Portfolio Choice and the Effects of Liquidity

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Abstract

This paper shows how to introduce liquidity into the well known mean-variance framework of portfolio selection. Either by estimating mean-variance liquidity constrained frontiers or directly estimating optimal portfolios for alternative levels of risk aversion and preference for liquidity, we obtain strong effects of liquidity on optimal portfolio selection. In particular, portfolio performance, measured by the Sharpe ratio relative to the tangency portfolio, varies significantly with liquidity. Moreover, although mean-variance performance becomes clearly worse, the levels of liquidity on optimal portfolios obtained when there is a positive preference for liquidity are much lower than on those optimal portfolios where investors show no sign of preference for liquidity.

1. Introduction

It is clear that liquidity is a very complex concept. In principal, we may think about liquidity as the ease of trading any amount of a security without affecting its price. This already suggest that liquidity has two key dimensions; its price and quantity characteristics[1]. It is very common to proxy these two dimensions by the relative bid-ask spread and depth respectively[2].

Generally speaking, liquidity has been mostly discussed on a direct microstructure context, where the main concern is to understand the effects of the market design on liquidity. However, there has also been an interest on the relationship between liquidity and the behavior of asset prices. In particular, a very important research connects the cross-sectional relationship between expected return and risk to microstructure issues by explicitly recognizing the level of liquidity on the asset pricing model. Most papers employ the relative bid-ask spread as a measure of the level liquidity, and study the existence of a liquidity premium on stock returns. A classic example of this literature is Amihud and Mendelson (1986), who show that expected stock returns are an increasing function of illiquidity costs, and that the relationship is concave due to the clientele effect[3]. Another classic paper is Brennan and Subrahmanyam (1996), who use Kyle's (1985) lambda estimated from intraday trade and quote data, as the proxy for the level of liquidity. Their evidence is also consistent with a positive illiquidity effect. Finally, a closely related literature analyzes information risk, rather than the level of liquidity, as the determinant of the cross-sectional variation of stock returns. The paper by Easley, Hvidkjaer, O'Hara (2002), show that information does affect asset prices, and O'Hara (2003) argues that symmetric information-based asset pricing models do not work because they assume that the underlying problems of liquidity and price discovery have been solved. She develops an asymmetric information asset pricing model that incorporates these effects, and shows how important informed-based trading becomes to explain the cross-sectional of stock prices.

[1] A very intuitive but also rigorous discussion on the two dimensions of liquidity may be found in Lee, Mucklow and Ready (2003). Moreover, there are at least two nice surveys on liquidity. The paper by Amihud, Mendelson and Pedersen (2004) which covers a discussion not only on stocks, but also on bonds and options, and the paper by Pascual (2003) which also discusses key econometric issues on estimating liquidity. Moreover, a general and relevant survey on microstructure is provided by Biais, Glosten and Spatt (2005).

[2] An empirical application of both dimensions to Spanish data may be found in Martínez, Rubio and Tapia (2005).

[3] The longer the holding period, the lower compensation investors require for the costs of illiquidity.

Interestingly, once we recognize that liquidity varies over time, as documented by Chordia, Roll and Subrahmanyam (2002), researchers have become interested in analyzing liquidity as an aggregate risk factor, and basically study whether aggregate liquidity affects the stochastic discount factor. Along these lines, Pastor and Stambaugh (2003), Acharya and Pedersen (2005), using US data, show significant pricing effects of liquidity as a risk factor. On the other hand, Martínez, Nieto, Rubio and Tapia (2005), using Spanish data, compare alternative measures of aggregate liquidity risk. They employ the measures of Pastor and Stambaugh (2003), Amihud (2002), and the return differential between portfolios of stocks with high and low sensitivity to changes in their relative bid-ask spread. They show that when aggregate liquidity is measured as suggested by Amihud (2002), higher (absolute) liquidity-related betas lead to higher expected returns.

By jointly analyzing the previous empirical evidence, it seems reasonable to conclude that there is positive liquidity premium on stock returns. This, of course, suggests that optimal portfolio choices by investors should be affected by liquidity. Surprisingly, however, no academic attention has been paid to directly consider the impact of liquidity on the optimal portfolio formation process. This paper covers this gap by extending the well known mean-variance approach to solve for the optimal portfolio problem based on the simultaneous trade-off between mean-variance and liquidity.

The paper employs two approaches to better understand the effects of liquidity on the optimal portfolio choices of investors. First, we solve for the mean-variance liquidity frontier by introducing an additional constraint on the traditional optimization problem. In particular, we obtain the mean-variance frontier subject not only to the typical constraint that the portfolio has a minimum required average return, but also subject to the constraint that our optimal portfolio has a minimum level of liquidity. Secondly, we directly solve for the optimal portfolio by changing the traditional objective function, where the expected portfolio return is penalized by the variance of the portfolio given a level of risk aversion. In this case, we also place some weight on the preference for liquidity we assume on investors. This implies that we are able to find the optimal portfolios for (simultaneously) different levels of risk aversion and preference for liquidity. Hence, we can easily analyze the impact of the two preference parameters on the optimal decision of investors.

Although, it seems that our specific sample period has a significant effect on our empirical evidence, we find strong support for the impact of liquidity on portfolio choice. In fact, we show that, independently of risk aversion, mean-variance optimal portfolios have higher Sharpe ratios when the preference for liquidity is not taken into account. However, it is also the case that these portfolios are always characterized by higher relative bid-spreads and, therefore, they may be considered as less liquid portfolios.

This paper is organized as follows. Section two discusses the data employed in the paper. Section three presents the optimization problem imposing a restriction on the required liquidity level and reports the corresponding empirical results, while Section four discusses alternative characteristics of optimal portfolios for different levels of risk aversion and the preference for liquidity. Finally, Section five concludes.

2. Data

We employ daily rates of returns on 29 stocks trading in the Spanish Stock Market from January 1996 to December 2000[4]. We also collect daily relative bid-ask spreads for the available 29 individual stocks. From daily returns, and the corresponding compounding given the number of trading days for each month in our sample, we calculate monthly returns for each stock. From the daily relative bid-ask spreads, we calculate the average relative bid-ask spread for each month and each stock. The monthly three-month Treasury bill rate is employed as the risk-free rate in the optimization problems, where we always use monthly data.

From the 29 stocks in the sample, 20 have always been part of the Spanish Ibex-35 index. They are the stocks with the highest trading volume and, in general, they have the largest capitalization among Spanish stocks. In order to have a representative sample for our exercise, we also collect data on 9 stocks which have never been on the Ibex-35 index. Table 1 contains monthly average returns, monthly volatility and the average

[4] *A priori*, the actual sample period employed in the paper is not especially relevant. We use it as a simple illustration of potential consequences of liquidity on portfolio choice.

relative bid-ask spreads of all stocks in the sample. Panel A displays average data on the Ibex-35 stocks, while Panel B contains the rest of stocks. As expected, in most cases the stocks in Panel B tend to have higher relative bid-ask spreads than the stocks in Panel A. The average bid-ask spread for stocks in the Ibex-35 is 0.31 percent, while the average spread for the rest of stocks is 1.05 percent. Hence, by assuming that liquidity is correctly measured by the bid-ask spread, stocks in the Ibex-35 tend to be the most liquid stocks in the sample.

Table 2 displays some general relationships presented in our data. The discussion based on the results reported in this table facilitates the interpretation of some of the key results we discuss later in the paper. Panel A contains the sample characteristics by liquidity-sorted portfolios. Using the complete time period, all stocks are ranked by the average relative bid-ask spread. Three portfolios are then formed where the first one (*High Liquidity*) includes 10 stocks with the lowest average relative bid-ask spread, the second portfolio (*Medium Liquidity*) employs 9 stocks with intermediate spreads, while the third one (*Low Liquidity*) contains the 10 most illiquid stocks. Surprisingly, we observe that stocks with higher liquidity also have the highest average return and, even more important, the highest Sharpe ratio. On the other hand, highly illiquid stocks have on average higher mean returns than medium liquidity stocks, and lower volatility than any other stocks, leading to a Sharpe ratio higher than the Sharpe ratio of medium liquidity stocks. It seems that during the sample period, the largest most liquid stocks in the Spanish market present the best average performance. It is important to note that during the nineties, the size effect in Spain change surprisingly its sign. This is especially the case for the second part of the nineties which coincides with our sample period. Of course, highly liquid stocks also tend to be the stocks with the largest capitalization. Moreover, during the sample period, it is well known that stock markets around the world experienced a tremendously successful performance. It seems that investors tried to take advantage of this extraordinary performance by holding well known, highly liquid stocks.

Figure 1 contains graphically similar conclusions than the ones we observe from Panel A of Table 2. We first plot the mean-variance pairs of all the stocks in our sample. It is clear from the figure that stocks in the Ibex-35 index tend to have relatively higher average returns than stocks which have not belonged to the index, and also a not particularly high volatility. Panel B of Figure 1 displays the tangency portfolio from a traditional mean-variance optimization with a risk-free asset and non-negative weights using monthly data from stocks in the Ibex-35 index. The corresponding Sharpe ratio is 0.5265. On the other hand, Panel C contains the results using stocks which have never been in the Ibex-35 index. The slope of the ex-post capital market line is this case just 0.3839. As expected, given the results from Panel A of Table 2, the performance of stocks in the Ibex-35 during our sample period is better than the performance of non-index stocks.

Panel B of Table 2 displays the average relative bid-ask spreads for 6 portfolios based on intersections between stocks sorted by average return and volatility. In this case, all stocks are separately ranked by average return and volatility. Three portfolios are then formed according to either average return (*Low Average Return*, *Medium Average Return* and *High Average Return*) or volatility (*Low Volatility*, *Medium Volatility* and *High Volatility*). Then, 6 portfolios based on intersections are obtained. Finally, we report the average relative bid-ask spreads of the 6 intersection portfolios. It is interesting to observe that the expected relationship between average returns and liquidity is obtained for stocks with low volatility, at least for the extreme cases. However, the relationship is strongly reversed for medium and high volatility stocks. On the other hand, it is always the case than stocks with high volatility tend to have higher average relative bid-ask spreads independently of the average return. Finally, Panel C of Table 2 reports the average returns for 6 portfolios based on intersections between stocks sorted by average bid-ask spread and volatility. The construction is similar to the previous two panels, and the results tend to confirm our initial empirical evidence. Thus, highly liquid stocks with medium and high volatility also present quite high average returns.

3. The Mean-Variance Liquidity Constrained Frontier

In this section, we obtain the minimum variance frontier by imposing not only the traditional constraint on average return, but also an additional constraint on a minimum required level of liquidity.

The mean-variance liquidity constrained frontier is obtained by solving the following optimization problem:

$$\min_{\omega} \frac{1}{2} \omega' V \omega$$

$$\text{subject to} \begin{cases} \omega'\mu = \mu_p \\ \omega' RBAS = RBAS_p \\ \omega' 1_N = 1 \\ \omega \geq 0 \end{cases} \quad (1)$$

where V is the NxN variance-covariance matrix of monthly stock returns, μ is the N-vector of monthly mean returns, $RBAS$ is N-vector of relative bid-ask spreads[5], μ_p and $RBAS_p$ are the required levels of average return and liquidity on the minimum variance liquidity constrained portfolio, and ω are the non-negative weights of each stock on the minimum variance liquidity constrained portfolio[6].

Panel A of Figure 2 displays the three-dimensional mean-variance liquidity constrained frontier, while Panel B contains the mean-variance frontier for alternative levels of liquidity. For each portfolio liquidity level between 0.0014 and 0.0170, we obtain three different frontiers depending upon the simultaneous behavior of average return, volatility and liquidity. For high levels of liquidity, with bid-ask spreads between 0.0014 and 0.0060, the frontier moves as expected, since higher illiquidity implies higher average return. On other hand, when liquidity is low, with levels of the spread between 0.0075 and 0.0170, we obtain exactly the opposite results and the frontier moves in the unexpected direction. Thus, for extreme low levels of liquidity, our portfolio would also generate low average returns. Once again, small, highly illiquid stocks display a bad performance during the sample period. Finally, for medium levels of liquidity, with the relative bid-ask spread between 0.006 and 0.0075, the results in terms of average returns depend not only on illiquidity, but also on volatility. For low levels of volatility, we obtain the expected result for which the higher the illiquidity the

[5] These are calculated as the mean of relative bid-ask spreads over the corresponding sample month.
[6] Similar results are obtained when we use the measure of liquidity proposed by Amihud (2002) instead of the relative bid-ask spread.

higher the average return. However, this changes when we consider high levels of volatility for the intermediate levels of illiquidity.

These results suggest that it is important to simultaneously consider the interplay between average returns, volatility and illiquidity. This is an interesting result which already implies that liquidity as a characteristic plays a role on determining optimal portfolios.

To be more precise, we calculate the tangency portfolio for each efficient frontier given a level of liquidity. In particular, we maximize the Sharpe ratio as follows,

$$\max_{\omega} \frac{\mu_p - r_f}{\sigma_p}$$

$$\text{subject to} \begin{cases} \omega'\mu + \left(1 - \omega' 1_N\right) r_f = \mu_p \\ \sqrt{\omega' V \omega} = \sigma_p \\ \omega' RBAS = RBAS_p \\ \omega' 1_N = 1 \\ \omega \geq 0 \end{cases} \tag{2}$$

where r_f is the monthly risk-free rate and σ_p is the volatility of the portfolio.

The results contained in Table 3 are consistent with our previous discussion. For low levels of bid-ask spreads or high liquidity, the Sharpe ratio increases as a function of illiquidity. Thus, the behavior of the tangency portfolio is better on average the higher the illiquidity level imposed. In other words, illiquidity incorporates a premium on performance. However, the opposite results are obtained when illiquidity is high. In fact, the Sharpe ratio decreases the lower the level of liquidity. This, once again, reflects the extraordinary importance of being part of the Spanish Ibex.35 index, at least during the available sample period.

Given the empirical relevance of liquidity as a risk factor on recent asset pricing literature, we next analyze the portfolio performance in terms of the Sharpe ratio for alternative levels of aggregate liquidity. We calculate the market-wide liquidity by estimating the average relative bid-ask spread across all stocks and for each month in the sample period. We therefore have a time-series of aggregate liquidity. Then, we separate market-wide liquidity in three time periods depending upon the level of aggregate liquidity and we again solve the optimization problem given by expression (2) for each of the three sub-periods separately. The striking results are displayed in Figure 3. It turns out that market-wide liquidity plays a key role on the simultaneous relationship between average returns, volatility and liquidity. Contrary to the general results reported in Table 3, the Sharpe ratio increases monotonically with illiquidity as long as the aggregate level of liquidity in the market is high enough. In fact, for both, the high and medium levels of market-wide liquidity, it seems that illiquidity is compensated with a higher Sharpe ratio or better average performance. However, when market-wide liquidity is low, the Sharpe ratio shows a hump that peaks at around 0.0068. The results suggest that the unexpected results obtained in Table 3 may depend on the market-wide level of liquidity. At least from January 1996 to December 2000, the performance premium on illiquidity is generated as long as market-wide liquidity is sufficiently high. Unfortunately, this positive result seems to break down when market-wide liquidity becomes low.

4. Optimal Portfolios, Risk Aversion and the Preference for Liquidity

We now introduce explicitly risk aversion and preference for liquidity in the objective function of the investor. The optimization problem becomes[7],

$$\max_{\omega} \mu_p - \frac{\gamma}{2}\sigma_p^2 + \eta RBAS_p \Leftrightarrow \max_{\omega} \omega'\mu - \frac{\gamma}{2}\omega' V \omega + \eta RBAS_p$$
$$subject\ to \begin{cases} \omega' 1_N = 1 \\ \omega \geq 0 \end{cases} \qquad (3)$$

[7] As before, we obtain qualitatively the same results when we use the measure of liquidity proposed by Amihud (2002).

where $\gamma > 0$ is the risk aversion parameter and $\eta \geq 0$ represents the preference for liquidity. We solve the problem for several values of γ and η. In particular, we allow risk aversion to take values within the following set $\gamma \in \{1, 2, 2.188, 4, 5, 10, 20\}$ where the value of 2.188 is the risk aversion estimated by León, Nave and Rubio (2007) for the Spanish stock market from January 1988 to December 2004. On the hand, $\eta \in \{0, 0.00005, 0.0005, 0.005\}$. Of course, when $\eta = 0$, problem (3) reduces to the traditional mean-variance problem in a static framework.

The results are displayed in Figure 4, where we analyze four cases, Panel A to Panel D, in which we study the relationship between average return, volatility, illiquidity and Sharpe ratio of optimal portfolios respectively as a function of the risk aversion parameter and for alternative levels of η.

It is important to note that, as expected, and independently of the preference for liquidity, we observe a declining average return and volatility as risk aversion increases. This suggests that, as risk aversion becomes more important, the optimal portfolio becomes less risky and, consequently, average returns are also negatively affected. In fact, for low levels of η, it turns out that volatility decreases with risk aversion more rapidly than average returns as long as γ is lower than 10 (for $\eta = 0$) or lower than 5 (for $\eta = 0.00005$). As observed in Panel D, this implies an increasing Sharpe ratio up to either $\gamma = 10$ or $\gamma = 5$ depending upon the preference for liquidity.

When we do not place any weight on liquidity, $\eta = 0$, we find that optimal portfolios have higher average returns and lower volatility (at least for $\gamma > 3.5$) than cases in which $\eta > 0$. Alternatively, when the preference for liquidity becomes different from zero we tend to obtain optimal portfolios with lower average returns and higher volatility. The investor seems to be willing to accept lower returns in order to have more liquidity, but the consequence of this behavior is a higher volatility in the optimal portfolio. In any case, these results imply a higher Sharpe ratio or a better mean-variance performance of optimal portfolios when the investor has no preference for liquidity. In particular, the Sharpe ratio is around 0.50 when $\eta = 0$ and approximately 0.35 when $\eta > 0$. Of course, this favorable result is accompanied by lower levels of liquidity in the corresponding optimal portfolios. Thus, illiquidity (the relative bid-ask spread) is around 0.006 when $\eta = 0$ and around 0.0015 when $\eta > 0$. The relationship between the illiquidity of the optimal portfolios and risk aversion displays an inverted hump that peaks at $\gamma = 5$ when the investor has no preference for liquidity. Interestingly, the illiquidity level of these portfolios when $\eta = 0$ is reduced when risk aversion increases from $\gamma = 1$ up to $\gamma = 5$. These are precisely the levels of risk aversion for which the Sharpe ratio increases more rapidly and levels of risk aversion closer to the usual empirical estimates reported in literature[8].

5. Conclusions

This paper shows strong effects of liquidity on optimal portfolio selection. Complex simultaneous relations are found between average returns, volatility and liquidity that should probably be taken into account when selecting optimal portfolios. Portfolio performance, as measured by the Sharpe ratio relative to the tangency portfolio, varies significantly with liquidity. Moreover, this relationship depends upon the market-wide level of liquidity. As long as aggregate liquidity is high enough, the Sharpe increases with illiquidity suggesting that, on average, there is required illiquidity premium when taking optimal portfolio decisions. Finally, when the investor shows no preference for liquidity the performance of optimal portfolios is clearly better independently of the level of risk aversion. However, these portfolios display a much lower level of liquidity than the optimal portfolios obtained when recognizing explicitly preference on liquidity.

[8] When León, Nave and Rubio (2007) allow for asymmetric negative and positive shocks, the risk aversion coefficient for the Spanish stock markets becomes 3.40.

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Table 1
Sample Descriptive Statistics

Sample monthly average returns, monthly volatility and average relative bid-ask spread of 29 stocks trading at the Spanish Stock Exchange from January 1996 to December 2000. 20 of these stocks have always been part of the Spanish Ibex-35 Index, while the other 9 stocks have never been part of the Ibex-35.

Panel A: Ibex-35 Stocks	*Average Return*	*Volatility of Returns*	*Relative Bid-Ask Spread*
ABERTIS (ABE)	0,0115	0,0790	0,0038
ACERINOX (ACX)	0,0208	0,1032	0,0036
AGUAS BARCELONA (AGS)	0,0155	0,0801	0,0049
ALTADIS (ALT)	0,0245	0,0978	0,0029
AUTOPISTAS MARE NOSTRUM (AUM)	0,0165	0,0694	0,0043
BANCO BILBAO VIZCAYA ARGENTARIA (BBVA)	0,0357	0,0950	0,0015
BANKINTER (BKT)	0,0267	0,0997	0,0032
HIDROCANTABRICO (CAN)	0,0217	0,0757	0,0046
DRAGADOS (DRC)	0,0279	0,0985	0,0035
ENDESA (ELE)	0,0142	0,0783	0,0015
FOMENTO, CONSTRUCCIONES Y CONTRATAS (FCC)	0,0235	0,1006	0,0046
GAS NATURAL (GAS)	0,0195	0,1013	0,0036
IBERDROLA (IBE)	0,0180	0,0747	0,0027
CORPORACION MAPFRE (MAP)	0,0084	0,1021	0,0046
BANCO POPULAR (POP)	0,0205	0,0829	0,0021
REPSOL (REP)	0,0181	0,0719	0,0014
SANTANDER CENTRAL HISPANO (SAN)	0,0300	0,0982	0,0016
TELEFONICA (TEF)	0,0359	0,0987	0,0014
UNION FENOSA (UNF)	0,0328	0,0841	0,0029
UNIASA-PULEVA (UNI)	0,0507	0,1081	0,0075

Panel B: No Ibex-35 Stocks	*Average Return*	*Volatility of Returns*	*Relative Bid-Ask Spread*
AZUCARERA EBRO (ASA)	0,0091	0,1086	0,0117
AZKOYEN (AZK)	0,0192	0,1605	0,0111
CEPSA (CEP)	0,0104	0,0738	0,0068
CORTEFIEL (CTF)	0,0164	0,0985	0,0097
BANCO GUIPUZCOANO (GUI)	0,0201	0,0613	0,0125
BANCO PASTOR (PAS)	0,0201	0,0619	0,0075
PROSEGUR (PSG)	0,0134	0,2326	0,0124
UNIPAPEL (UPL)	0,0159	0,1002	0,0170
ZARDOYA OTIS (ZOT)	0,0177	0,0677	0,0057

Table 2
Liquidity, Average Returns and Volatility

Panel A: Sample Characteristics by Liquidity-Sorted Portfolios. Using the complete time period from January 1996 to December 2000, all stocks are ranked by the average relative bid-ask spread. Three portfolios are then formed where the first one (*High Liquidity*) includes 10 stocks with the lowest average relative bid-ask spread, the second portfolio (*Medium Liquidity*) employs 9 stocks with intermediate spreads, while the third one (*Low Liquidity*) contains the 10 most illiquid stocks.

	Average Return	*Volatility of Returns*	*Sharpe Ratio*	*Average Relative Bid-Ask Spread*
High Liquidity	0,0256	0,0580	0,3762	0,0021
Medium Liquidity	0,0184	0,0591	0,2474	0,0041
Low Liquidity	0,0193	0,0504	0,3080	0,0102

Panel B: Average Relative Bid-Ask Spreads for 6 Portfolios based on Intersections between Stocks Sorted by Average Return and Volatility. Using the complete time period from January 1996 to December 2000, all stocks are separately ranked by average return and volatility. Three portfolios are then formed according to either average return (*Low Average Return*, *Medium Average Return* and *High Average Return*) or volatility (*Low Volatility*, *Medium Volatility* and *High Volatility*). Then, 6 portfolios based on intersections are obtained. Finally, we report the average relative bid-ask spreads of the 6 intersection portfolios.

Average Relative Bid-Ask Spread	*Low Volatility*	*Medium Volatility*	*High Volatility*
Low Average Return	0,0041	0,0073	0,0114
Medium Average Return	0,0060	0,0021	0,0061
High Average Return	0,0046	0,0023	0,0051

Panel C: Average Returns for 6 Portfolios based on Intersections between Stocks Sorted by Average Bid-Ask Spread and Volatility. Using the complete time period from January 1996 to December 2000, all stocks are separately ranked by average relative bid-ask spread and volatility. Three portfolios are then formed according to either average relative bid-ask spread (*High Liquidity*, *Medium Liquidity* and *Low Liquidity*) or volatility (*Low Volatility*, *Medium Volatility* and *High Volatility*). Then, 6 portfolios based on intersections are obtained. Finally, we report the average returns of the 6 intersection portfolios.

Average Return	*Low Volatility*	*Medium Volatility*	*High Volatility*
High Liquidity	0,0168	0,0299	0,0267
Medium Liquidity	0,0165	0,0217	0,0181
Low Liquidity	0,0171	0,0164	0,0217

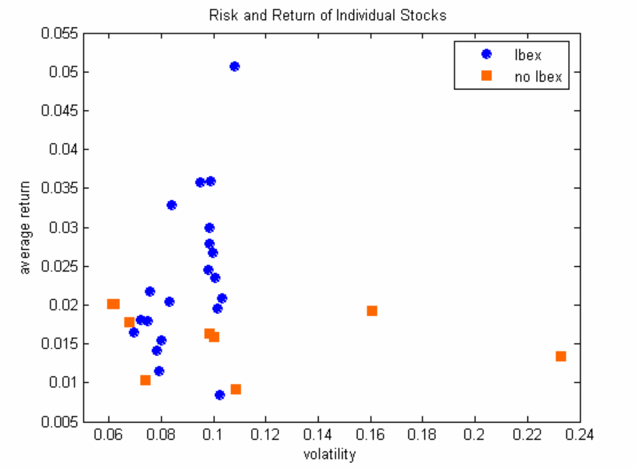
Table 3
Sharpe Ratios for Alternative Levels of Liquidity

To obtain the tangency portfolio we maximize the Sharpe ratio by a given level of average returns, volatility and liquidity. The table contains the volatility, average return and Sharpe ratio for alternative levels of liquidity measured by the relative bid-ask spread. The results use monthly returns from January 1996 to December 2000.

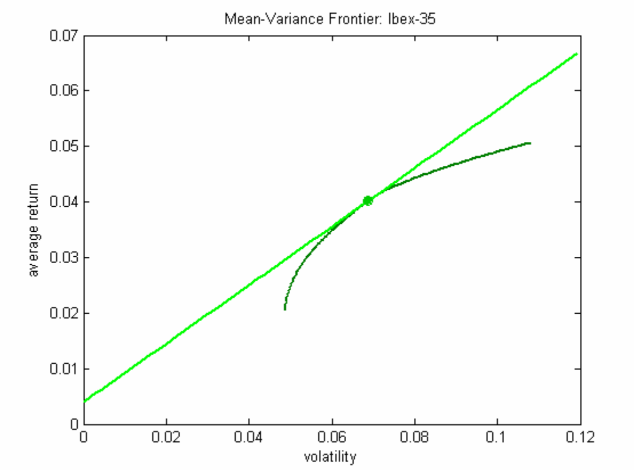
Relative Bid-Ask Spread for Tangency Portfolios	*Volatility for Tangency Portfolios*	*Average Return for Tangency Portfolios*	*Sharpe Ratio for Tangency Portfolios*
0,001	0,096	0,036	0,334
0,003	0,064	0,036	0,498
0,005	0,057	0,035	0,545
0,007	0,050	0,032	0,563
0,010	0,050	0,029	0,507
0,012	0,055	0,027	0,417
0,015	0,070	0,022	0,252
0,017	0,099	0,017	0,127

Figure 1
Monthly Average Return and Volatility of Sample Stocks
January 1996-December 2000

Panel A: Average Returns and Volatility for 29 Stocks



Panel B: Mean-Variance Frontier for 20 stocks which have always been at Ibex-35



Panel C: Mean-Variance Frontier for 9 stocks which never been at Ibex-35

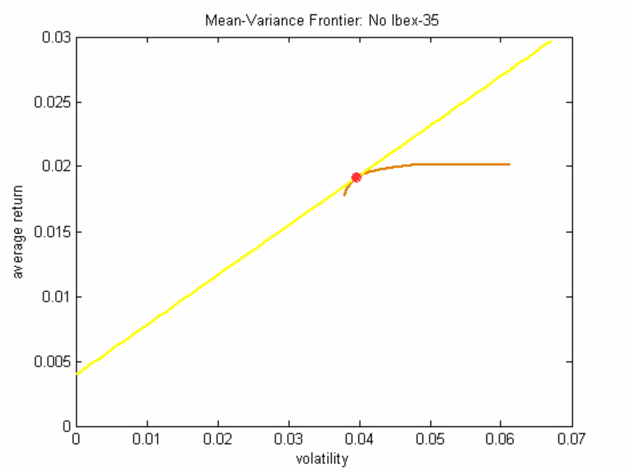
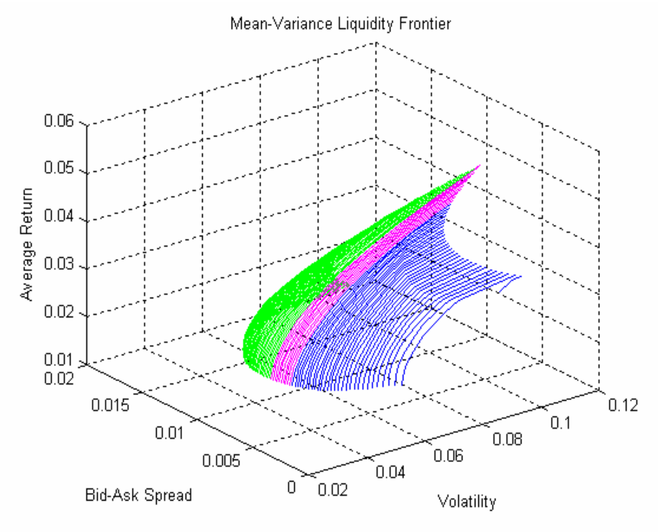


Figure 2
The Mean-Variance Liquidity Constrained Frontier
January 1996-December 2000

Panel A: Mean-Variance Liquidity Constrained Frontier



Panel B: Mean-Variance Liquidity Constrained Frontier For Alternative Levels of Liquidity as Measured by the Average Relative Bid-Ask Spread (RBAS)

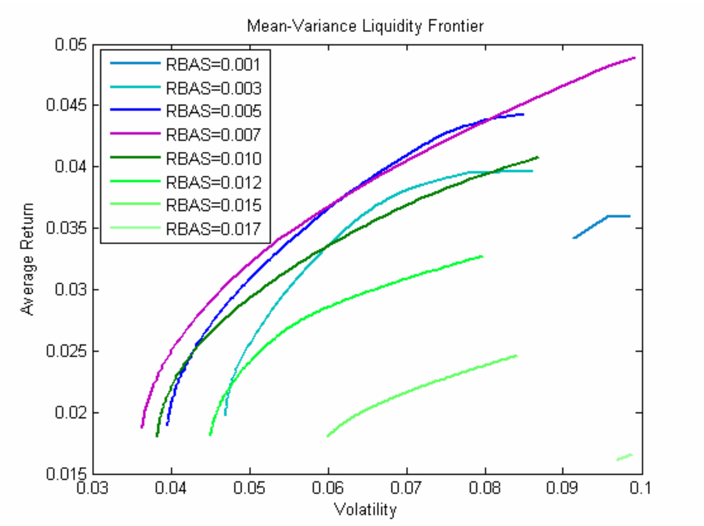


Figure 3
The Sharpe Ratio for Alternative Levels of Aggregate Illiquidity
January 1996-December 2000

This figure represents the relationship between the Sharpe ratio and illiquidity once the sample period has been divided in three sub-periods classified according to the aggregate level of illiquidity

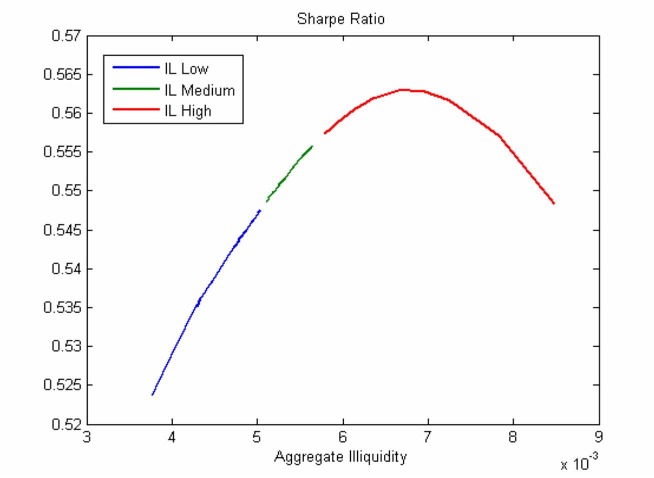
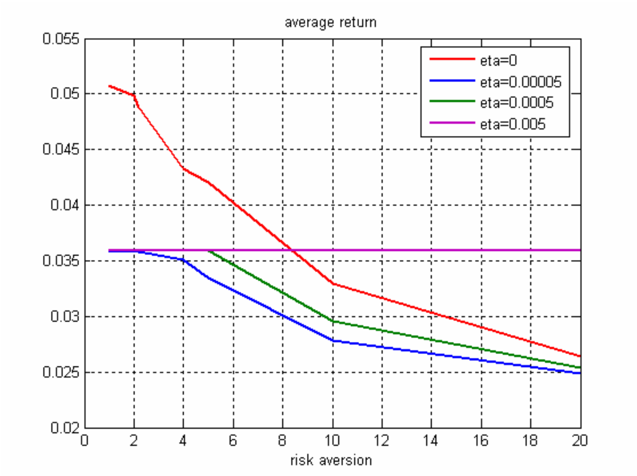
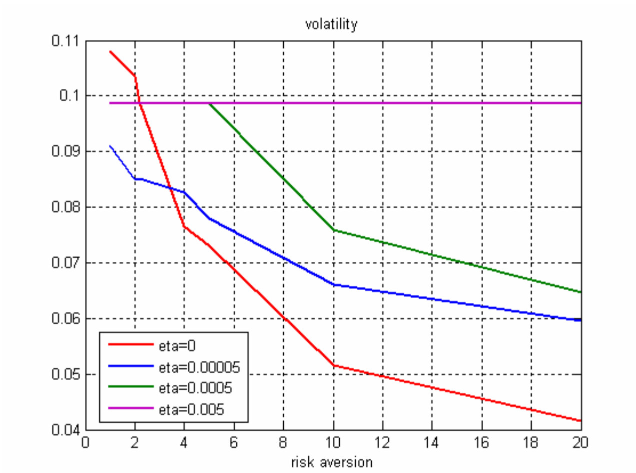


Figure 4
Characteristics of Optimal Portfolios for Alternative Levels of Risk Aversion and Preference for Liquidity
January 1996-December 2000

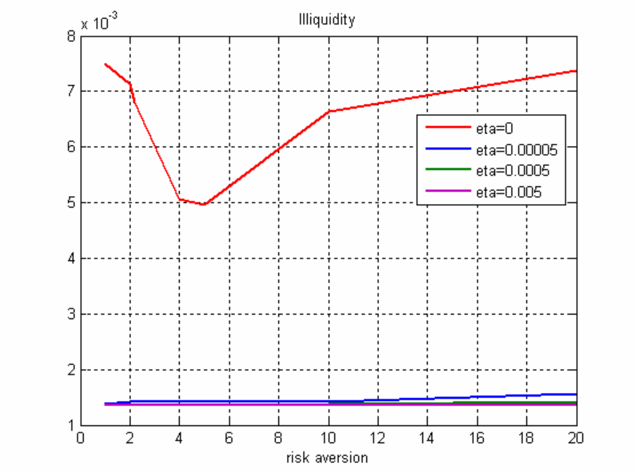
Panel A: Average Return of the Optimal Portfolio as a Function of Risk Aversion and Preference for Liquidity



Panel B: Volatility of the Optimal Portfolio as a Function of Risk Aversion and Preference for Liquidity



Panel C: Illiquidity of the Optimal Portfolio as a Function of Risk Aversion and Preference for Liquidity



Panel D: Sharpe Ratio of the Optimal Portfolio as a Function of Risk Aversion and Preference for Liquidity