Economics Working Paper 107

Human Capital, Heterogeneous Agents and Technological Change*

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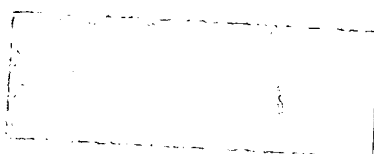.March 1995

Abstract

Technological change, the distribution of acquired skills and income inequality are determined by the human capital investment decisions of agents endowed with different abilities. Education entails a time to build process that enables workers to irreversibly shift from unskilled to skilled categories. A larger potential supply of skilled labor has a market size effect that encourages human capital investments. An increase in the skill premium leads to skill acquisition; the expanded skilled labor supply partially offsets the initial premium rise. Policies that impose too high minimum schooling requirements, overly long schooling periods, overexpanded bureaucracy, an inefficient number of teachers, and a distorted allocation of teachers between primary and higher education can permanently slow down growth.

1 Introduction

The literature on endogenous economic growth emphasizes two main features that can explain long run growth: human capital accumulation and technological change. Yet it is difficult to find models in which these two aspects interrelate in a way that fits the actual experience of labor markets with workers that differ in skill levels generating an unequal income distribution. Most models in which human capital is accumulated endogenously (Uzawa, 1965; Lucas, 1988) are based on the notion of an infinitely-lived representative agent. Workers can accumulate human capital ad infinitum through the continuous allocation of time to education instead of going to work. The pioneering model of endogenous innovation (Romer, 1990) incorporated a key distinction between unskilled and skilled labor but took as exogenous the amounts of skilled and unskilled labor in the economy.

Previous work has often yielded results that minimize the human capital accumulation effect of economic policies and changes in the environment. For instance, Grossman and Helpman (1991) obtain that higher wages or skilled labor wage premia do not strengthen the incentives for skill acquisition. Recent work by Stokey (1994a, 1994b), Ciccone (1994) and Eicher (1993) has introduced endogenous skill acquisition in growth models with homogeneous agents. In these models, agents have the same discounted value of utility so that there are no welfare redistribution effects from economic policies or growth.

Stokey (1994a, 1994b) analyzed transitional dynamics in a model with agents that acquire skills by spending goods on education. Formally, she develops a two capital-goods model in which human capital is distinguished from physical capital because goods invested in human capital accumulation are assumed to have decreasing returns in generating human capital but there are no decreasing returns in the physical capital investment process. The presence of decreasing returns puts a limit to human capital accumulation so that there is no sustained growth in the steady state.

Ciccone (1994) develops a physical and human capital model in which increasing returns in the average level of human capital arises through endogenous comparative advantage. In this model, the individual return to human capital is increasing in the economy's average level of human capital because of the interdependence between this average level and the rate of technological change. There is neither a separate research sector nor a

patents market. Eicher (1993) develops an overlapping generations model of endogenous human capital investment through a privately-financed university system. The model yields a positive effect of population (market size) on human capital accumulation and economic growth because a larger market size augments the pool of funds available to finance education.

This paper develops a Romer-styled model of product innovation that focuses on the interaction between human capital and the commercial research sector. In our framework, heterogeneous agents differ in abilities and are differentiated ex post in terms of skills acquired through education. This allows us to analyze within-country income distribution effects and the role of the introduction of an explicit educational sector. The rate of economic growth depends on the potential supply of skilled labor rather than on population size or exogenously-given amounts of human capital. Public education policies can lead to a resource allocation pattern in which additional human capital can reduce the long run growth rate.

Skill acquisition decisions entail comparing total schooling costs with the discounted value of future wages. Following Bailén (1994), the skill acquisition decision is related to the ratio of skilled-to-unskilled labor wages and a parameter representing individual abilities. In equilibrium, skilled workers will be those with greater abilities that justify bearing the foregone opportunity costs from attending school during a discrete period of time. The model puts a cap on human capital accumulation through finite lifetime. Each generation is forced to reconstruct the level of human capital achieved by its predecessors by spending time at school. Intertemporal utility maximization is motivated through an inheritance motive à la Barro and Becker.

Human capital heterogeneity is captured by focusing on the decision to educate or not to educate made by heterogeneous workers rather than on how much time they spend at school. The intuition of the results obtained from models with homogeneous workers that hinge on the optimal amount of time spent at school is that schooling time is insensitive to such changes as variations in the skill premium (Findlay and Kierzkowski, 1983). This intuition is consistent with historical evidence. Historically, and internationally, time spent at school has been relatively constant -a University degree takes four or five years, a Ph.D. around five. On the other hand, these models do not capture the wide cross sectional disparity between the shares of the population in each acquired skill class. A Haitian Ph.D. has spent a similar amount of time at school and has incurred in similar human capital invest-

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ments as a Japanese or German one. It is only that there are so many less Ph.D.s in Haiti.

Our model takes the period of human capital investment as exogenously given, so that it plays the role of a fixed cost that causes education not to be optimal for the less able or disadvantaged (the optimal schooling period is obtained in the Appendix). Once heterogeneous workers have the possibility of shifting from unskilled to skilled categories, the previously mentioned perplexing results on human capital and growth disappear. Higher skill premia will induce a larger set of workers to acquire greater skills, resulting in a higher level of human capital and faster growth.

The paper offers some explanations for the difficulty in rationalizing the lack of a consistent correlation between the level of human capital and the rate of economic growth (Lucas, 1993; Benhabib and Spiegel, 1994). The empirical evidence shows a long term increase in educational levels and time spent at school, but a corresponding increase in the economy's growth rate, as predicted by Lucas-Uzawa models, Romer-type and other popular growth models, is not always observed.

In our model, there are three effects derived from an increase of the time spent at school. First, as usually obtained, the productivity of skilled workers increases. Second, for a given life span, the fraction of the lifetime spent working declines, reducing the effective supply of skilled workers. Third, the opportunity cost of education increases with the time of specialization, reducing the proportion of workers that choose to become skilled. The overall effect on human capital derived from these combined factors is ambiguous.

If we introduce an explicit education sector (as in Sections 5-6) there is an additional factor explaining the nonmonotonicity of the relation between human capital and growth. This factor relates to the trade off arising from the necessity of setting aside some human capital (teachers and bureaucrats which supports educators' tasks) to create new human capital. A greater number of teachers increases the stock of human capital but not necessarily the human capital devoted to the research activities that generate growth in this model. Both primary and higher education teachers increase the productivity of high-skill workers. We obtain that the growth-maximizing proportion of basic and higher education teachers is determined by their relative effects on the productivity of high-skill workers.

The growth and human capital effects of the education bureaucracy hinges on its size. When bureaucracy is small relative to teachers, an increase in

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its size exerts a positive effect on human capital and growth. However, an overextended bureaucratic sector both reduces the effective supply of human capital and growth.

Sections 2 and 3 develop the basic model of research and time-to educate learning process. Section 4 analyses the economy's balanced growth path equilibrium. Section 5 extends the model to consider educational costs consisting of both students' time and educators' time, introduces a public education system and develops conditions under which an expanded educational sector can reduce growth. Section 6 considers an educational system consisting of both education bureaucrats and teachers and obtains a golden rule bureaucrat-teacher ratio. The conclusion reviews relevant empirical evidence and discusses possible extensions.

2 The Model: Production and Consumption Sectors

The model consists of two sectors: a monopolistically competitive manufacturing sector that produces differentiated consumption goods and a competitive research sector that develops the blueprints that differentiate consumption goods among themselves. There are three inputs: skilled labor, unskilled labor and technology. Growth is due to technological change produced in the R&D sector. Physical capital is not considered.

2.1 The research sector

The aggregate research production function is specified as a function of the stock of skilled labor, $H(t)$, and the knowledge derived from the productive experience in the sector which is assumed to be equal to the number of goods $N(t)$,

$$dN(t) = \frac{H(t)}{a}N(t)dt \qquad (1)$$

and $a > 0$ is (the inverse of) the research productivity parameter.

The increasing returns to scale research sector is composed of many competitive firms that design new commercial goods. All these firms are endowed with the same technology and utilize the stock of knowledge $N(t)$ as

a free good that is the source of external economies in research. The representative research firm sells infinitely-lived patent rights on a new good, obtaining a revenue P(n,t). Since knowledge is a free good, the total cost of blueprints equals $H(t)w_H(t) = a\frac{w_H(t)}{N(t)}\frac{dN(t)}{dt}$, where $w_H(t)$ is the skilled labor nominal wage rate, that will be shown to be constant along the balanced growth path. Hence, the unit cost of blueprints implied by (1) is given by $\frac{aw_H}{N(t)}$. The profits of the representative research firm R are thus given by $\Pi_R(n,t) = P(n,t) - \frac{aw_H}{N(t)}$. In order to specify the patents' price $P(n,t)$ and skilled labor wages we must examine manufacturing sector's wages and workers' skill acquisition behavior.

2.2 The manufacturing sector

The patents conferring the rights to commercialize a new good are purchased by a single manufacturing firm that operates in a monopolistically competitive market for differentiated goods. The representative firm produces a quantity $c(n,s)$ of consumption variety n at instant s. The amount of profits $\Pi(n,t)$ obtained from the commercialization of a good n invented at t is given by

$$\Pi(n,t) = \int_t^\infty e^{-\int_t^s r(\tau)d\tau}\pi(n,s)ds, \tag{2}$$

where $r(\tau)$ represents the instantaneous interest rate at time τ, and $\pi(n,s)$ represents firm's profits at time s.

The representative manufacturing firm utilizes unskilled labor $l(t)$ to produce output under a linear technology, which is shared by all manufacturing firms:

$$c(n,t) = l(t). \tag{3}$$

Profits at time s, $\pi(n,s)$ are given by

$$\pi(n,s) = p[c(n,s)]c(n,s) - w_L l(s), \tag{4}$$

where w_L is the unskilled labor nominal wage that will be shown to be constant along the balanced growth path and $c(n,t)$ is the quantity of good n sold at a price $p[c(n,t)]$. Since the linear technology (3) implies that the marginal cost is equal to w_L, the first order condition of the firms' profit maximization problem is:

$$p[c(n,t)] = \frac{\epsilon[c(n,t)]}{\epsilon[c(n,t)] - 1}w_L, \tag{5}$$

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where $\epsilon[c(n,t)]$ represents the elasticity of the demand for any good n. This pricing formula is the standard markup over marginal cost obtained under monopolistic competition.

2.3 Research and manufacturing linkages

The research sector produces blueprints through an increasing returns to scale production function of skilled labor and the stock of public knowledge derived from the development of new goods. It is assumed that the stock of public knowledge capital derived from learning is equal to the economy's cumulative experience in developing new goods. This implies the existence of knowledge spillovers in research.

The manufacturing sector utilizes only one input, unskilled labor, and has a linear production function, which serves to stress that our results do not hinge on increasing returns in manufacturing. The extreme assumption that the research sector does not use unskilled labor while manufacturing does not use skilled labor can be easily relaxed. In general, what is necessary for the results is that the research sector is appropriately more human capital intensive than the manufacturing sector. The model can be generalized to incorporate the existence of spillovers from the research to the manufacturing sector (as in Romer, 1990), or learning within manufacturing (Young, 1993).

In Romer (1990), the allocation of skilled labor between manufacturing and research plays a key role in determining the growth rate. In our model, a similar role is played by the allocation of workers between unskilled and skilled categories via schooling. Education provides an indirect mechanism for interindustry factor reallocation.

2.4 Consumers' behavior

There is a given number of finitely-lived consumers that share the same altruistic preferences in which offspring's welfare enters into the utility function. Preferences are thus represented by the following infinite horizon utility function:

$$U(t_0) = \int_{t_0}^{\infty} e^{-\rho(t-t_0)} ln D(t) dt, \tag{6}$$

where $\rho > 0$ is the intertemporal discount factor applying to a consumer and its offspring's utility. D(t) is a differentiated goods' utility index, given by

$$D(t) = [\int_0^{N(t)} c(n,t)^\alpha dn]^{1/\alpha}, \tag{7}$$

where $N(t)$ is the amount of varieties available at t and $0 < \alpha < 1$ is a preferences' parameter. This index implies that the elasticity of substitution between any two goods is $\epsilon(n,n') = 1/(1-\alpha)$. The elasticity of substitution is greater than one but lower than infinity meaning that goods are imperfect substitutes. Consumer goods enter symmetrically in the utility index and have the same cost function, so that they will be consumed in equal amounts c(t) and $D(t) = N(t)^{\frac{1-\alpha}{\alpha}} N(t)c(t)$.

Notice that for any given aggregate consumption amount, $N(t)c(t)$, increases in the available variety $N(t)$ imply greater consumers' welfare. In the steady state, the level of aggregate consumption output remain constant, and growth takes place as a result of increases in the utility derived from growing levels of the variety term $N(t)^{\frac{1-\alpha}{\alpha}}$.

The intertemporal aggregate budget constraint is given by

$$\int_{t_0}^\infty e^{-\int_{t_0}^t r(s)ds} E(t)dt \leq A(t_0), \tag{8}$$

where

$$E(t) = \int_0^{N(t)} p(n,t)c(n,t)dn$$

represents nominal expenditures at t, and $A(t_0)$ is the present value of the stream of factor incomes, plus the value of initial asset holdings.

The consumers' intertemporal maximization problem can be decomposed into two stages. In the first, households maximize static welfare by allocating their budgets among available consumer goods, given the expenditure $E(t)$. In the second stage, consumers determine the flow of expenditure that maximizes their intertemporal welfare. The solution of the static allocation problem is given by

$$c^d(n,t) = \frac{p(n,t)^{-\epsilon}E(t)}{\int_0^{N(t)} p(n',t)^{1-\epsilon}dn'} = \frac{E(t)}{N(t)p(t)}, \tag{9}$$

where the last equality arises because of the symmetry of costs and the utility index (7), which implies that prices and consumption amounts must be the same for all goods.

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The solution of the intertemporal maximization problem implies that the stream of nominal expenditures follows

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho.$$

In this model it is convenient (see Grossman and Helpman 1991) to normalize prizes so that the nominal expenditure remains constant throughout time. Using the previous expression, we obtain:

$$\frac{\dot{E}(t)}{E(t)} = 0,$$

or

$$r(t) = \rho. \tag{10}$$

Furthermore, consumption goods' prices are normalized so that nominal expenditure is always $E(t) = P_D(t)D(t) = P_D(t)N(t)^{\frac{1-\alpha}{\alpha}}N(t)c(t) = 1$, where $P_D(t)$ is an ideal price index associated with the consumption goods index $D(t)$. Thus, in this set up nominal expenditures serve as the economy's numeraire. In growth equilibria, real expenditure will keep expanding which implies that there is an accompanying price deflation, $-g_P = \frac{1-\alpha}{\alpha}g_N$. The real interest rate equals the rate of time preference ρ minus the inflation rate g_P ($\rho - g_P > \rho$). Notice also that in growth equilibria there are savings even if there is no capital and all currently produced goods are consumed. Savings take place through increases in the value of firms' shares all of which are owned by consumers.

3 Heterogeneous Workers' Skill Acquisition Decisions

An individual's career decision entails the dichotomous choice of whether to participate in the labor market as an unskilled or skilled worker. Those individuals who decide to be unskilled work all their life. If they choose to be skilled, they must devote a period S of their lives for skill acquisition. We assume that the schooling period is given and common for all workers that choose to be skilled.

In Sections 3 - 4 we consider the self-schooling case, in which only student's own time enters as input into education. In Section 5, we extend our analysis to consider educators' time costs besides students' time costs. There is a complete credit market that finances students' consumption but there are no explicit parental investments in children as in Becker, Murphy and Tamura (1990). Leisure, on the job training, and retirement are not considered in this paper. There are no intergenerational externalities in the form of spillovers from present generation's education to future generations' productivities, such as that studied by Stokey (1991).

The economy's population, denoted by M, is assumed to be uniformly distributed on the age interval $[0, T]$. This assumption implies a given life expectancy and zero population growth. Along the balanced growth path equilibrium, labor market age structure will also remain constant. The measure of the individuals that exit the labor market equals the measure of new labor market entrants that just finished school. The latter, in turn are compensated for by a new generation entering schooling. The economy's labor market structure remains unchanged even if the identities of workers change all the time. We proceed to search for the labor market equilibrium for this stationary population structure.

Agents are assumed to be heterogeneous with respect to their educational achievement, that depends on their individual abilities. The ability is assumed to be dynasty-specific and publicly known. Figure 1 describes the distribution of skills in the population. There is an interval $[0, i^+]$ of individual abilities, and the density of individuals with a given parameter i is constant and equal to m. The total population is $M = mi^+$. For any given cohort, of size M/T, there is also a uniform distribution of the parameter $i \in [0, i^+]$ among the cohort's members. The density of individuals from a given cohort endowed with the productivity parameter i is thus $m/T = M/(i^+T), \forall i$.

The decision on specialization depends on the innate ability parameter because the benefits from education are taken to be a function of that parameter. If an individual endowed with the ability parameter i chooses to be skilled, she or he obtains a wage income $w_H i^\beta h(S)$ between times $t+S$ and $t+T$. The parameter $\beta \in (0, 1)$, assumed to be common for all individuals, could be interpreted as representing the contribution of social environment to earning power. The function $h(S)$ represents the contribution of S schooling years to wage earnings. If a worker does not choose to acquire skills, she or he

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obtains a wage income w_L, independent of his innate productivity parameter i. This formulation implies that the productivity parameter is only effective if the worker chooses to acquire skills. All unskilled work is homogeneous but skilled workers are heterogeneous, both with respect to unskilled workers and among themselves.

Initially, we assume that the only cost of skill acquisition is students' time devoted to education (the case of spending teachers' time in education is treated in Section 5). In this case, an individual endowed with an ability parameter i chooses to specialize if the discounted skilled labor wage income obtained in the period $T - S$ is greater than the unskilled labor wage income obtained working during the entire duration T of her or his life. Algebraically, schooling is chosen if:

$$\int_{t+S}^{t+T} e^{-\rho(s-t)} w_H i^\beta h(S) ds \geq \int_t^{t+T} e^{-\rho(s-t)} w_L ds. \qquad (11)$$

This condition implies that the ability parameter of skilled workers must be greater than i^*, where i^* verifies

$$i^* = \left[\frac{1 - e^{-\rho T}}{h(S)[e^{-\rho S} - e^{-\rho T}]}\right]^{1/\beta} \left[\frac{w_L}{w_H}\right]^{1/\beta} \equiv \left[\frac{G(S)}{h(S)}\right]^{1/\beta} \left[\frac{w_L}{w_H}\right]^{1/\beta}. \qquad (12)$$

The optimal i^* is obtained by equating the effective skilled wage, $h(S) w_H i^\beta$, with the foregone unskilled wages, $G(S) w_L$, adjusted for the different unskilled and skilled labor working periods, T and T-S (see Figure 2).

Because the foregone unskilled wage opportunity cost $G(S) \equiv \frac{1-e^{-\rho T}}{e^{-\rho S}-e^{-\rho T}} >$ 1, we can easily verify that, in equilibrium, a positive supply of skilled labor implies $w_H i^\beta h(S) > w_L$. That is, the wage income of skilled workers must exceed the income of unskilled workers in equilibrium. Notice, also, that individuals with low enough i are more productive as unskilled than as skilled labor ($w_H i^\beta h(S)$ is less than w_H).

The supply of unskilled labor is given by the measure of the individuals with an ability parameter lower than the critical level i^*, that is, workers with $i \in [0, i^*]$. Since the population is uniformly distributed on this interval, the aggregate supply of unskilled labor is $L^* = m[i^* - 0] = mi^*$. Using (12), we obtain:

$$L^s = mi^* = m\left[\frac{G(S)}{h(S)}\right]^{1/\beta} \left[\frac{w_L}{w_H}\right]^{1/\beta}. \qquad (13)$$

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The workers that decide to become skilled are those whose ability indexes exceed the critical level i^*, that is, those workers endowed with an ability parameter $i \in [i^*, i^+]$. The skilled labor supplied by any working cohort is simply the human capital contributed by the $m[i^+ - i^*]/T$ among them that achieved schooling. Since skilled individuals are heterogeneous in terms of effective labor supplied, the working cohort supply equals $\frac{m}{T} h(S) \int_{i^*}^{i^+} i^\beta di$. The economy's supply of skilled labor is obtained by adding up the supply of human capital over the $T - S$ working cohorts:

$$H^S = \frac{T - S}{T} mh(S) \int_{i^*}^{i^+} i^\beta di = M^+(S) - u(S) \frac{mh(S)}{\beta + 1} [\frac{G(S)}{h(S)}]^{\frac{\beta+1}{\beta}} [\frac{w_L}{w_H}]^{\frac{\beta+1}{\beta}}$$

(14)

where $M^+(S) = \frac{T-S}{T} m \frac{(i^+)^{\beta+1}}{\beta+1} h(S)$ is the supply of skilled labor if all workers choose to acquire skills and $u(S) = \frac{T-S}{T}$ is the fraction of time devoted to work by skilled workers. Since S is the only policy variable, we do not write the remaining arguments m, T, i^+ and β, of the function $M^+(\cdot)$. We will take $M^+(S)$ as a parameter representing the potential skilled labor market size in the economy.

The potential supply of skilled labor $M^+(S)$ is obtained by determining the effective supply of the economy's skilled labor if all workers become skilled. The value of $M^+(S)$ depends on the population density m, the parameter i^+ (which is related to average ability $i^+/2$), the social environment parameter β, the fraction of time devoted to productive activities by skilled workers, $u(S) = \frac{T-S}{T}$, and the productivity effect of devoting S years to schooling, h(S). Notice that the potential supply of skilled labor $M^+(S)$ is not merely a function of the population size parameter m but also of labor market structure and the productivity of education.

Recall that population is uniformly distributed on the age interval $[0,T]$, that skilled workers are employed a fraction $u(S) = \frac{T-S}{T}$ of their lives, and that in the steady state the skill structure of the population is constant. Therefore, we can sustain a steady state in which neither the ratio w_H/w_L nor the skilled workers ability interval $[i^*, i^+]$ vary. In this stationary equilibrium, the aggregate supply of skilled labor will be greater the higher the wage ratio w_H/w_L. Next section pins down the equilibrium relative wage rate and determines the critical ability level i^*, the steady state's unskilled and skilled labor supplies, and the associated growth rate.

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4 The Equilibrium

This section determines the balanced growth path equilibrium along which all the real variables grow at the same constant rate. Equation (1) implies that the growth rate of the number of goods is equal to $\frac{\dot{N}(t)}{N(t)} = H/a = g$. In order to compute g we need to specify the demand for skilled labor and utilize the labor market equilibrium conditions to specify the equilibrium value of human capital H.

4.1 Equilibrium in the Unskilled Labor and Consumption Goods Markets

The linear technology represented by the production function (3) implies that the demand for unskilled labor $L^d(t)$ is equal to the aggregate supply of manufacturing output. Because the supply of output must be equal to the demand for consumption goods, the equilibrium in the unskilled labor market follows immediately from the equilibrium in the consumption goods market.

Recall that, since goods are symmetric in terms of costs and preferences, all differentiated goods must be priced equally. Substituting the value of the elasticity $\epsilon[c(n,t)] = \frac{1}{1-\alpha}$ into the pricing equation (5) yields the common differentiated goods' price $p(n,t) = p(t) = w_L/\alpha$. Substituting this result into the demand function (9) and using the condition $E(t) = 1$, $\forall t$, we obtain that the consumption demand for each variety equals $c^d(t) = \frac{E(t)}{N(t)p(t)} = \frac{\alpha}{N(t)w_L}$, which also equals the demand for unskilled labor $l^d(t)$. Hence, the aggregate consumption demand is given by $C^d(t) = N(t)c^d(t)$, and is equal to the demand for unskilled labor $N(t)l^d(t)$:

$$L^d(t) = N(t)[\frac{\alpha}{N(t)w_L}] = \frac{\alpha}{w_L} \tag{15}$$

The unskilled labor market equilibrium condition is obtained by equating (15) to the supply of unskilled labor given by equation (13).

4.2 Equilibrium in the Patents Market

The sector that designs new goods sells infinitely-lived patent rights to manufacturers. In a perfectly competitive patents market, the patent price

$P(n,t) = P(t)$, $\forall n$, must equal the stream of profits $\Pi(t)$ earned by the monopolistically competitive manufacturing firm. Using equations (2), (3), (4), (10), and the pricing equation $\alpha p[c(n,t)] = w_L$, we obtain

$$P(t) = \int_t^\infty e^{-\rho(s-t)}(1-\alpha)p(n,s)c(n,s)ds.$$

Imposing the normalization $E(t)=1$ $\forall t$, we have $p(n,s)c(n,s) = \frac{1}{N(t)}$. As long as $dN(t)/dt > 0$, competition among research firms leads to an equilibrium in which the price of a patent $P(t)$ is equal to the cost of the blueprint, $\frac{aw_H}{N(t)}$:

$$\int_t^\infty e^{-\rho(s-t)}(1-\alpha)\frac{1}{N(s)}ds = \frac{aw_H}{N(t)}.$$

Using the balanced growth path equation for the number of varieties, $N(s) = N(t)e^{g(s-t)}$, and performing integration yields

$$\frac{(1-\alpha)}{(\rho+g)N(t)} = \frac{aw_H}{N(t)}. \qquad (16)$$

Notice the negative relationship between the skilled labor wage rate w_H and the growth rate g derived from the patents market equilibrium. An increase in w_H raises research costs, reducing the rate of blueprint creation $dN(t)/dt$ that satisfies the equality between discounted manufacturer's profits and the cost of developing a new variety. This negative relationship is depicted as curve $w_H{}^{patent}$ in Figure 3.

4.3 Equilibrium in the Skilled Labor Market

The demand for skilled labor $H^d(t)$ is derived from the research sector technology expressed by (1). The constant growth rate condition $\frac{\dot{N}(t)}{N(t)} = g$ and the equality (16) yield

$$H^d(t) = a\frac{\dot{N}(t)}{N(t)} = ag = \frac{(1-\alpha)}{w_H} - a\rho. \qquad (17)$$

The supply of skilled labor is given by (14) and depends positively on the relative wage ratio w_H/w_L. The equilibrium in this market can be calculated

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by equating the supply of skilled labor $H^s(t)$ and the demand $H^d(t)$ in (14) and (17)):

$$H^S = M^+(S) - u(S)\frac{mh(S)}{\beta+1}[\frac{G(S)}{h(S)}]^{\frac{\beta+1}{\beta}}[\frac{w_L}{w_H}]^{\frac{\beta+1}{\beta}} = ag = H^d. \qquad (18)$$

4.4 The Equilibrium Growth Rate

In order to solve for the growth rate along the balanced growth path, we substitute the unskilled labor market equilibrium condition $L^s = m[\frac{G(S)}{h(S)}]^{\frac{1}{\beta}}[\frac{w_L}{w_H}]^{\frac{1}{\beta}}$ $= \frac{\alpha}{w_L} = L^d$ into the supply of skilled labor (14), obtaining

$$M^+(S) - \frac{u(S)G(S)\alpha}{(\beta+1)w_H} = ag. \qquad (19)$$

This equation, depicted by the $w_H{}^{labor}$ curve in Figure 3, shows the existence of a positive relationship between the skilled workers' wage w_H and the growth rate g compatible with the labor market equilibrium condition (notice that we combined the skilled and unskilled labor market equilibrium conditions to obtain (19)). Higher skilled labor wages raise workers' incentives to become skilled, increasing the supply of skilled labor, the amount of labor available for the research sector and, thus, the growth rate.

Figure 3 illustrates the determination of the growth rate resulting from the equilibrium conditions (16) and (19) in the patents and labor markets. There are two curves: the $w_H{}^{patent}$ curve depicts the negative relationship between the researchers' cost, w_H, and the growth rate. The $w_H{}^{labor}$ curve reflects the positive efect of skilled labor wages on the supply of human capital, and, hence, on growth. The intersection of the curves gives us the growth rate and skilled labor nominal wages (recall that real wages increase due to deflation).

Algebraically, the growth rate is given by:

$$g = \frac{(1-\alpha)(\beta+1)M^+(S) - u(S)G(S)\alpha a\rho}{a[(1-\alpha)(\beta+1) + u(S)G(S)\alpha]}, \qquad (20)$$

where $M^+(S)$ is equal to $u(S)m\frac{(i^+)^{\beta+1}}{(\beta+1)}h(S)$.

The endogenously determined growth rate depends positively on the potential supply of skilled labor $M^+(S)$. In the pioneering models of endogenous

technological change, the relevant variable was the population size (Grossman and Helpman, 1991) or the exogenous supply of human capital (Romer, 1990). In this model, human capital is endogenously supplied, and depends on the population size parameter m, but also on the labor market participation rate $u(S) = \frac{T-S}{T}$, the opportunity cost of becoming skilled $G(S) = \frac{1-e^{-\rho T}}{e^{-\rho S} - e^{-\rho T}}$, the productivity of education $h(S)$, and other parameters such as those representing population capabilities i^+ and the social environment β.

The growth rate depends on the structure of the labor market. A longer specialization period S has three effects. First, a positive growth effect derives from an increase in skilled workers' productivity h(S). Second, a negative growth effect results from the reduction of the fraction of time devoted to work by each skilled worker, $u(S) = \frac{T-S}{T}$. Finally, an increase in the opportunity cost of acquiring skills, G(S), also has a negative growth effect. The reason is that a higher schooling opportunity cost implies an increase in the critical ability parameter i^* and induces a reduction in the number of individuals who choose to become skilled, $m[i^+ - i^*]$. The impact on growth of an increase in the specialization period is ambiguous because it depends on the overall result derived from the interaction of these three different effects.

The growth rate depends negatively on the preferences' elasticity parameter α, because a higher α means less love for variety and implies that firms can charge a lower markup on the marginal costs of manufacturing a good. The rate of technological change depends negatively on the research cost parameter a, and is negatively related to the discount rate ρ. These properties and their intuition are the familiar ones in the endogenous growth literature.

5   Can Higher Educational Expenditures Reduce Growth?

This Section introduces the existence of a universal public education system. We will show that, even if educational expenditures improve workers' productivity, it is possible to obtain an equilibrium in which augmenting the resources allocated to education implies more human capital but slower growth.

We take government education policies as exogenously determined. Bailén

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and Rivera-Batiz (1995a) study optimal educational policies under Ramsey wage taxation. They compare the growth and welfare properties of this solution with the results obtained under an equilibrium in which education is privately financed.

We extend the previous model in two directions. First, teachers' time is considered as an input into education besides students' time. This implies a trade-off between human capital devoted to research and teaching. Second, two different levels of education are introduced: basic and higher levels.

The economy's labor market is described as follows. All workers are legally bound by law to stay a minimum period S_1 of their lifetimes at school. Notice that this legal requirement introduces a distortion because very low ability agents are forced to undertake minimum schooling that they may not voluntarily choose. Workers can decide between this minimum period of schooling (that enables them to work in the manufacturing sector, and earn a wage rate w_L per unit of basic-skill labor), or to stay an additional period of time at school, so that the amount of education years is $S_2 > S_1$.

5.1   Education Spending and Growth

Workers who choose the higher educational level can work as teachers or researchers. The government decides the number of teachers in the basic and higher education levels, $H_E{}^1$ and $H_E{}^2$, and finances the whole educational system through lump-sum taxes. The productivity of basic-skill workers increase with the amount of basic school level educators in a proportion $f(H_E{}^1)$, whereas the productivity of high-skill workers depends on basic and high education teachers in a proportion $f(H_E{}^2, H_E{}^1)$ (high-skill workers receive both basic and higher level education). We assume, as before, that the productivity of workers in the manufacturing sector is independent of their individual ability parameter i. Thus, the workers who decide to stay the minimum period at school S_1 earn a wage $f(H_E{}^1)h(S_1)w_L$ between $t + S_1$, $t + T$; whereas the workers that choose the schooling period S_2 earn $f(H_E{}^2, H_E{}^1)w_H i^\beta h(S_2)$ between $t + S_2$ and $t+T$.

Combining the labor market clearing conditions with the equilibrium in the patent market (that requires $\frac{1-\alpha}{\rho+g} = a w_H$), we obtain that the long run growth rate of this economy is

$$g = \frac{\Gamma[f(H_E{}^2, H_E{}^1)M^+(S_2) - (H_E{}^1 + H_E{}^2)] - \Lambda\rho}{a\Gamma + \Lambda}, \qquad (21)$$

where $\Gamma = (1 - \alpha)(\beta + 1)u(S_1)$, $\Lambda = u(S_2)G(S_1, S_2)\alpha a$, and

$$G(S_1, S_2) = (e^{-\rho S_1} - e^{-\rho T})/(e^{-\rho S_2} - e^{-\rho T}).$$

An increase in the allocation of workers to the educational sector ($H_E{}^1$ or $H_E{}^2$) has two opposite effects on growth. First, the productivity of labor goes up, increasing the effective supply of high-skill workers. Second, the fraction of high-skill workers available for the research sector is smaller. The total effect on growth of a higher number of educators depends on the efficiency of public educational expenditures and the $M^+(S_2)$ component of potential high-skill labor.

It is easy to verify that an increase in the number of basic education teachers $H_E{}^1$ has a positive effect on growth only if the value of $M^+(S_2)$ is higher than $1/f_{H_E{}^1}(H_E{}^2, H_E{}^1)$. Analogously, an increase in the number of higher level educators has a positive growth effect if $M^+(S_2) \geq 1/f_{H_E{}^2}(H_E{}^2, H_E{}^1)$. If $M^+(S_2)$ is small enough (because either m or $h(S_2)$ is small), then for given educators' marginal productivities, $f_{H_E{}^1}(H_E{}^2, H_E{}^1)$ and $f_{H_E{}^2}(H_E{}^2, H_E{}^1)$, an increase in the number of educators reduces long run growth. This happens because, when m or $h(S_2)$ are small, the productivity effect on high-skill workers of raising the number of educators affects a lower number of workers or less efficient workers. Notice that this reduction in growth is compatible with an expansion in the effective supply of skilled workers. This result suggests the possibility of educational investments that could reduce the growth rate as depicted in Figure 4.

In this framework it is possible to obtain the relative proportion of basic to higher education teachers that maximizes growth. This proportion is obtained by equating the marginal productivities of both kinds of educators in the higher education level, $f_{H_E{}^2}(H_E{}^2, H_E{}^1) = f_{H_E{}^1}(H_E{}^2, H_E{}^1)$. Hence, basic school educators have a role on growth because of their effect on the productivity of high-skill labor. The growth-maximizing proportion of basic to higher education teachers depends only on their relative effect on the productivity of high-skill workers.

Finally, we have that an increase in the resources devoted to education entail income distribution effects. The initial effect of a greater amount of

educators is to raise the ratio of high-skill to low-skill labor wages. This raises the amount of workers who decide to acquire more skills. In the long run, the supply of high-skill labor augments and this partially offsets the initial greater wage premia. Since education entails a time to build process, the short run supply of high-skill labor remains constant at a level $H^s = \bar{H}$. Then the high-skill labor market equilibrium condition becomes $\bar{H} = \frac{1-\alpha}{w_H} - a\rho + H_E{}^1 + H_E{}^2$. Thus, the high-skill labor wage rate that clears this market is given by $w_H = \frac{1-\alpha}{\bar{H}-(H_E{}^1+H_E{}^2)+a\rho}$, which increases when the government raises the amount of educators. In the long run, the supply of high-skill labor H^s rises because of the higher skill premia. The increase in H^s partially offsets the initial rise in the skill premia (for the dynamics of this process, see Bailén and Rivera-Batiz (1995b)).

Education investments also have an output level effect because the productivity of all workers -and, in particular, the productivity of basic-skill workers- increases with the number of basic education teachers. Thus, an expansion in the number of basic education teachers leads to an increase in the supply of consumption output, even in the case in which the amount of variety grows at a slower rate. The positive output level effect means that basic education investments are not necessarily negative from the welfare's point of view, even if they decrease the long run growth rate.

5.2 Minimum Schooling Requirements

Another instrument of public education policies is the requirement of a minimum schooling period. For many countries, the fixing of minimum schooling requirements is key to their educational policies. In this framework, we obtain that an educational policy consisting of raising the minimum schooling period S_1 has ambiguous consequences for the stock of human capital and the economy's growth rate. Laws establishing minimum requirements have two opposite growth effects. First, the opportunity cost $G(S_1, S_2)$ of becoming high-skill worker is reduced, inducing more workers to acquire skills. This is so because of the reduction in the additional education time length required to acquire more skills. The consequence is an increase in the supply of human capital and the economy's growth rate.

The second effect of raising S_1 is to reduce $(T - S_1)/T$ and thus the effective supply of basic-skill labor. As a consequence, the wage of basic-skill workers increases, reducing skill premia and the incentives for acquiring more

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skills.

The overall effect of a higher S_1 depends on the relative importance of the opportunity cost reduction and effective low-skill labor supply reduction. Notice that the increase in S_1 raises $h(S_1)$ but it has no growth effect because a higher $h(S_1)$ has two offsetting effects. First, a higher $h(S_1)$ raises the return $f(H_E^1)h(S_1)w_L$ of low-skill workers, which increases the incentives of remaining low-skilled relative to becoming high-skilled. Second, a higher $h(S_1)$ increases the supply of basic-skill labor, lowers the basic-skill labor wage rate w_L, and lowers the skill premia w_H/w_L. These two opposite effects cancel out against each other.

6 Education Bureaucracy

We extend the previous model by considering that the education process requires supporting personnel, coordinators and other administrators, that is, a bureaucratic structure. Education bureaucrats facilitate educators' tasks by increasing their productivity. However, education bureaucracy is subject to diminishing returns and absorbs part of the human capital stock, reducing the effective supply of educators and researchers. We find that there exists a critical ratio between bureaucrats and educators that maximizes growth. If government employs too many bureaucrats, growth will be negatively affected.

The total amount of high-skill labor in the public education sector, H_P, is assumed to be exogenously determined by the government. High-skill workers in the public sector can directly provide educational services as educators (H_E^1 and H_E^2) or can support teaching activities as education bureaucracy (H_B). The basic and high public education system productivities, $f(H_E^1, H_B)$ and $f(H_E^2, H_E^1; H_B)$, depend positively on the amount of high-skill workers employed as education bureaucrats. In particular, we assume that the productivity effect on high-skill workers is measured by $f(H_E^2, H_E^1; H_B) = b(H_B)(H_E^1 + H_E^2)$, where $b'(H_B) > 0$, $b''(H_B) < 0$. This implies that an increase in the number of educators raises the productivity of all workers in the economy. On the other hand, a larger bureaucracy is necessary to augment the marginal productivity of educators. To capture large bureaucracy inefficiency, we assume that teachers' support and coordination activities present diminishing returns, because marginal bureaucratic

activities are less important than primary ones.

Under the previous conditions, the economy's rate of technological change is given by

$$g = \frac{\Gamma[b(H_B)(H_E{}^1 + H_E{}^2)M^+(S_2) - (H_E + H_B)] - \Lambda\rho}{a\Gamma + \Lambda}.$$ (22)

We obtain that education bureaucrats have two opposite effects on growth. On one hand, a larger bureaucracy makes educators more productive, raises the potential supply of high-skill labor and leads to faster growth. On the other hand, a greater number of bureaucrats reduces the amount of high-skill labor that can be employed in education or in research. Next we proceed to discuss the relationship between bureaucrats and educators that maximizes long run growth.

6.1 A Golden Rule for Education Bureaucracy

Differentiating the growth expression (22) with respect to H_B, we obtain that the amount of bureaucrats that maximizes the growth rate must verify

$$b'(H_B) = \frac{1}{(H_E{}^1 + H_E{}^2)M^+(S_2)}.$$

Differentiating (22) with respect to the number of basic or high education teachers, we have

$$1 = \frac{1}{b(H_B)M^+(S_2)}.$$

Combining the previous conditions, we obtain that the growth-maximizing relationship between education bureaucrats and educators is

$$\frac{b(H_B)}{b'(H_B)} = H_E{}^1 + H_E{}^2.$$ (23)

In particular, if $b(H_B) = AH_B{}^\gamma$, this relationship becomes $H_B = \gamma(H_E{}^1 + H_E{}^2)$.

Equation (23) shows that the number of education bureaucrats that maximizes growth can be written as a function of the amount of educators

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$H_E = H_E{}^1 + H_E{}^2$. The ratio between education bureaucrats and educators depends only on the productivity of bureaucracy, and is independent of the rest of the economic variables, such as the market size or the periods of schooling S_1 and S_2. Notice that, for a given amount of high-skill labor employed in the education sector $\bar{H}_P$, $b(H_B)H_E = b(\bar{H}_P - H_E)H_E$. Differentiating with respect to H_E we obtain that the ratio between education bureaucracy and educators that maximizes growth also maximizes the effective supply of human capital. This result suggests that an increase in the amount of high-skill labor employed in the public education sector (H_P) that overly raises the bureaucracy-teachers ratio can reduce the effective supply of human capital and slow down growth.

7   Conclusion

Modern growth theory places education and human capital formation as the central mechanism underlying economic expansion. This paper provides a microfoundation for human capital formation and technological change in a setting of heterogeneous workers. It models human capital accumulation that takes the form of an increase in the proportion of the workers that pay the fixed training costs of becoming skilled. High ability workers will find it profitable to invest in education while the less able will choose to remain unskilled. A larger potential supply of skilled labor has a positive market size effect. This effect raises the demand of skilled relative to unskilled labor and increases the wages of skilled labor relative to those received by unskilled workers. The higher wage premium effect augments the benefits of education and expands the ability range for which schooling is profitable. The consequent expansion in the stock of human capital provides a push to growth besides diffusion of ideas, scale effects in manufacturing, and other benefits of a larger market size.

This paper has emphasized a number of relationships that have interesting empirical counterparts. First, our model suggests that skill premia are positively correlated with investments in education. For instance, the South Korean experience shows that the ratio of the wages of college, university graduates and over to the wages of high school graduates gradually increased from 1.75 to 2.18 between 1971 and 1985, then declined to 1.92 in 1988. At the same time, the percentage of the population enroled in post-

secondary schools and universities rose more than six times (Rivera-Batiz, 1995). In addition, Westphal et al. (1985) reports that the number of Korean scientists and engineers increased from 6.9 per million population to 22 in the 1970s. Second, not only the level of human capital but also its composition and structure matters. Data gathered by Westphal et al. show that, in 1978, the ratio of engineering students to total post secondary education population was 26 percent in rapidly-growing Korea, while it was below 15 percent in Argentina, Brazil and Mexico. Murphy, Shleifer and Vishny (1991) provide evidence that the allocation of talent towards low-efficiency activities can slow down growth. Third, the model and empirical evidence provided by Benhabib and Spiegel (1994) suggest that there is no clear monotonic relationship between the level of schooling and growth. The seemingly improved schooling levels do not seem to entail higher rates of economic growth in most advanced countries and others like Philippines and many Latin-American countries.

The analysis suggests that it is possible to obtain efficiency gains and faster growth by reducing overextended schooling periods. In fact, Spanish educational reforms recently reduced the length of the higher education schooling period from five to four years. In the early nineties, a similar reform in Argentina reduced the length of many college careers from six to five years. These reforms were intended to augment the effective supply of human capital by improving the efficiency of the educational system.

We have stressed the role of education bureaucracies. Empirical evidence (see Hanusheck, 1992) shows that an expansion in the American education system has been compatible with a reduction in scores achievements. Average SAT scores declined from about 960 in 1966 to less than 900 in 1990. On the other hand, the ratio of education bureaucracy to instructional staff expenditures surged from sixty-four to over one hundred twelve per cent. Our model can help explain why an increase in the number of educators can be associated with educational performance decline when the bureaucracy-instructor ratio rises sufficiently.

The model has interesting implications for the dynamics of endogenous growth processes in which there are time lags arising from time spent at school. The dynamics hinge on the response of individuals to the observation of higher skilled premia. A greater number of workers will choose to go for schooling over remaining unskilled. Since workers need a period to specialize, an increase in the relative wage of skilled workers neither implies

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an immediate increase in the supply of skilled labor nor faster growth. The short run effect is to keep unchanged the supply of skilled workers and reduce the supply of unskilled workers, and hence output. The dynamics underlying this "austerity" period are analyzed in Bailén and Rivera-Batiz (1995b).

This paper has studied the determinants of the economy's potential for generating human capital. If growth works through human capital accumulation, the human capital effects studied here should be important. Common intuition, and experience, tell us that the economics of efficient and effective education provision hinge on the heterogeneous character of the labor force ex ante, and on the market forces that play a role in determining ex post heterogeneity. After all, it is largely by converting the masses of unskilled labor forces into skilled workers that education has worked to elevate levels of living and generate take offs in economic growth.

References

Bailén, José M.: "Basic Research, Product Innovation, and Growth," Economics Working Paper No. 88, Universitat Pompeu Fabra, (1994).

Bailén, José M., and Rivera-Batiz, Luis A.: "Education Policy, Talent Allocation, and Growth," *mimeo*, Universitat Pompeu Fabra, (1995a).

Bailén, José M., and Rivera-Batiz, Luis A.: "The Dynamics of An Endogenous Growth Model With Education Time Lags," Universitat Pompeu Fabra, in preparation, *mimeo*, (1995b).

Becker, Gary S., Murphy, Kevin M., and Tamura, Robert: "Human Capital, Fertility, and Economic Growth", *Journal of Political Economy*, vol. 98, No. 5, (1990): S12-S37.

Benhabib, Jess, and Mark M. Spiegel: "The Role of Human Capital in Economic Development. Evidence from Aggregate Cross-Country Data," *Journal of Monetary Economics* 34 (1994): 143-173.

Ciccone, Antonio: "Human Capital Accumulation, Endogenous Comparative Advantadge, and Technological Change", Universitat Pompeu Fabra and University of California, Berkeley, *mimeo*, November (1994)

Eicher, Theo S.: "Interaction between Endogenous Technological Change and Endogenous Human Capital," Columbia University, *mimeo*, June (1993).

Findlay, Ronald., and Kierzkowski, Henryk: "International Trade and Human Capital: A Simple General Equilibrium Model," *Journal of Political Economy* 91, No.6 (1983):957-978.

Grossman, Gene M.; and Helpman, Elhanan: *Innovation and Growth in the Global Economy*, Cambridge, Massachussets: MIT Press (1991).

Hanushek, Eric A., Rivkin, Steven G., and Jamison, Dean T.: "Improving Educational Outcomes while Controlling Costs," *Carnegie-Rochester Conference Series on Public Policy* 37 (1992): 205-238.

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Lucas, Robert E., Jr.: "On the Mechanics of Economic Development," *Journal of Monetary Economics* 22, No 17, (1988):3-42.

Lucas, Robert E.. Jr.: "Making a Miracle," *Econometrica* 61, No 2, (1993):251-272.

Murphy, Kevin M., Shleifer, Andrei., and Vishny, Robert W.: "The Allocation of Talent: Implications for Growth," *Quarterly Journal of Economics* 106, No 2 (1991):503-530

Rivera-Batiz, Luis A.: " Convergence and Labor Markets," *mimeo*, Universitat Pompeu Fabra, (1995)

Romer, Paul M.: "Endogenous Technological Change," *Journal of Political Economy* 98, No 5, (1990): S71-S102.

Stokey, Nancy L.: "Human Capital, Product Quality and Growth," *Quarterly Journal of Economics* 106 (1991):587-616.

Stokey, Nancy L.: "Free trade, Factor Returns, and Factor Accumulation," Working Paper, University of Chicago, (1994a).

Stokey, Nancy L.: "NAFTA and Mexican Development," *mimeo*, University of Chicago, (1994b).

Uzawa, Hirofumi.: "Optimum Technical Change in an Aggregative Model of Economic Growth," *International Economic Review* 6 (1965):18-31.

Westphal, L.E., Kim, L., and Dahlman, C.J.: "Reflections on the Republic of Korea's Acquisition of Technological Capability," in N. Rosenberg and C.J. Dahlman (eds), *International Technology Transfer*, New York: Praeger, (1985):167-221.

Young, Alwyn: "Invention and Bounded Learning by Doing," *Journal of Political Economy* 101, No 3, (1993):443-472.

Appendix I

Consumers' Problem

A. Static Problem

The representative consumer maximizes (6) subject to (8). The Lagrangian of the problem is

$$L = c^{\alpha} - \lambda[pc - E].$$

The first order conditions are

$$\frac{\partial L}{\partial c} = \alpha c^{\alpha-1} - \lambda p = 0 \rightarrow c(n,t) = [\frac{\lambda p}{\alpha}]^{-\epsilon} \qquad (24)$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \int_0^{N(t)} p(n,t)c(n,t)dn = E(t) \qquad (25)$$

Substituting (24) into (25) we obtain

$$E(t) = [\frac{\lambda}{\alpha}]^{-\epsilon} \int_0^{N(t)} p(n',t)^{1-\epsilon}dn' \Rightarrow$$
$$[\frac{\lambda}{\alpha}]^{-\epsilon} = \frac{E(t)}{\int_0^{N(t)} p(n',t)^{1-\epsilon}dn'} \qquad (26)$$

and substituting (26) into (24) we obtain the demand functions (9).

Dynamic Problem

Let $E(t) = P_D(t)D(t)$ be nominal expenditures, where $P_D(t)$ is a price index correspondent to D(t). Thus $lnD(t) = lnE(t) - lnP_D(t)$. Substituting this expression into (6) we determine the indirect utility function. Given the constraint (8), the Lagrangian of the problem is

$$L^* = e^{-\rho(t-t_0)}[lnE(t) - lnP_D(t)] - \lambda(t_0)[e^{-\int_{t_0}^t r(s)ds}E(t) - A(t_0)].$$

Differentiation with respect to E(s) and $\lambda(t_0)$ yields the first order conditions

$$\frac{e^{-\rho(t-t_0)}}{E(t)} = \lambda(t_0)e^{-[R(t)-R(t_0)]} \qquad (27)$$

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plus the budget constrain (8). Notice that R(t) - $R(t_0)$ is the solution of the interest rate integral. Differentiating logarithmically with respect to time t we have

$$\frac{\dot{E}(t)}{E(t)} = \dot{R}(t) - \rho = r(t) - \rho. \tag{28}$$

Appendix II

Human Capital Supply

A. Determination of i^*

A worker decides to become skilled if inequality (11) holds. The left side of the inequality is

$$\int_{t+S}^{t+T} e^{-\rho(s-t)} w_H h(S) i^\beta ds = \frac{e^{-\rho(s-t)}}{-\rho} \Big|_{t+S}^{t+T} w_H h(S) i^\beta$$

$$= [\frac{e^{-\rho S} - e^{-\rho T}}{\rho} w_H h(S) i^\beta].$$

The right side is given by

$$\int_{t}^{t+T} e^{-\rho(s-t)} w_L ds = [\frac{1 - e^{-\rho T}}{\rho} w_L].$$

Comparing both expressions and solving for i^*, we determine the expression (12).

Optimal Schooling Period S

Skilled workers choose the schooling period S that maximizes their discounted revenues, given by

$$\int_{t+S}^{t+T} e^{-\rho(s-t)} w_H h(S) i^\beta ds = [\frac{e^{-\rho S} - e^{-\rho T}}{\rho} w_H h(S) i^\beta].$$

Differentiating with respect to S, we obtain the first order condition

$$-\rho[\frac{e^{-\rho S}}{\rho} w_H h(S) i^\beta] + [\frac{e^{-\rho S} - e^{-\rho T}}{\rho} w_H h'(S) i^\beta] = 0$$

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which implies

$$\frac{h'(S)}{h(S)} = \frac{\rho}{1 - e^{-\rho(T-S)}}.$$

Notice that the previous result means that the optimal choice of S is independent of the individual ability parameter i. This happens because a greater i raises both the revenues and the opportunity costs (foregone effective wages) of schooling in the same proportion. Thus, we can determine the supply of skilled labor as a function of individuals' abilities, leaving the schooling period as exogenously determined by the government, that can choose S optimally (this fact does not change the measure of individuals that decide to acquire skills).

Appendix III

Self Education Case: the Growth Rate

Given equations (14) and (17) (that determine the supply and demand for human capital), the equilibrium in the market for skilled labor implies

$$M^+(S) - u(S)h(S)\frac{m}{\beta+1}[\frac{G(S)w_l}{h(S)w_h}]^{\frac{\beta+1}{\beta}} = \frac{(1-\alpha)}{w_h} - a\rho.$$

Using unskilled labor market equilibrium condition, (13) = (15), and substituting into the previous equation, the supply of human capital can be writen as

$$M^+(S) - \frac{u(S)h(S)}{\beta+1}\frac{G(S)w_l}{h(S)w_h}m[\frac{G(S)w_l}{h(S)w_h}]^{\frac{1}{\beta}} = M^+(S) - \frac{u(S)}{\beta+1}\frac{G(S)w_l}{w_h}\frac{\alpha}{w_l} =$$

$$= M^+(S) - \frac{u(S)G(S)}{\beta+1}\frac{\alpha}{w_h}.$$

The equilibrium condition in the patents market implies that the wage rate for the skilled workers is $w_h = \frac{1-\alpha}{a(\rho+g)}$. Substituting w_H into the previous equation, we obtain that the supply of human capital is equal to

$$M^+(S) - \frac{u(S)G(S)a(\rho+g)\alpha}{(1-\alpha)(\beta+1)}.$$

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The demand function for skilled labor is $H^d = a\frac{\dot{N}(t)}{N(t)} = ag$. Then, the equality between the supply and demand for human capital implies

$$M^+(S) - \frac{u(S)G(S)a(\rho + g)\alpha}{(1 - \alpha)(\beta + 1)} = ag.$$

Solving for g, we obtain the expression for the growth rate (20).

Appendix IV

Public Education Sector: Equilibrium Growth

The supply of high-skill labor is given by integration over the set of workers endowed with an ability parameter i greater than $(\frac{G(S_1,S_2)f(H_E^1)h(S_1)w_L}{f(H_E^2,H_E^1)h(S_2)w_H})^{1/\beta}$, where

$$G(S_1, S_2) = (e^{-\rho S_1} - e^{-\rho T})/(e^{-\rho S_2} - e^{-\rho T}).$$

The effective high-skill labor supply equals

$$u(S_2)f(H_E^2, H_E^1)h(S_2)m\int_{i^*}^{i^+} i^\beta di,$$

where $u(S_2) = \frac{T-S_2}{T}$. The demand for high-skill labor is the sum of the amount of high-skill labor allocated to the educational sector by the government, $H_E = H_E^1 + H_E^2$, and the amount of high-skill labor demanded by profit-maximizing research firms, given by (17). Consequently, the high-skill labor market will be in equilibrium if

$$f(H_E^2, H_E^1)[M^+(S_2) - u(S_2)\frac{mh(S_2)}{\beta + 1}(\frac{G(S_1, S_2)f(H_E^1)h(S_1)w_L}{f(H_E^2, H_E^1)h(S_2)w_H})^{\frac{\beta+1}{\beta}}]$$

$$= H_E^1 + H_E^2 + \frac{1 - \alpha}{w_H} - a\rho.$$

The potential supply of high-skill labor is $f(H_E^1, H_E^2)M^+(S_2)$, where $M^+(S_2)$ is equal to $u(S_2)mh(S_2)\frac{(i^+)^{\beta+1}}{\beta+1}$. The equilibrium condition for the basic-skill labor market is given by

$$\frac{T - S_1}{T}f(H_E^1)h(S_1)m(\frac{G(S_1, S_2)f(H_E^1)h(S_1)w_L}{f(H_E^2, H_E^1)h(S_2)w_H})^{1/\beta} = \frac{\alpha}{w_L}.$$

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From this equilibrium condition, we obtain

$$f(H_E)m(\frac{G(S_1,S_2)h(S_1)w_L}{h(S_2)w_H})^{1/\beta} = \frac{\alpha}{w_L u(S_1)h(S_1)}$$

where $u(S_1) = \frac{T-S_1}{T}$.

Using this condition, the supply of high-skill labor is given by

$$f(H_E)M^+(S_2) - f(H_E)\frac{u(S_2)m}{\beta+1}(\frac{G(S_1,S_2)h(S_1)w_L}{h(S_2)w_H})^{\frac{1+\beta}{\beta}} =$$

$$f(H_E)M^+(S_2) - f(H_E)\frac{u(S_2)m}{\beta+1}(\frac{G(S_1,S_2)h(S_1)w_L}{h(S_2)w_H})^{\frac{1}{\beta}}(\frac{G(S_1,S_2)h(S_1)w_L}{h(S_2)w_H}) =$$

$$f(H_E)M^+(S_2) - \frac{u(S_2)}{u(S_1)}\frac{\alpha}{w_L h(S_1)}(\frac{G(S_1,S_2)h(S_1)w_L}{h(S_2)w_H})$$

$$= f(H_E)M^+(S_2) - \frac{u(S_2)}{u(S_1)(\beta+1)}\frac{\alpha G(S_1,S_2)}{w_H}),$$

$(u(S_2)=\frac{T-S_2}{T})$. The demand for skilled labor is equal to $\frac{1-\alpha}{w_H} - a\rho + H_E^1 + H_E^2 = ag + H_E^1 + H_E^2$. Using the patents market equilibrium condition $w_H = \frac{1-\alpha}{\rho+g}$, substituing this condition into the supply of skilled labor and solving for g, we obtain the equilibrium growth rate expression (21).

Appendix V

The Dynamics

In this Appendix we examine the model's dynamic equilibrium equations. We obtain that the dynamics of the economy can be expressed by a system of difference-differential equations.

We begin with labor market equilibrium. An individual born in a period s and endowed with an ability parameter i decides to acquire skills if the discounted wages of skilled labor $W_H(s)$ are sufficiently high relative to the discounted wages of unskilled workers $W_L(s)$, that is, if

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$$i^*(s) \geq \frac{W_L(s)}{h(S)W_H(s)}^{1/\beta} = [\frac{\int_s^{s+T} e^{-\rho(u-s)} w_L(u) du}{h(S) \int_{s+S}^{s+T} e^{-\rho(u-s)} w_H(u) du}]^{1/\beta}. \quad (29)$$

The supply of skilled labor at time t is given by the number of workers born at time $s \in [t\text{-}(T\text{-}S), t]$ endowed with a parameter of ability i^* higher than the ratio $[W_L(s)/W_H(s)]^{1/\beta}$ of discounted wage earnings at instant s. Since the total number of workers (distributed in T cohorts) endowed with a given ability parameter i is m, the number of workers endowed with an ability parameter higher than i^* belonging to each cohort is $\frac{m}{T}[i^+ - i^*]$. The effective supply of skilled workers is determined by aggregating the supplies of skilled labor made by each generation of workers born at $s \in [t, t\text{-}(T\text{-}S)]$, that is

$$H^s(t) = \frac{m}{T} h(S) \int_{t-T}^{t-S} \int_{i^o(s)}^{i^+} i^\beta di ds.$$

Solving this integral and using equation (29), we determine the supply of skilled labor function

$$H^S = \frac{m}{T} h(S)[(T - S)\frac{(i^+)^{\beta+1}}{\beta + 1} - \frac{1}{\beta + 1}[\int_{t-T}^{t-S} \frac{W_L(s)}{h(S)W_H(s)}^{\frac{\beta+1}{\beta}} ds]. \quad (30)$$

From the technology expressed in equation (1), we know that the rate of innovation $\frac{\dot{N}(t)}{N(t)} \equiv g(t)$ is equal to $H(t)/a$. Differentiation of the supply of skilled labor with respect to time t yields

$$\dot{H}(t) = -\frac{m}{T(\beta + 1)}[[\frac{W_L(t - S)}{W_H(t - S)}]^{\frac{\beta+1}{\beta}} - [\frac{W_L(t - T)}{W_H(t - T)}]^{\frac{\beta+1}{\beta}}] \quad (31)$$

that is, we have that the supply of human capital (and, thus, the economy's rate of innovation) increases if the ratio of discounted wage earnings at $t\text{-}S$ $W_H(t - S)/W_L(t - S)$, is higher than the ratio at $t\text{-}T$, $W_H(t - T)/W_L(t - T)$. This happens because at each instant t the generation born at $t\text{-}T$ disappears, and so its human capital. At the same time, the generation born at $t\text{-}S$ begins to supply effective human capital. Therefore, since the population does not grow, the total supply of human capital is increased if there are more workers who decide to become skilled at $t\text{-}S$ than workers who decide

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to become skilled at t-T, that is, if the ratio of discounted wage at t-S is higher than the ratio at t-T. Notice that the increase of the supply of human capital *does not depend* on the ratio of discounted wages at t.

On the other hand, the supply of unskilled labor is made by all individuals endowed with an ability parameter lower than i^* born between t-T and t. Using (29) we obtain

$$L^s(t) = \frac{m}{T} \int_{t-T}^{t} [i^*(s) - 0] ds = \frac{m}{Th(S)} \int_{t-T}^{t} [\frac{W_L(s)}{W_H(s)}]^{1/\beta} ds. \qquad (32)$$

This expression means that the supply of unskilled labor (and the supply of manufactured goods) increases if the ratio $W_L(t)/W_H(t)$ is higher than the ratio $W_l(t - T)/W_h(t - T)$. As we can see, contrary to the innovation rate, the supply of output at t *depends* on the ratio of discounted wages at t.

We need to determine now the dynamic equations for the demand for skilled and unskilled labor. Skilled labor is demanded by research firms, who sell the patent rights on new goods to monopolistically competitive firms that commercialize the new goods. The equilibrium condition for the patent market implies that the value of the manufacturing firm $V(t)$ must be equal to the price of the patent, equal to the cost of producing a new blueprint, that is

$$V(t) = \int_{t}^{\infty} e^{-\rho(s-t)} \frac{1 - \alpha}{N(s)} ds = \frac{aw_h(t)}{N(t)}. \qquad (33)$$

Let $Z(t) = V(t)N(t)$ be equal to the aggregate value of all manufacturer firms in this economy. The derivative of Z(t) with respect to time yields $\dot{Z}(t) = \dot{V}(t)N(t) + V(t)\dot{N}(t)$. By equation (33) $\dot{V}(t) = \rho V(t) - \frac{1-\alpha}{N(t)}$, and the technology expressed in (1) means $\dot{N}(t) = \frac{H(t)N(t)}{a}$. Furthermore, the equilibrium condition for the patents market means $N(t)V(t) = aw_H(t)$. Hence, $\dot{Z}(t) = a\dot{w}_H(t)$. Using these conditions, we determine

$$\dot{w}_H(t) = [\rho + \frac{H(t)}{a}]w_H(t) - \frac{1 - \alpha}{a}, \qquad (34)$$

where *H(t)* is the supply of human capital given by (30).

The equilibrium condition in the unskilled labor market implies that the supply of unskilled labor (32) must be equal to the demand for this factor, $\frac{\alpha}{w_L(t)}$. From this condition we obtain

$$\dot{w}_L(t) = \frac{m w_L(t)^2}{T\alpha}[[\frac{W_L(t-T)}{W_H(t-T)}]^{1/\beta} - [\frac{W_L(t)}{W_H(t)}]^{1/\beta}]. \tag{35}$$

Finally, we need to know the dynamic behavior of the discounted skilled and unskilled wages $W_H(t)$ and $W_L(t)$. Differentiation of these two integrals yields

$$\dot{W}_H(t) = \rho W_H(t) + e^{-\rho T} w_H(t+T) - e^{-\rho S} w_H(t+S) \tag{36}$$

$$\dot{W}_L(t) = \rho W_L(t) + e^{-\rho T} w_L(t+T) - w_L(t). \tag{37}$$

Equations (31), (34) - (37) form a system of difference-differential equations. The system can be studied by numerical methods (see Bailén and Rivera-Batiz, 1995b). In general, since H(t) is bounded, that is, $0 \leq H(t) \leq M^+(S)$, if we begin with an arbitrary set of initial conditions $\{\{W_H(u), W_L(u)\}_{t-T-1}^{t-1}, w_H(t-1); w_L(t-1)\}$, it is possible to obtain either a no-growth solution or bounded growth with oscillations in the rate of innovation. If we begin at the unique balanced growth path equilibrium, though, oscillatory behavior will not arise.
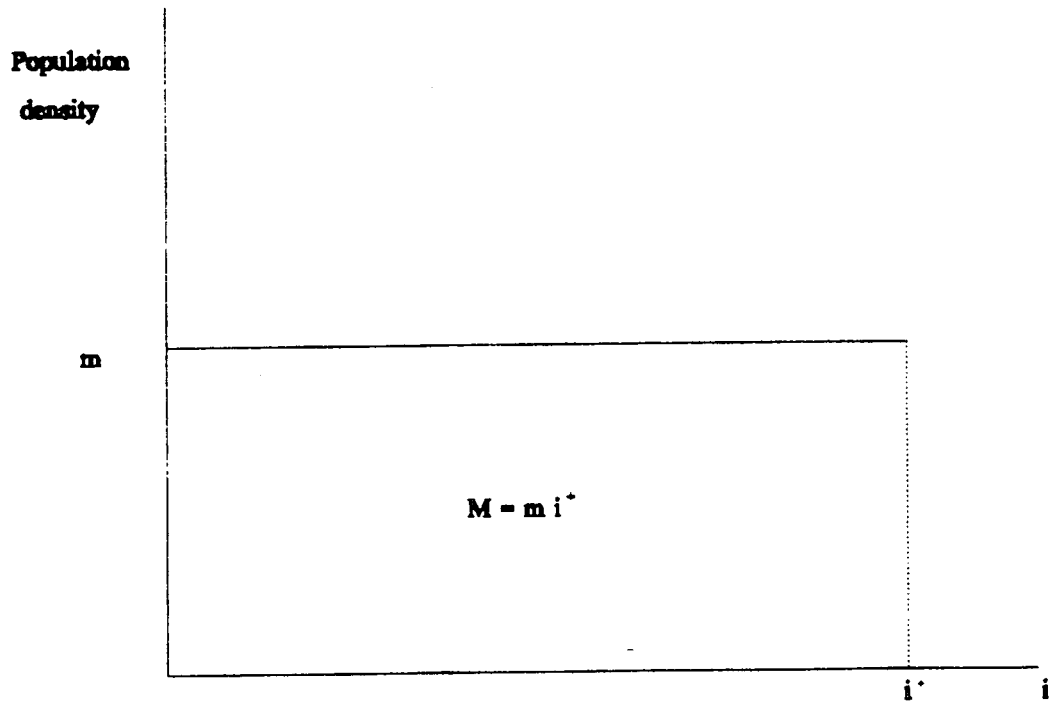
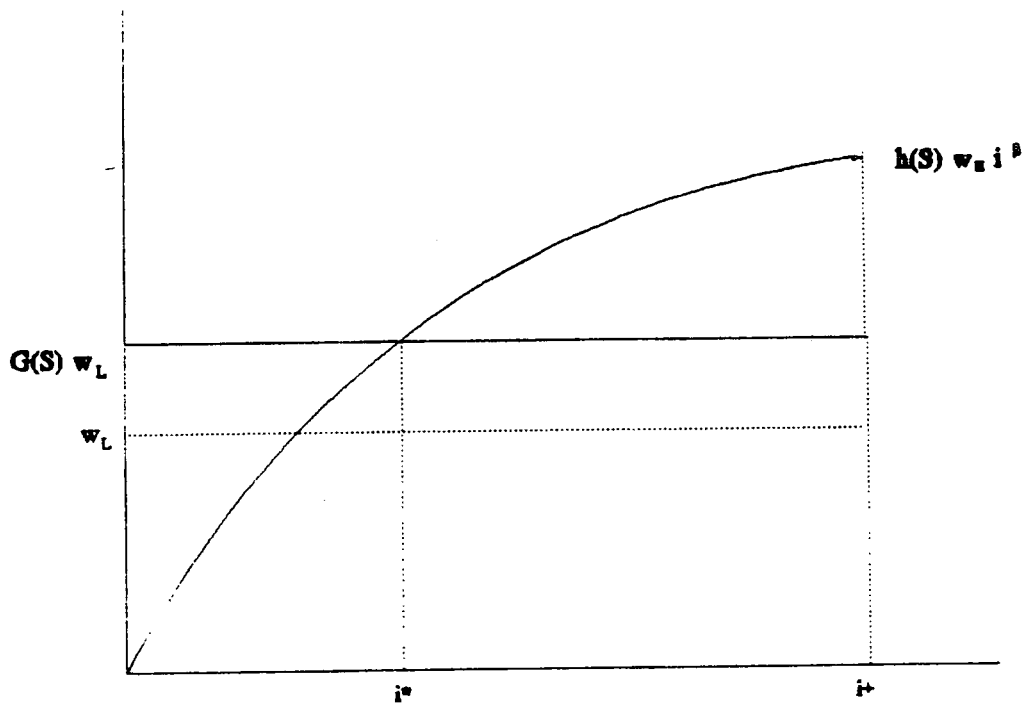**Figure 1:** Distribution of Skills in the Population

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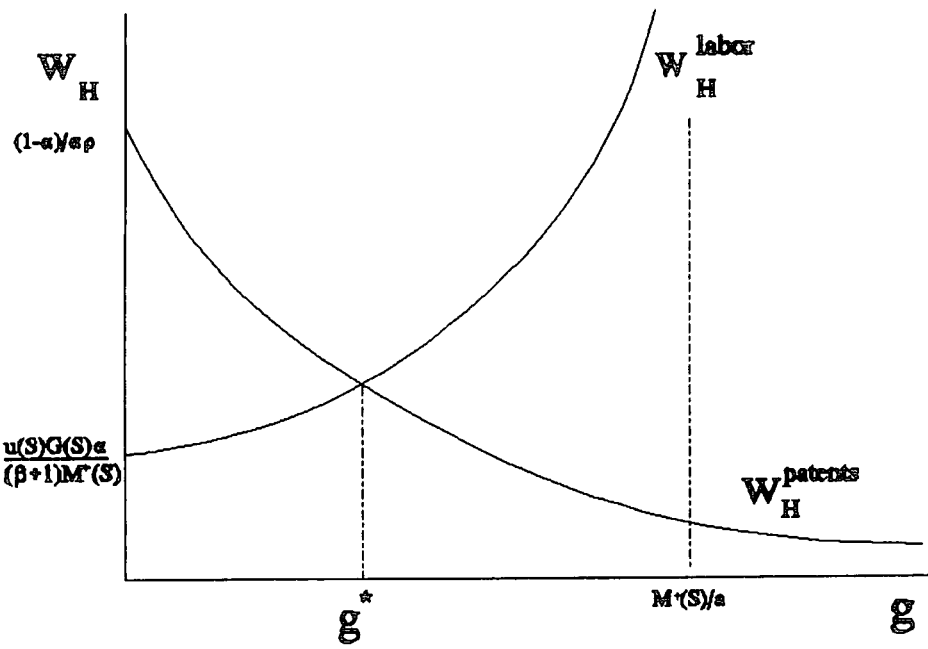
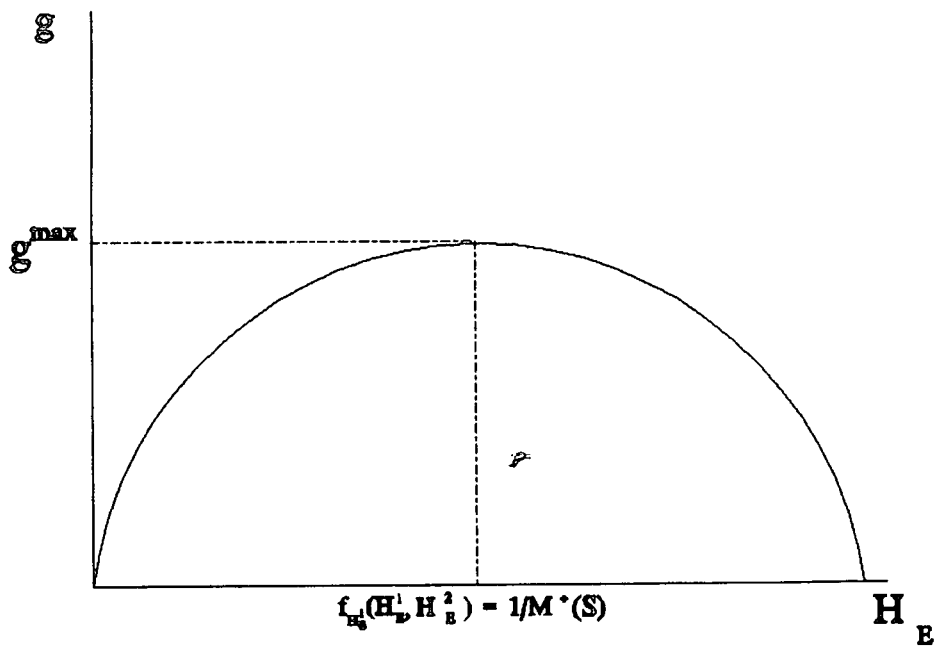**Figure 3:** Relationship between Growth and Skilled Workers' Wage Rate

Figure 4: Relationship between Growth and the Number of Educators

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