taken by good entrepreneurs that the interest of the equilibrium lies, since these contracts must be incentive compatible. What are the properties of these contracts?

First of all, suppose that entrepreneurs have no wealth, so that the use of collateral is impossible. In that case, the only way of preventing bad entrepreneurs from applying to the good contract is by sufficiently restricting the amount of investment allowed by the latter. In the absence of entrepreneurial wealth, then, separation requires rationing the investment of good entrepreneurs relative to the full-information economy.

Separation through the rationing of investment, though, is clearly costly. What if entrepreneurs have some positive level of wealth? In that case, efficient screening requires that good entrepreneurs pledge all of their wealth as collateral: this is of no cost to them, whereas it raises the cost of funds for bad entrepreneurs because they lose this collateral more often. As entrepreneurial wealth increases, then, an increasing fraction of the screening will take place through higher rates of collateralization and a decreasing fraction of it will take place through the distortion of investment. Eventually, if entrepreneurial wealth increases enough, it reaches a level W^* that enables good entrepreneurs to separate themselves only by providing collateral.

Our previous analysis suggests that separation is very costly for good entrepreneurs when W is low, and becomes less costly when W is high. And, when separation is costly, our economy as stated so far might fail to have an equilibrium. This will be the case whenever there is a pooling allocation that Pareto dominates the separating allocation of Proposition 1. The reason is the usual one in models of screening: if a pooling allocation Pareto dominates the separating equilibrium, then banks can profitably deviate from the latter to the former while attracting all entrepreneurs. But the pooling allocation can never be an equilibrium either: once in it, banks always have an incentive to engage in cream-skimming by designing a deviating contract that profitably lures good entrepreneurs away from the pool.⁸

Indeed, it can be formally shown that – provided that the proportion of good entrepreneurs in the economy is sufficiently high – a separating equilibrium will fail to exist in our economy whenever entrepreneurial wealth is low. In particular, there is a well-defined function $W(r)$ that determines, for each interest rate, the minimum level of entrepreneurial wealth that is necessary in order for an equilibrium to exist.⁹ If the mix of entrepreneurs in the economy is sufficiently high, however, $W(r) > 0$ for all r and an equilibrium fails to exist whenever $W = 0$. The following lemma, adapted

⁸In particular, banks could lure good entrepreneurs away from the pool by offering contracts that entail less investment and lower interest rate payments than the pooling contract.

⁹Naturally, $W'(r) < 0$, since high interest rates lead to low levels of equilibrium investment and hence separation can be attained with relatively low levels of wealth

from Martin (2009), establishes this formally:

Lemma 1. *Let $\bar{p} = \lambda^G p^G + \lambda^B p^B$ denote the average probability of success of all entrepreneurs in the economy. If $\bar{p} > \bar{p}_0 = \frac{\alpha^B p^B}{\alpha^G}$, then the separating allocation of Proposition 1 is always dominated by a pooling allocation when $W = 0$. In this case, our economy fails to have an equilibrium.*

4 Welfare-enhancing interventions by the planner

Consider the baseline economy analyzed so far. In particular, assume an economy in which $\bar{p} > \bar{p}_0$ and $W = 0$.¹⁰ According to the previous analysis, such an economy does not have an equilibrium. Consider a central planner that seeks to maximize social welfare subject to an incentive compatibility constraint: its problem can then be interpreted as the constrained maximization of G profits. Such a planner, even if subject to constraints (E) and (NCS), can clearly offer pooling contracts whenever these dominate the separating equilibrium. But there is even more that a planner can do. Indeed, there is no reason for which it cannot engage in cross-subsidization between different types of contracts. It is well known that, by doing so, it can typically increase welfare beyond what can be attained through pure pooling or separating allocations. We now explore some of these alternatives.

4.1 Transfers: relaxing NCS

Consider first the case of a planner that, besides offering different contracts, is able to resort to direct transfers to cross-subsidize between them. One way to do this is simply to collect resources from applicants to the G contract in order to distribute them among applicants to the B contract. Since we restrict ourselves to the case in which $W = 0$, though, this cannot be done directly. What the planner can do, however, is to increase the interest payments charged to entrepreneurs who apply to the G contract in order to distribute the proceeds among entrepreneurs applying to the B contract. We refer to such a scheme as a *direct-transfer scheme*. Under such a scheme, and letting T stand for the (fixed) surcharge charged to applicants to the G contract in the event of success,

¹⁰The assumption by which $W = 0$ is made to simplify the exposition and is not necessary for our results. In the current section, we consider different schemes through which the planner can engage in cross-subsidization between different contracts: since entrepreneurs have no wealth, the optimal degree of cross-subsidization is fully undertaken with borrowed resources, i.e. through credit. If entrepreneurs had some positive level of wealth, a fraction of the optimal cross-subsidization would be undertaken directly with entrepreneurial resources, while the remaining part would be undertaken through credit in the form that we analyze below.

the planner's problem may be formulated as follows:

$$\max_{I^G, I^B, T} \alpha^G p^G f(I^G) - I^G - p^G T$$

s.t.

$$\alpha^B p^B f(I^G) - \frac{p^B}{p^G} r I^G - T p^B \leq \alpha^B p^B f(I^B) - r I^B + T \frac{\lambda^G p^G}{\lambda^B}, \quad (9)$$

$$\alpha^G p^G f(I^G) - r I^G - T p^G \geq \alpha^G p^G f(I^B) - \frac{p^G}{p^B} r I^B + T \frac{\lambda^G p^G}{\lambda^B}, \quad (10)$$

where Equations (9) and (10), which already incorporate a zero-profit constraint for the planner, respectively represent the incentive compatibility constraints of bad and good entrepreneurs. The following Lemma characterizes the solution to the planner's problem.

Proposition 2. *Consider our baseline economy when $W = 0$ and $\bar{p} > \bar{p}_0$. Under a direct transfer scheme, the allocation chosen by the planner entails a positive transfer $\hat{T} > 0$. Moreover, such an allocation entails an investment of \hat{I}^j by entrepreneurs of type $j \in \{B, G\}$, where \hat{I}^j is defined by:*

$$\begin{aligned} \hat{I}^B &= I^{B*} \\ f'(\hat{I}^G) &= \frac{r}{p^G \alpha^G - (\alpha^B - \alpha^G) p^B \frac{\lambda^B}{\lambda^G}} \end{aligned}$$

This allocation Pareto dominates the competitive equilibrium.

Proof. See Appendix. □

Proposition 2 implies that the competitive equilibrium of our economy can always be improved upon if the central planner can resort to transfers. The intuition for this result is as follows: if the planner can cross-subsidize between different contracts, it can ask applicants to the G contract for a fixed fee in the event of success, the proceeds of which are then distributed among applicants to the B contract. If the proportion of good to bad types in the economy is high (in particular, so as to satisfy $\bar{p} > \bar{p}_0$), such a mechanism demands few resources from each good entrepreneur whereas it increases the cost of mimicking for bad entrepreneurs. In other words, such a scheme enables good entrepreneurs to make a payment in order to “relax” their incentive constraint.¹¹ Clearly, this solution always outperforms the separating equilibrium of Proposition 1. But it also outperforms a pure pooling allocation because, whereas the latter requires bad entrepreneurs to overinvest relative to their efficient level of investment, the use of cross-subsidization avoids this

¹¹In particular – under the direct transfer scheme – each marginal increase in T increases the cost of G types by p^G , whereas it increases the cost of mimicking for B types by $\frac{\lambda^G p^G}{\lambda^B} + p^B$.

distortion while nonetheless expanding investment by good entrepreneurs.¹²

The planner intervention analyzed in this section lends itself to a very natural interpretation. We remind the reader that, according to Bisin and Gottardi (2006) and others, the problem of adverse selection can be thought of as a consumption externality. In our setting, bad entrepreneurs constrain the choices of their good counterparts through the incentive compatibility constraint: since they do not internalize this effect, competitive equilibria are inefficient. According to this logic, the inefficiency could be eliminated if only good entrepreneurs could pay the bad ones to internalize the externality that they generate. But this is exactly what the separating contracts with transfers allow, making it possible for good entrepreneurs to “pay” those who apply to the bad loan contract. This relaxes the incentive compatibility constraint of loan contracts and allows good entrepreneurs to expand their investment in equilibrium.

4.2 Credit lines and collateral: preserving NCS with additional funds

As has been shown, a central planner can outperform the competitive equilibrium of our economy by engaging in direct transfers among different contracts. We now show that there is an alternative way in which the planner can achieve the same level of investment. It consists in first lending money to all entrepreneurs and then requiring applicants to G contracts to pledge these resources as collateral. This essentially amounts to splitting the loan in two. First, entrepreneurs have access to an “official credit line” from which they can draw funds and, second, they apply to separating contracts like the ones described in Proposition 1. The interesting aspect of this scheme is that, as we later argue, it is fully compatible with maintaining the assumption of NCS.

We refer to this scheme as a *loan-collateralization scheme*. In it, the planner first offers to lend all entrepreneurs an amount W_T through a credit line: in order to break even, it charges an interest rate of $(1/\bar{p})$ per unit of credit provided in the event of success. It then offers separating contracts in the manner of Proposition 1. The existence of the first loan, though, now makes it possible for entrepreneurs to pledge up to W_T as collateral when applying to these contracts. Under such a

¹²In fact, note that G profits must necessarily increase with respect to the pooling allocation, since the planner could always choose to implement the latter with the appropriate transfers and loans.

scheme, the planner's problem may be formulated as follows:

$$\max_{I^G, I^B, W_T} \alpha^G p^G f(I^G) - I^G + \left(1 - \frac{p^G}{\bar{p}}\right) W_T$$

s.t.

$$\alpha^B p^B f(I^G) - \frac{p^B}{p^G} I^G - \left(1 - \frac{p^B}{p^G}\right) W_T \leq \alpha^B p^B f(I^B) - r I^B, \quad (11)$$

$$\alpha^G p^G f(I^G) - r I^G \geq \alpha^G p^G f(I^B) - \frac{p^G}{p^B} I^B, \quad (12)$$

where Equations (9) and (10) represent the respective incentive compatibility constraints of B and G entrepreneurs when applying to the separating contracts. The following Lemma characterizes the solution to the planner's problem.

Proposition 3. *Consider our baseline economy when $W = 0$ and $\bar{p} > \bar{p}_0$. Under a loan-collateralization scheme, the allocation chosen by the planner entails a positive loan through the credit line $W_T = \widehat{W} > 0$. Moreover, such an allocation entails an investment of \widehat{I}^j by entrepreneurs of type $j \in \{B, G\}$, where \widehat{I}^j is defined by:*

$$\begin{aligned} \widehat{I}^B &= I^{B*} \\ f'(\widehat{I}^G) &= \frac{r}{p^G \alpha^G - (\alpha^B - \alpha^G) p^B \frac{\lambda^B}{\lambda^G}} \end{aligned}$$

Proof. See Appendix. □

Relative to the separating contracts of Proposition 1, bad entrepreneurs benefit from the loan-collateralization scheme because they obtain an amount \widehat{W} at an effectively cross-subsidized rate of (p^B/\bar{p}) .¹³ How about good entrepreneurs? Once again, since the marginal cost of the funds it provides is effectively equal to $(p^G/\bar{p} - 1)$, the planner's first loan might not seem like a good idea for them. On the other hand, though, it benefits these entrepreneurs indirectly by increasing their resources at the time of applying to the loan contracts. Since good entrepreneurs are more willing than bad ones to pledge them as collateral, these additional resources enable them to obtain better contractual terms. Ultimately, the ratio between the cost of increasing W_T and the benefit it reports by increasing the cost of mimicking for B types is exactly the same as the corresponding ratios under the direct transfer scheme. Hence, the planner is able to achieve the same level of investment under the former than under the latter.

¹³Although the loan-collateralization scheme always increases the expected profits of entrepreneurs with bad projects, it must be verified that these always have enough resources to fulfill their obligations in the event of success (see Appendix).

What is going on? From an economic perspective, the loan-collateralization scheme enables good entrepreneurs to “buy” an efficient screening technology. In this model, good entrepreneurs can be screened by distorting their investment or by pledging wealth as collateral: of the two, the latter is costless whereas the former is not. If the initial problem is one of scarcity of the resource that allows for efficient screening, the planner can help by allowing good entrepreneurs to purchase more of it. Although directly costly because they must pay a premium for these additional funds, the intervention is indirectly beneficial for good entrepreneurs because it enables them to be screened more efficiently by banks: in other words, the benefits of improved screening are higher than its cost.

Before concluding our description of the loan-collateralization scheme, we remind the reader that it attains the same level of investment as the direct transfer scheme. Given that this is the case, why have we bothered in characterizing both? The reason is that they differ in one crucial aspect. The direct transfer scheme requires the planner to offer separating contracts with cross-subsidization, and it is therefore not decentralizable as a competitive equilibrium: in the terminology of the introduction, it suffers from the problems of cross-subsidization and cream-skimming. The loan-collateralization scheme, however, is resilient to these problems. In the first stage there is cross-subsidization but no separation, since the planner lends the same amount to all entrepreneurs. In the second stage, there is separation but no cross-subsidization, since the planner offers a pair of separating contracts designed in the manner of Proposition 1. In principle, then, there is promise that such a scheme can be decentralized as a competitive equilibrium. And this is where we turn our attention next.

5 Efficiency restored: the case of the missing market

We are now ready to show our main result. We do so by modifying the baseline economy of Section 2: while we preserve competition among our original banks under the assumptions of exclusivity and no cross-subsidization, we introduce a new financial market in which entrepreneurs can raise additional resources. We henceforth refer to the original credit market as the “exclusive market”, and to the new financial market as the “non-exclusive market”.

As in the exclusive market, competition in the non-exclusive market is modeled as a two-stage game of screening a la Rothschild and Stiglitz (1976). In this market, N non-exclusive intermediaries collect deposits from savers and compete to lend resources to entrepreneurs.¹⁴ In the first stage,

¹⁴Exactly as banks in the exclusive market, non-exclusive intermediaries are assumed to be risk neutral and competitive: on the deposit side, they take the gross interest factor on deposits as given and they compete on the loan market by designing contracts.

these intermediaries design contracts (I, R, c) , in the second stage entrepreneurs apply to these contracts and all applications are accepted. This market, then, looks exactly like its exclusive counterpart, with one crucial difference: entrepreneurs are allowed to simultaneously apply to contracts offered by different intermediaries. It can be therefore be thought of as a market in which it is difficult for contract providers to monitor an agent's trades, so that exclusivity is impossible to enforce. We make the natural assumption that each non-exclusive intermediary can nonetheless monitor an individual agent's application to its own contract, so that it is impossible for any given entrepreneur to apply more than once to a contract offered by an individual intermediary. The reader can therefore think of our modified economy as consisting of a first set of markets in which all trades can be observed and verified, and a second set of markets in which individuals can engage in hidden trades.¹⁵

As for the timing of the non-exclusive market, we assume that it opens Today after contracts are posted – but before applications take place – in the exclusive market. In other words, competition in credit markets Today is now assumed to work as follows: (i) contracts are posted in exclusive markets; (ii) contracts are posted in non-exclusive markets; (iii) entrepreneurs apply to non-exclusive contracts, and all applications are accepted, and; (iv) entrepreneurs apply to exclusive contracts, and all applications are accepted.¹⁶ An equilibrium of the modified economy is thus defined as follows:

Definition 2. *Given our modified economy with $W = 0$ and $\bar{p} > \bar{p}_0$, an equilibrium is defined as a set of exclusive and non-exclusive contracts, $C^E(W) = \{(I^B, R^B, c^B), (I^G, R^G, c^G)\}$ and $C^{NE}(W) = \{(I_N^B, R_N^B, c_N^B), (I_N^G, R_N^G, c_N^G)\}$ such that:*

1. *Exclusive contracts satisfy the feasibility, incentive-compatibility and zero-profit conditions laid out in Definition 1.*
2. *Non-exclusive contracts satisfy feasibility and the zero-profit condition of non-exclusive intermediaries.*
3. *No exclusive bank or non-exclusive intermediary can profit by offering alternative contracts.*

¹⁵We wish to stress one important fact to the reader: the establishment of a non-exclusive market like the one just described does not require changing any of the informational or technological assumptions of our economy. In fact, these contracts display the same level of contingency as exclusive contracts offered by banks and are not therefore “new” assets in any fundamental way.

¹⁶As is commonly the case in economies of adverse selection, this timing is not innocuous, and the precise manner in which competition is modelled will affect the type of competitive equilibrium that emerges. One appealing feature of this timing, though, is that it is consistent with our contracts being optimal. In particular, non-exclusive intermediaries cannot condition their contracts on the applications entrepreneurs make in the exclusive market, because these happen after non-exclusive credit is allocated.

Before proceeding to show our main results, there are two important observations regarding any equilibrium of our modified economy. First, as implicitly stated in Definition 2, any credit provided through the exclusive market must in the form of separating contracts: as mentioned earlier, no equilibrium can pool entrepreneurs in these contracts, since this would provide incentives for profitable deviations through cream-skimming. Second, in any equilibrium in which some credit is provided through non-exclusive contracts, these must be of the pooling type. Indeed, besides trivial equilibria in which only bad entrepreneurs obtain credit in the non-exclusive market at an interest rate of $(1/p^B)$, there can be no separation in these contracts: if there was any such separation, bad entrepreneurs would always have an incentive to mimic their good counterparts in order to be cross-subsidized.

In sum, any equilibrium of our modified economy in which credit is granted through both, exclusive and non-exclusive markets, must entail separation in the former and pooling in the latter. Why then, do G entrepreneurs bother to raise funds at all in non-exclusive market? The reason, as we have mentioned in the previous section, is that non-exclusive borrowing provides resources to be used as collateral in the exclusive financial market. In fact, as we now show, the combination of these two markets makes it possible to attain the constrained optimal allocation as a competitive equilibrium.

Proposition 4. *Consider our modified economy when $W = 0$ and $\bar{p} > \bar{p}_0$. The constrained optimal allocation characterized in Proposition 3 can be decentralized as a competitive equilibrium of this economy. In such an equilibrium:*

1. *Each non-exclusive intermediary offers a unique contract given by,*

$$(I_N, R_N, c_N) = \left(\frac{\widehat{W}}{N}, \frac{1}{\bar{p}}, 0 \right). \quad (13)$$

2. *Each exclusive bank offers a pair of contracts*

$$\left\{ \left(I^{B^*}, \frac{1}{p^B}, 0 \right), \left(\widehat{I}^G, R^G(\widehat{W}), \frac{\widehat{W}}{\widehat{I}^G} \right) \right\}, \quad (14)$$

where the G contract satisfies incentive compatibility and the bank's zero-profit condition as stated in Definition 1.

3. *An entrepreneur of type $j \in \{B, G\}$ applies to all the non-exclusive contracts, and to one exclusive contract from the pair in Equation (14) corresponding to his type.*

Proof. See Appendix. □

The competitive equilibrium described in Proposition 4 reproduces the allocation attained through the loan-collateralization scheme described in the previous section. In this equilibrium, all entrepreneurs borrow a total of \widehat{W} units from the non-exclusive market. While bad entrepreneurs simply keep these resources, good entrepreneurs pledge them as collateral when applying to the separating contracts offered by the exclusive banks.

Given the contracts offered, this is the optimal strategy for all entrepreneurs. Bad entrepreneurs borrow from the non-exclusive market at a profit because they do so at a cross-subsidized rate of interest. Good entrepreneurs, by definition of the constrained optimal allocation, maximize their profits by using both markets in order to expand their overall level of borrowing and investment.

It remains to be shown, however, that there are no profitable deviations for contract designers. Note first that, in the equilibrium of Proposition 4, no individual contract is expected to yield negative profits. Hence, as we have already anticipated in our discussion of the loan-collateralization scheme, the equilibrium is resilient to the problem of cross-subsidization, i.e. to deviations that entail the withdrawal of contracts by their designers. Any deviation, then, must be associated to the problem of cream-skimming, i.e. it must entail the design of alternative contracts that attract only good entrepreneurs. In the case of non-exclusive intermediaries, such deviation is clearly non-existent. There is no way in which they can make profits by seeking to attract good entrepreneurs: given the non-exclusive nature of their contracts, any funds that they offer at an interest rate below $(1/p^B)$ will attract bad entrepreneurs as well.

The crucial part of the argument, then, consists in showing that there are no profitable deviations for exclusive banks either. After all, the standard arguments for cream-skimming would seem to apply to our modified economy as well. In the equilibrium of Proposition 4, it is directly costly for good entrepreneurs to borrow from the non-exclusive market but beneficial for bad entrepreneurs to do so. What prevents exclusive banks from designing G contracts that entail a smaller loan size but require a lower level of collateral – and hence of cross-subsidization – as well? Why doesn't such a deviation, which destroys the pooling equilibrium in our baseline economy, destroy the equilibrium outlined in Proposition 4?

The answer is that, in the efficient equilibrium outlined above, all cross-subsidization is undertaken through non-exclusive markets. This simple feature drastically limits the type of profitable deviations that exclusive banks can design. To see this, consider a pooling allocation in our baseline economy: such an allocation cannot be an equilibrium because good entrepreneurs can be lured away from it. But this is only possible because contracts are exclusive: bad entrepreneurs, when deciding whether to follow good entrepreneurs in their deviation, know that doing so requires them to give up the cross-subsidization that they obtain in the pooling allocation. Simply put, bad entre-

preneurs must choose: they *either* stay at the pooling and receive the benefits of cross-subsidization *or* they follow good entrepreneurs in their deviation. In the equilibrium of Proposition 4, though, this reasoning does not apply. Since exclusive banks cannot monitor what entrepreneurs do in the non-exclusive market, they cannot condition their contracts on such actions. At most, they can require an applicant to show up with a certain level of collateralizable resources in order access a given contract. This makes it very difficult to attract only good entrepreneurs by designing contracts that entail less investment but also require less collateral: now, bad entrepreneurs can do *both*, follow the good ones in their deviation *and* obtain cross-subsidization in the non-exclusive market.

Indeed, as we show in the Appendix, any attempt at cream-skimming on behalf of exclusive banks will ultimately attract all entrepreneurs. The reason is as follows: suppose that an exclusive bank offers a separating contract that entails a lower level of investment but requires a lower level of collateral as well. If designed properly, such a contract will be attractive to good entrepreneurs because it will enable them to borrow less from the non-exclusive market, which is costly for them. However, any such contract will necessarily also be attractive to bad entrepreneurs, since it enables them to keep the resources \widehat{W} that they obtain in the non-exclusive market without requiring them to fully pledge these resources as collateral, which is costly for them. To clarify the argument, we also show in the Appendix that the inability to contract on the non-exclusive trades lies at the heart of the argument. If this restriction is lifted, bad entrepreneurs would have to choose between being cross-subsidized in the non-exclusive market or following good entrepreneurs in their deviation, the standard cream-skimming argument would apply once more, and the efficient equilibrium of Proposition 4 would unravel.

This result, and the logic that underlies it, is somewhat surprising. It implies that, contrary to what is commonly thought regarding environments of asymmetric information, allowing agents to engage hidden trades might be welfare-enhancing in the presence of adverse selection.¹⁷ In our setup, hidden trades are beneficial because they restrict the type of deviations that can be designed by exclusive banks, which are ultimately at the heart of the traditional inefficiency result. In a sense, then, the presence of the non-exclusive market “disciplines” its exclusive counterpart and buttresses the existence of an efficient equilibrium.¹⁸

¹⁷For an analysis of how hidden trading might be problematic in environments of moral hazard, see Bisin and Guaitoli (2004).

¹⁸In this sense, our result is related to the literature that stresses the possible role of hidden trades in disciplining governments. See, for example, Bisin and Rampini (2006).

5.1 Discussion

Overall, the constrained optimal allocation in the economy requires some separation and some cross-subsidization. In Section 4.2, we argued that this allocation could be attained through a loan-collateralization scheme. Such a scheme entailed both cross-subsidization and separation, but never in the same loan: the planner would pool all entrepreneurs in a first loan, and it would then separate them through standard contracts. It is this feature of the scheme that allows it to be decentralized as a competitive equilibrium. Such an equilibrium entails pooling of all entrepreneurs in the non-exclusive market. Any proceeds obtained in this market, though, are pledged as collateral in the market for exclusive loan contracts in order to achieve separation.

We have shown that there is an equilibrium of the modified economy that attains constrained efficiency, but are there other equilibria? Without providing a formal treatment of this question, we conjecture that the answer depends crucially on what is assumed regarding price-taking behavior by exclusive contract designers. In particular, if these anticipate the effects of their posted contracts on the working of the non-exclusive market, the unique equilibrium is indeed the one characterized in Proposition 4.

To see this, note first that entrepreneurs must necessarily borrow from the non-exclusive market in any equilibrium of our economy with $W = 0$ and $\bar{p} > \bar{p}_0$. If they do not, we are back in the case of the baseline economy, which fails to have an equilibrium. Hence, the only way in which an equilibrium can differ from the efficient one of Proposition 4 is by entailing too much or too little cross-subsidization in the non-exclusive market.

Suppose, for example, that exclusive banks expect each non-exclusive intermediary to offer a unique contract given by

$$\left(\frac{W_0}{N}, \frac{1}{\bar{p}}, 0 \right),$$

for $W(r) < W_0 < \widehat{W}$, where $W(r)$ is defined as in Section 3.2. In this case, cross-subsidization in the non-exclusive market is expected to be inefficiently low, and exclusive banks do not have an incentive to offer the separating contract that requires \widehat{W} units of collateral. There is, however, an equilibrium in which exclusive intermediaries post G contracts that require W_0 units of collateral and entrepreneurs effectively borrow this amount from non-exclusive intermediaries. By the same token, it is possible to construct equilibria in which – relative to the efficient equilibrium – entrepreneurs borrow too much from the non-exclusive market.

These equilibria, though, can only survive if exclusive banks take non-exclusive contracts as given. If they anticipate the effect of their own contracts on those offered by non-exclusive intermediaries, however, only the efficient equilibrium survives. Take the example with inefficiently low

cross-subsidization developed above: if an exclusive bank happens to post the separating contract requiring \widehat{W} units of collateral, the non-exclusive intermediaries will have an incentive to provide \widehat{W} units of credit. But, anticipating this, an exclusive bank will in fact have the incentive to deviate and post the aforementioned contract. A similar argument rules out equilibria in which the degree of cross-subsidization in non-exclusive market is inefficiently high.

This discussion only highlights what Hellwig (1987) has already stressed: the way in which competition is modeled is crucial in determining the set of equilibria in economies of adverse selection. Regardless of the particular modeling choice that is made, though, the main finding of this paper remains valid. Suppose, for example, that instead of modeling the exclusive and non-exclusive markets as games of screening à la Rothschild-Stiglitz, we had modeled them both as a set of perfectly competitive markets in which all possible contracts could be traded. In such a setting, exclusive and non-exclusive contracts would be traded simultaneously. Would the constrained optimal allocation be an equilibrium of such an economy? If in equilibrium agents trade exclusive contracts as in Equation (14), it is clearly optimal for them to borrow \widehat{W} from the non-exclusive market according to Equation (13). But if entrepreneurs raise \widehat{W} in the non-exclusive market according to Equation (13), the only equilibrium in the contract markets will be one in which agents trade contracts as in Equation (14). This last statement follows directly from the findings of Dubey and Geanakoplos (2002), who showed that – when modeled in a fully competitive fashion – models of screening à la Rothschild-Stiglitz have a unique equilibrium that corresponds to the separating equilibrium of Proposition 1.^{19,20}

In the presence of adverse selection in the credit market, then, the competitive equilibrium will typically be inefficient. It is well known that there is a welfare-enhancing intervention by the planner that consists in directly cross-subsidizing some contracts at the expense of others. We have shown that the same allocation can be attained by an alternative scheme that consists in lending funds to entrepreneurs and then offering separating contracts that rely only on collateralization. This last scheme, though, can be decentralized as a competitive equilibrium in a very natural way. The only reason for which this has been thought to be unfeasible in standard models of adverse selection in the credit market is because of the assumption by which entrepreneurs can only borrow

¹⁹Once again, we have chosen not to pursue this modeling alternative in order to simplify the exposition as much as possible. Modeling competitive markets for contracts as in Dubey and Geanakoplos (2002), for example, requires the explicit introduction of off-equilibrium beliefs. While the substance of our argument would be fundamentally unchanged, developing it properly in such a setting would require the reader to pay greater attention to notation and to technical considerations.

²⁰A caveat is in order. Whereas the exact manner in which the exclusive and non-exclusive markets are modeled is not really important for our main result, the timing in which they operate might be. If both markets are assumed to be simultaneous, for example, the contracts that we consider may no longer be optimal. In particular, it might be optimal for non-exclusive intermediaries to make their contracts contingent on the actions that entrepreneurs undertake in the exclusive market. In our setting, this issue does not arise.

from one source. If this assumption is removed, efficiency is attainable in equilibrium.

6 Concluding Remarks

The present paper has analyzed a standard setting of adverse selection in credit markets. In particular, we have studied an economy in which entrepreneurs are privately informed about the quality of their investment opportunities and they need to borrow from banks. Conventional wisdom suggests that, in such settings, competitive equilibria will typically be inefficient. The reason for this can essentially be framed as one of externalities: under adverse selection, the incentive compatibility constraint restricts allocations across types, so that decisions made by one type of agents constrain the choices available to others. Achieving efficiency in this environment requires this externality to be internalized: this could be done, for example, by allowing for transfers among types of agents. Such a solution, however, cannot be decentralized as a competitive equilibrium because it requires that some contracts yield positive expected profits while other yield expected losses. Obviously, there is no incentive to offer the latter in equilibrium.

We have argued that the conventional wisdom for this class of environments rests on one implicit assumption: entrepreneurs can only borrow from banks. If an additional non-exclusive market is added, so that entrepreneurs can obtain funds beyond those offered by banks, we show that the constrained efficient allocation is an equilibrium of the economy. We have provided an intuition for this result: if good entrepreneurs can distinguish themselves in the eyes of banks by pledging collateral, it might be beneficial for them to raise more of it through the non-exclusive market. Of course, doing so is costly. If entrepreneurs with good projects raise funds by borrowing from the non-exclusive market, bad entrepreneurs also have an incentive to do so and benefit from the ensuing cross-subsidization. Good entrepreneurs, then, face a trade-off: borrowing from the non-exclusive market is directly costly because it entails cross-subsidization of bad borrowers, but it is indirectly beneficial because it allows them to raise collateral and relax the incentive compatibility constraint in the exclusive market for bank loans. We show that there is an equilibrium of our economy in which this trade-off is exploited optimally to attain the efficient levels of investment. In such an equilibrium, there is pooling of all borrowers in the non-exclusive market and separation of borrowers with different types in the exclusive market for bank loans.

Our paper complements existing results in the literature on adverse selection by showing that, in certain settings, it is possible to attain efficiency only through the interaction of standard competitive markets. Admittedly, our result is substantially less general than the one of Bisin and Gottardi (2006): whereas their findings apply to canonical problems of adverse selection, our find-

ings thus far are circumscribed to credit markets in which collateral can be a useful screening device. In this sense, credit markets are particular in that they provide goods but also demand them for screening purposes: hence, the screening technology can in a sense be provided by markets themselves. Having said this, we feel that our finding gains in simplicity and intuition what it lacks in generality. Indeed, a general solution like the one proposed by Bisin and Gottardi (2006) requires the intervention of a planner to design the consumption rights, instrument their use, and distribute an initial endowment of these rights throughout the population. In our setting, though, efficiency is attained simply by letting firms borrow from two different markets at different rates. The mechanism by which efficiency is attained is simple and has realistic implications. Casual observation suggests that firms do borrow from various sources at the same time (banks and bonds are the simplest examples that come to mind). It remains to be seen whether the findings of this paper, by which efficiency can be attained in economies of adverse selection through the interaction of various competitive markets, can be generalized to a broader class of environments.

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7 Appendix

7.1 Direct Transfer Scheme (Proposition 2)

Taking into account his budget constraint, the planner’s problem may be formulated as follows:

$$\max_{I^G, I^B, T} \alpha^G p^G f(I^G) - I^G - p^G T$$

subject to

$$\begin{aligned} \alpha^B p^B f(I^G) - \frac{p^B}{p^G} I^G - T p^B &\leq \alpha^B p^B f(I^B) - I^B + T \frac{\lambda^G p^G}{\lambda^B} \\ \alpha^G p^G f(I^G) - I^G - T p^G &\geq \alpha^G p^G f(I^B) - \frac{p^G}{p^B} I^B + T \frac{\lambda^G p^G}{\lambda^B} \end{aligned}$$

We solve the problem by using only one incentive compatibility constraint and conjecturing that the other one will be slack. Once we derive the optimal contracts, we will prove this conjecture to be correct. It can be readily obtained that $\widehat{I}^B = I^{B*}$. Using β to denote the multiplier on the incentive compatibility constraints, the first order conditions with respect to I^G and T are:

$$\alpha^G p^G f'(\widehat{I}^G) - 1 = \beta [\alpha^B p^B f'(\widehat{I}^G) - \frac{p^B}{p^G}], \quad (15)$$

$$-p^G + \beta \left[\frac{\lambda^G p^G}{\lambda^B} + p^B \right] = 0, \quad (16)$$

where, in the last equality, we already incorporate the fact that any solution must entail $\widehat{T} > 0$. To see this, suppose that $\widehat{T} = 0$. In this case, the incentive compatible allocation would entail $\widehat{I}^G < I^{B*}$. From Equation (15), it follows that

$$\beta = \frac{\alpha^G p^G f'(\widehat{I}^G) - 1}{\alpha^B p^B f'(\widehat{I}^G) - \frac{p^B}{p^G}} > \frac{\frac{\alpha^G p^G}{\alpha^B p^B} - 1}{1 - \frac{p^B}{p^G}} > \frac{p^G \lambda^B}{\bar{p}},$$

which, together with Equation (16), implies that any solution must entail $T > 0$. Hence, in the optimal solution,

$$\beta = \frac{p^G}{\bar{p}} \lambda^B = \frac{\frac{p^G}{\bar{p}} - 1}{1 - \frac{p^B}{p^G}}, \quad (17)$$

and \widehat{I}^G is implicitly given by

$$\begin{aligned} \alpha^G p^G f'(\widehat{I}^G) - 1 &= \frac{p^G}{\bar{p}} \lambda^B \left[\alpha^B p^B f'(\widehat{I}^G) - \frac{p^B}{p^G} \right], \\ f'(\widehat{I}^G) &= \frac{1}{p^G \alpha^G - (\alpha^B - \alpha^G) \frac{\lambda^B}{\lambda^G} p^B}. \end{aligned}$$

It remains to be shown that this allocation is incentive compatible for G entrepreneurs, so that:

$$\alpha^G p^G f(\widehat{I}^G) - \widehat{I}^G - \widehat{T} \frac{p^G}{\lambda^B} \geq \alpha^G p^G f(I^{B*}) - \frac{p^G}{p^B} I^{B*} = \chi. \quad (18)$$

Suppose that this is not the case. This implies that, in the (T, I^G) space, the constrained optimal allocation $(\widehat{T}, \widehat{I}^G)$ lies below the constraint given by Equation (18). But this is only possible if, at some point, the constraint intersects the G isoprofit given by

$$\alpha^G p^G f(I^G) - I^G - T p^G = \alpha^G p^G f(\widehat{I}^G) - \widehat{I}^G - \widehat{T} p^G,$$

which can clearly never happen.

7.2 Loan Collateralization Scheme (Proposition 3)

Under the loan-collateralization scheme, the planner's optimization problem may be written as follows,

$$\max_{I^G, I^B, T} \alpha^G p^G f(I^G) - I^G + \left(1 - \frac{p^G}{\bar{p}}\right) W_T,$$

subject to the incentive compatibility constraints

$$\alpha^B p^B f(I^G) - \frac{p^B}{p^G} I^G - \left(1 - \frac{p^B}{p^G}\right) W_T \leq \alpha^B p^B f(I^B) - r I^B, \quad (19)$$

$$\alpha^G p^G f(I^G) - r I^G \geq \alpha^G p^G f(I^B) - \frac{p^G}{p^B} I^B, \quad (20)$$

Once again, it is immediate to see that $\widehat{I}^B = I^{B^*}$. Using β to denote the multiplier on the incentive compatibility constraints, the first order conditions with respect to I^G and T are:

$$\alpha^G p^G f'(\widehat{I}^G) - 1 = \beta \left[\alpha^B p^B f'(\widehat{I}^G) - \frac{p^B}{p^G} \right], \quad (21)$$

$$-\left(1 - \frac{p^G}{\bar{p}}\right) + \beta \left(1 - \frac{p^B}{p^G}\right) = 0, \quad (22)$$

where, in the last equality, we already incorporate the fact that any solution must entail $\widehat{W} > 0$. This can be verified through the same arguments we have invoked in the previous section. Hence, in the optimal solution,

$$\beta = \frac{\frac{p^G}{\bar{p}} - 1}{1 - \frac{p^B}{p^G}} = \frac{p^G}{\bar{p}} \lambda^B, \quad (23)$$

and \widehat{I}^G is implicitly given by

$$f'(\widehat{I}^G) = \frac{1}{p^G \alpha^G - (\alpha^B - \alpha^G) \frac{\lambda^B}{\lambda^G} p^B}.$$

That, in this allocation, the incentive compatibility constraint of good entrepreneurs is slack follows directly from observing that $\widehat{I}^G > I^{B^*}$.

7.3 Feasibility of the loan-collateralization scheme

It needs to be shown that entrepreneurs with bad projects, who benefit from this scheme in expected terms, have the resources to pay back the first-stage loan in the event that they are successful. Formally, this implies that,

$$\alpha^B f(I^{B^*}) - \frac{1}{p^B} \cdot I^{B^*} + \widehat{W} \cdot \left(1 - \frac{1}{\bar{p}}\right) \geq 0.$$

Replacing the expression for \widehat{W} from the incentive compatibility constraint of bad entrepreneurs, it can be shown that a sufficient condition for this inequality to be satisfied is that

$$\frac{\alpha^B f(I^{G*}) - \frac{1}{p^G} \cdot I^{G*}}{\alpha^B f(I^{B*}) - \frac{1}{p^B} \cdot I^{B*}} \leq 1 + \frac{1 - \frac{p^B}{p^G}}{\frac{p^B}{\bar{p}} - p^B}.$$

7.4 Existence of an Efficient Equilibrium (Proposition 4)

As we mention in the main body of the paper, there are clearly no profitable deviations for entrepreneurs: given the contracts offered, their optimal strategy is to apply to all of the non-exclusive contracts and – in the case of good entrepreneurs – to pledge these resources as collateral when applying to the exclusive contracts. Therefore, any deviation must emerge from contract designers themselves.

Given the contracts posted by exclusive banks, non-exclusive intermediaries have no profitable deviations. Any contract posted by them with an interest rate lower than $(1/p^B)$ would necessarily attract bad entrepreneurs, since it would be costless for them to obtain the implied cross-subsidization. In the best of cases, then, they can hope to attract all entrepreneurs by offering contracts with an interest rate of $(1/\bar{p})$. But, in the efficient equilibrium, good entrepreneurs are already obtaining all the funds that they desire at this rate, and so no profitable deviations exist.

The subtle part of the argument therefore lies on the possible deviations of exclusive banks. Can they design a contract (I_1, R_1, W_1) that manages to attract only good entrepreneurs by requiring to borrow less in the non-exclusive market? Formally, any such deviation must satisfy

$$\alpha^G p^G f(I_1^G) - I_1^G > \alpha^G p^G f(\widehat{I}^G) - \widehat{I}^G + \left(1 - \frac{p^G}{\bar{p}}\right) \cdot (\widehat{W} - W_1), \quad (24)$$

and

$$\alpha^B p^B f(I_1^G) - \frac{p^B}{p^G} I_1^G - \left(1 - \frac{p^B}{p^G}\right) W_1 \leq \alpha^B p^B f(I^{B*}) - I^{B*}, \quad (25)$$

where the last equation represents the incentive compatibility constraint of bad entrepreneurs. Crucially, \widehat{W} does not appear in this constraint since, should they choose to apply to the deviating contract, bad entrepreneurs can still raise this amount in the non-exclusive market while only pledging a fraction (W_1/\widehat{W}) of it in the exclusive market.

Taking the efficient equilibrium as a starting point, any effective deviation must therefore satisfy

$$\frac{\alpha^G p^G f'(I^G) - 1}{\frac{p^G}{\bar{p}} - 1} = \frac{dW}{dI^G} \Big|_{\pi^G=0} \leq \frac{dW}{dI^G} \leq \frac{\alpha^B p^B f'(I^G) - \frac{p^B}{p^G}}{1 - \frac{p^B}{p^G}} = \frac{dW}{dI^G} \Big|_{\pi^B=0},$$

which is impossible, since

$$\frac{\alpha^G p^G f'(I^G) - 1}{\frac{p^G}{\bar{p}} - 1} \geq \frac{\alpha^B p^B f'(I^G) - \frac{p^B}{p^G}}{1 - \frac{p^B}{p^G}}, \quad (26)$$

for all $I^G \leq \widehat{I}^G$. At the constrained optimal allocation, Equation (26) holds with equality, whereas the inequality is strict whenever $I^G < \widehat{I}^G$. This proves that there exist no profitable deviations for exclusive banks, so that the constrained optimal allocation is indeed an equilibrium.

To see that this relies on the fact that it is impossible to monitor trades in the non-exclusive market, consider that this was not the case. Then, exclusive banks could design deviations in which entrepreneurs are required to borrow no more than W_1 in the non-exclusive markets. This would change the relevant incentive compatibility constraint for bad entrepreneurs to

$$\alpha^B p^B f(I_1^G) - \frac{p^B}{p^G} I_1^G + \left(\frac{p^B}{p^G} - \frac{p^B}{\bar{p}} \right) W_1 \leq \alpha^B p^B f(\widehat{I}^G) - \frac{p^B}{p^G} \widehat{I}^G + \left(\frac{p^B}{p^G} - \frac{p^B}{\bar{p}} \right) \widehat{W}, \quad (27)$$

since by choosing to follow good entrepreneurs they would now have to decrease their non-exclusive borrowing by $(\widehat{W} - W_1)$. Hence, any effective deviation would need to satisfy

$$\frac{\alpha^G p^G f'(I^G) - 1}{\frac{p^G}{\bar{p}} - 1} = \frac{dW}{dI^G} \Big|_{\pi^G=0} \leq \frac{dW}{dI^G} \leq \frac{\alpha^B p^B f'(I^G) - \frac{p^B}{p^G}}{\frac{p^B}{\bar{p}} - \frac{p^B}{p^G}} = \frac{dW}{dI^G} \Big|_{\pi^B=0},$$

which is indeed possible at the constrained optimal allocation.