

Measuring Time-Varying Economic Fears with Consumption-Based Stochastic Discount Factors

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Abstract

This paper analyzes empirically the volatility of consumption-based stochastic discount factors as a measure of implicit economic fears by studying its relationship with future economic and stock market cycles. Time-varying economic fears seem to be well captured by the volatility of stochastic discount factors. In particular, the volatility of recursive utility-based stochastic discount factor with contemporaneous consumption growth explains between 9 and 34 percent of future changes in industrial production at short and long horizons respectively. They also explain ex-ante uncertainty and risk aversion. However, future stock market cycles are better explained by a similar stochastic discount factor with long-run consumption growth.

1. Introduction

It is well known that risk-neutral probabilities are objective (or physical) probabilities adjusted upward (downward) if they are associated with states with high (low) marginal utility of consumption. Hence, objective probabilities are just risk-neutral probabilities multiplied by some risk aversion adjustment which depends on the actual preferences of the representative investor. The interaction of these two sets of probabilities with the volatility of stochastic discount factors, as a way of measuring time-varying aggregate economic fears, is the focus of this work.

In their seminal paper, Breeden and Litzenberger (1978) show that the risk-neutral density can be recovered from option prices as long as the market is dynamically complete. On the other hand, the existence of risk aversion means that risk-neutral densities will probably differ from the actual density from which realizations of returns are drawn. Several procedures have been proposed to obtain comparable risk-adjusted densities. Jackwerth (2000) recognizes a changing risk-neutral probability density function while imposing a stationary objective density function. This is problematic and leads to the well known pricing kernel puzzle. To avoid this debatable assumption, Bliss and Panigirtzoglou (2004) assume risk-aversion function stationarity and estimate implied preference parameters from power and exponential utility functions. Finally, Alonso, Blanco and Rubio (2006) extends this work using habit preferences and Benzoni, Dufresne and Goldstein (2005) argue that the pricing kernel puzzle and the volatility smirk can be rationalized if the agent has recursive preferences and if the aggregate dividend and consumption processes are driven by a persistent stochastic growth variable that can jump.¹

Contrary to this literature, this paper explores empirically the theoretical results underlying objective and risk-neutral probabilities without relying on option data, except for motivating the procedures employed along the presentation and some additional robustness analysis. In a recent theoretical paper, Bakshi, Chen and Hjalmarsson (2004) (BCH hereafter) define a distance between the risk-neutral and the objective probability measures, which can be related to the volatility of the defining stochastic discount factor (SDF). By arguing that the BCH distance captures economic

¹ Note that both papers propose state dependent utility functions. A complete theoretical discussion of why state dependence in fundamentals and preferences are necessary to explain risk aversion puzzles are discussed by Chabi-Yo, Garcia and Renault (2005).

fears, and given the association between their distance and the volatility of the defining SDF, our paper analyzes empirically the volatility of consumption-based SDFs to measure investors' implicit recession fears.

In particular, we employ several sensible consumption-based SDF candidates and discuss whether their volatilities are able to predict future economic cycles. Thus, we analyze the empirical link between the ex-ante economic fears about macroeconomic fundamentals and the ex-post economic cycle in the financial market and the economy. The empirical exercise is performed using data from Spain, one of the largest industrial economies in Europe. Moreover, a robustness analysis of the results using data from the better-known U.S. market is also performed.

The paper shows that recursive preferences and long-run aggregate consumption risk are important when measuring time-varying economic fears. More precisely, the volatility of the SDF based on recursive preferences and contemporaneous consumption growth tends to be especially high just before macroeconomic recessions, while the volatility of SDFs based simultaneously on long-run consumption growth and recursive preferences seems to be particularly high before persistent decreases in the stock market. The volatility of a habit-based SDF is also significant when predicting both macroeconomic recessions and stock market falls at short-horizons. Interestingly, the recursive utility-based SDFs are also able to explain significantly uncertainty and risk aversion in the stock market. It should be noted that we do not pursue to compare SDFs from the traditional asset pricing point of view. We just want to study whether the volatility of reasonable SDFs is able to predict future economic cycles. Of course, the forecasting performance of the alternative specifications employed in the paper may be different. This is how the comparison of the proposed consumption-based SDFs should be understood.

This paper is organized as follows. Section 2 discusses the theoretical framework that relates risk-neutral and objective probability distributions with the volatility of SDFs. Section 3 presents the stochastic discount factor specifications analyzed in the paper, while Section 4 contains a description of data and some initial empirical results using the Hansen-Jagannathan (1991) volatility bound. Section 5 selects the appropriate consumption-based stochastic discount factors, and Section 6 discusses how well these specifications capture macroeconomic and stock market

recessions. Section 7 discusses additional results using U.S. data, and Section 8 concludes with a summary of our findings.

2. A Distance Metric between the Risk-Neutral and Objective Probability Distributions and the Volatility of the Stochastic Discount Factor

There are well known economic episodes, like the stock market crash in 1987, the Asian currency crisis during the summer of 1997, the Russian default in the summer of 1998, the Gulf wars, or the terrorist attack on September 11th, 2001, in which the left-tail of the risk-neutral density becomes considerably fatter than the corresponding left-tail of the risk-adjusted counterpart.

Figure 1 compares estimated probability density functions for two different expiration days for the European-style Spanish equity option contract on the IBEX-35 futures for Spanish at a four-week horizon. Panel A shows density functions estimated with option prices of 24/8/2001; i.e., before the terror attacks of September 11th. On that day, all densities have a similar shape. Naturally, risk-adjusted densities appear (slightly) shifted to the right.² Similarly, Panel B shows probability density functions estimated with option prices of 21/9/2001, which reflect the impact on market prices of the events of September 11th. Compared with panel A, the probability mass of the tails, and especially on the left tail, is much higher reflecting the higher uncertainty. As expected by the definition of risk-neutral probabilities, risk-adjusted densities display lower left-skewness than those of the risk-neutral density, pointing out that the latter distribution overstates poor states of nature, especially during stress economic periods. Marginal utility is higher in those scenarios and this is precisely what is introduced into the estimated risk-neutral densities. Figure 2 contains the difference between the monthly probabilistic mass assigned to the 10 percent left tail of the risk-neutral and the power risk-adjusted density functions from October 1996 to December 2004.³ It is quite

² Using data from October 1996 to December 2004, preference parameters for power and exponential utility functions are implicitly estimated by searching for the optimal level of risk aversion that maximize the predicting ability of the resulting risk-adjusted densities. See Alonso, Blanco and Rubio (2006) for details.

³ The power utility employs a risk aversion coefficient of 1.67, which is the implied estimate obtained by Alonso, Blanco and Rubio (2006) for the same time period.

striking to observe how these differences are time-varying with a clear increasing pattern for every potentially damaging economic episode.

Given this evidence, we may also argue that, in absolute values, these crash fears may cause a positive overall gap or distance between the risk-neutral and objective probability measures. For a given percentage of the tails of the density functions, the potential economic downturn increases more the probabilistic mass assigned to the left tail of the risk-neutral density over the risk-adjusted density than the probabilistic mass assigned to the right tail of the risk-adjusted density over and above the risk-neutral counterpart. This suggest that the overall distance taken in absolute value between the risk-neutral and objective probability measures may be well suited to proxy for economic fears of investors. Interestingly, the theoretical results provided by BCH (2004) formalize an overall distance measure between the two probability sets. Moreover, they also show that their overall distance between both measures is associated with the volatility of any empirically sound SDF.

In order to describe their metric,⁴ consider an economy endowed with a probability space $(\Omega, \mathfrak{F}, P)$ where Ω denotes the state space and \mathfrak{F} is the tribe of subsets of Ω that are events and can therefore be assigned a probability. We denote P and Q as the objective and risk-neutral probability measures respectively. These two measures are probabilistically equivalent since they share exactly the same null events, yet assign different (positive) probability masses to the same event.

Under no arbitrage opportunities, there exists a strictly positive SDF, M , such that the price of any financial asset between any two time periods t and $t+1$ is given by

$$p_{jt} = E_t^P (X_{t+1}M_{t+1}) \quad (1)$$

where p_{jt} is the price of asset j at time t , X_{t+1} is the future payoff of asset j , and E_t^P is the conditional expectation with respect to the objective probability measure P . Alternatively, the price of the financial asset with respect to the risk-neutral probability Q is

⁴ Our presentation is slightly different and much shorter than the discussion in the original paper by BCH (2004).

$$p_{jt} = \frac{1}{R_f} E_t^Q (X_{t+1}) \quad (2)$$

where E_t^Q is the conditional expectation with respect to Q and R_f is the gross riskless-rate of interest between t and $t+1$.

The well known Radon-Nikodym derivative is a strictly positive random variable $\frac{dQ}{dP}$ with $E^P \left(\frac{dQ}{dP} \right) = 1$. Then, the risk-neutral probability measure Q equivalent to P can be defined in terms of the Radon-Nikodym derivative through the definition of expectation with respect to Q given by $E^Q (X) = E^P \left(\frac{dQ}{dP} X \right)$ for any random variable X . We choose a particular equivalent probability measure Q such that $\frac{dQ}{dP} = R_f M_{t+1}$.⁵ Given that $E_t^P (M_{t+1}) = 1/R_f$, it must be true that,

$$dQ = \frac{M_{t+1}}{E^P (M_{t+1})} dP \quad (3)$$

BCH (2004) define the distance between P and Q as

$$D_0 (P, Q) \equiv \int_{\Omega} |dQ(X) - dP(X)| \quad (4)$$

This distance will be zero if and only if P and Q assigns the same probability mass to every given event belonging to \mathfrak{T} in the state space Ω . Substituting the expression (3) into (4) we obtain,

$$D_0 (P, Q) = E_t^P \left| \frac{M_{t+1}}{E_t^P (M_{t+1})} - 1 \right| = R_f E_t^P \left| M_{t+1} - \frac{1}{R_f} \right| \quad (5)$$

Then, the absolute distance between both probability measures is completely determined by the expectation under the objective probability of the absolute difference

⁵ One can check that, under no arbitrage, this random variable is strictly positive and of expectation 1.

between M and $1/R_f$.⁶ It is now convenient to scale $D_0(P, Q)$ by R_f , and

denote $D_I(P, Q) \equiv D_0(P, Q)/R_f$, so that $D_I(P, Q) = E_t^P \left| M_{t+1} - \frac{1}{R_f} \right|$.

By recalling that $\|X\| \equiv \sqrt{E(X^2)}$, we finally apply Hölder's inequality to the distance probability measures to obtain,⁷

$$0 \leq D_I(P, Q) \leq \left\| M_{t+1} - \frac{1}{R_f} \right\| = \left\| M_{t+1} - E_t^P(M_{t+1}) \right\| = \sigma(M) \quad (6)$$

where $\sigma(M)$ is the standard deviation of the stochastic discount factor M . Hence, the volatility of the stochastic discount factor provides an upper bound for the distance between the risk-neutral and objective probability distribution, up to a constant of proportionality. This implies that a higher volatility of the defining SDF is not necessarily accompanied by a larger distance between the probability measures and, therefore, by increasing economic fears from investors. The concrete relationship between the volatility of any sensible SDF and the distance between probabilities becomes an empirical issue. In any case, we expect that, at the beginning of stressed economic periods, the volatility of reasonable SDFs should increase to reflect the overall larger absolute gap between the risk-neutral and objective probability measures. This is, therefore, the main hypothesis to be investigated by our empirical analysis below. In fact, by employing the Hansen and Jagannathan (1991) volatility bound, and assuming a mean-reverting process for the volatility of the SDF, Brennan, Wang and Xia (2004) show a strong counter-cyclical behavior of the volatility of the SDF under the ICAPM framework of Merton (1973). Note that we are interested in studying

⁶ This is the case since R_f is just a scaling factor.

⁷ For the probability space $(\Omega, \mathfrak{F}, P)$, $L^2(\Omega, \mathfrak{F}, P)$ denotes the space of the random variable with finite second moment, $E[|X|^2] < \infty$. For any two random variables X and Y belonging to $L^2(\Omega, \mathfrak{F}, P)$, Hölder's inequality establishes that, $E|XY| \leq [E|X|^2]^{1/2} [E|Y|^2]^{1/2}$.

whether the volatility of sensible SDFs predicts future economic cycles, rather than analyzing contemporaneous pricing relationships as in Brennan, Wang and Xia (2004).

3. Consumption-based Stochastic Discount Factors

Despite the fact that nondurable consumption growth betas have repeatedly failed to explain the cross-sectional variation of average returns, the recent U.S. and Spanish evidence has shown that the covariance of returns with consumption growth over the quarter of the return and many following quarters explains a considerable variation of expected returns.⁸ The main reason is that consumption is slow to adjust to returns. This is a very important result of the modern asset pricing literature because it maintains consumption as a primary determinant of the utility function of the representative agent.

At the same time, in a completely different setting, it has also recently been shown that small and value firms are more pro-cyclical than large and growth firms with respect to the growth rate of durable consumption.⁹ This suggests that durable versus nondurable consumption growth rates is a pro-cyclical state variable that accentuates the counter-cyclical behavior of marginal utility. Moreover, the inclusion of durable consumption can be done under recursive utility where the return of market equity wealth is part of the stochastic discount factor. Once again, this allows a higher volatility of the stochastic discount factor relative to specifications where only consumption growth is employed.¹⁰

Finally, habit persistence has shown to be a key preference representation in asset pricing modeling.¹¹ The reason is the extra volatility in marginal utility of consumption obtained throughout the behavior of the so called surplus consumption ratio which is the percentage difference between consumption and the level of habits.

We now briefly discuss the alternative SDFs employed in this paper. The well known SDF under power utility is given by

⁸ See Parker and Julliard (2005) and Márquez and Nieto (2007) for the US and Spanish markets respectively.

⁹ See Yogo (2006).

¹⁰ See Campbell (1996).

¹¹ See Campbell and Cochrane (1999), Li (2001) and Chen and Ludvigson (2004).

$$M_{t+1} = \beta \frac{U'(C_{t+1})}{U'(C_t)} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \quad (7)$$

where C_t is aggregate *per-capita* non-durable consumption as calculated in time t , $U'(C_t)$ is marginal utility, β is the subjective discount factor or impatience parameter, and γ is the coefficient of relative risk aversion. The set of parameters to be estimated is given by $\theta = \{\beta, \gamma\}$.

Parker and Julliard (2005) keep marginal utility of consumption as the key aggregate risk factor. They argue that consumption growth rates and stock returns do not covary contemporaneously as preferences in (7) indicate because agents' consumption takes time to respond to changes in wealth. The cost of adjusting consumption to current circumstances is greater than the cost of adjusting investment in financial assets. Furthermore, marginal utility of consumption is related to other slow-adjusting factors such as changes in labor earnings or property investments. Hence, they suggest measuring asset risk as the covariance between returns and consumption growth rate not only in the period to which returns refer, but also in several periods forward. They refer to this as ultimate consumption risk. They propose the following SDF

$$M_{t+1}^S = \beta^{S+1} \frac{R_{ft+1,t+1+S} U'(C_{t+1+S})}{U'(C_t)} \quad (8)$$

Under the power specification, the SDF takes the form

$$M_{t+1}^S = \beta^{S+1} R_{ft+1,t+1+S} \left(\frac{C_{t+1+S}}{C_t} \right)^{-\gamma} \quad (9)$$

where C_{t+1+S}/C_t is the consumption growth rate between t and $t+1+S$, and $R_{ft+1,t+1+S}$ is the risk-free rate corresponding to the same horizon. In the empirical specification below, we follow the finding of Márquez and Nieto (2007) who verify that, in the Spanish case, a three-year frame (S is 11 quarters) is the most appropriate

time lag for conciliating the consumption growth rate with current returns on equity assets.¹²

Contrary to the previous specifications, the SDF under recursive utility has the advantage of separating relative risk aversion and the elasticity of intertemporal substitution. Moreover, this SDF not only incorporates consumption growth but also the return on the market portfolio. In particular, under recursive utility, the contemporaneous SDF is given by,

$$M_{t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\eta} \right]^\kappa R_{mt+1}^{\kappa-1} \quad (10)$$

where η is the elasticity of intertemporal substitution, $\kappa \equiv \frac{1-\gamma}{1-1/\eta}$, and R_{mt} is the return on the market portfolio at any time t . The set of parameters to be estimated is given by $\theta = \{\beta, \gamma, \eta\}$.

Similarly, the specification under ultimate consumption risk and recursive utility becomes,

$$M_{t+1}^S = \left[\beta^{S+1} \left(\frac{C_{t+1+S}}{C_t} \right)^{-1/\eta} \right]^\kappa R_{mt+1+S}^{\kappa-1} R_{ft+1+S} \quad (11)$$

Yogo (2006) incorporates durable consumption to the marginal utility using a recursive preferences specification in which both types of consumption are not separable. The idea is, as usual, to increase the volatility of marginal consumption. The contemporaneous SDF is given by,

$$M_{t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\eta} \left(\frac{1-\alpha + \alpha (D_{t+1}/C_{t+1})^{(\rho-1)/\rho}}{1-\alpha + \alpha (D_t/C_t)^{(\rho-1)/\rho}} \right)^{(\eta-\rho)/(\eta(\rho-1))} R_{mt+1}^{1-1/\kappa} \right]^\kappa \quad (12)$$

¹² Somewhat surprisingly, the same time lag is found by Parker and Julliard with U.S. data.

where D_t is aggregate *per-capita* stock of durable consumption as calculated in time t , α is the expenditure share of the durable consumption good, and ρ is the elasticity of substitution between durable and non-durable consumption. Hence, the set of parameters is given by $\theta = \{\beta, \gamma, \eta, \alpha, \rho\}$.

As before, this paper analyzes the durable consumption-based asset pricing model under the perspective of ultimate consumption risk. In this case, the SDF becomes,

$$M_{t+1}^S = \left[\beta^{S+1} \left(\frac{C_{t+1+S}}{C_t} \right)^{-1/\eta} \left(\frac{1 - \alpha + \alpha (D_{t+1+S}/C_{t+1+S})^{(\rho-1)/\rho}}{1 - \alpha + \alpha (D_t/C_t)^{(\rho-1)/\rho}} \right)^{(\eta-\rho)/(\eta(\rho-1))} \right]^\kappa \times R_{mt+1+S}^{\kappa-1} R_{ft+1+S} \quad (13)$$

Finally, the SDF under the external habit persistence model of Campbell and Cochrane (1999) is given by,

$$M_{t+1} = \beta \left(\frac{SC_{t+1} C_{t+1}}{SC_t C_t} \right)^{-\gamma} \quad (14)$$

where H_t is the level of habits and $SC_t = \frac{C_t - H_t}{C_t}$ is a state variable known as “surplus consumption ratio” that allows to capture dependencies among states of nature. It is important to point out that SC_t is a recession indicator; it is low after several quarters of consumption declines and high in booms. It should be noted that the recognition of habits eliminates the need of including long-run consumption growth rates in the SDF. The nature of habits should be playing the equivalent role of ultimate consumption risk.

Under this specification, relative risk aversion changes with the surplus consumption ratio,

$$AR(SC_t) = \frac{\gamma}{SC_t} \quad (15)$$

Hence, with recessions, as consumption falls toward habit, people become less willing to tolerate further falls in consumption and they become more risk averse.

Next, we define the habit formation process. Level of habits can be written as a function of past consumption. We use consumption growth rates to ensure that the function is stationary to get

$$H_t = C_t g\left(\frac{C_{t-1}}{C_t}, \dots, \frac{C_{t-L}}{C_t}\right) \quad (16)$$

A reasonable function that guarantees $H_t < C_t$ is the following:

$$g(x) = h\left(1 + e^{-x}\right)^{-1} \quad (17)$$

where h is the global habit persistence parameter,

$x = \left(\delta \frac{C_{t-1}}{C_t} + \delta^2 \frac{C_{t-2}}{C_t} + \dots + \delta^L \frac{C_{t-L}}{C_t}\right)$, with $0 \leq h \leq 1$, $0 \leq \delta \leq 1$, and it is verified

that $0 \leq g(x) \leq 1$.

Therefore, the habit specification is given by

$$H_t = C_t h \left(1 + e^{-\left(\delta \frac{C_{t-1}}{C_t} + \delta^2 \frac{C_{t-2}}{C_t} + \dots + \delta^L \frac{C_{t-L}}{C_t}\right)} \right)^{-1} \quad (18)$$

In the actual estimation of this model, and to be consistent with ultimate consumption risk, L will be 12 quarters. The set of parameters to be estimated is $\theta = \{\beta, \gamma, h, \delta\}$.

4. Data and Some Preliminary Results

We have calculated quarterly real non-durable and durable Spanish consumption growth from 1962 to 2003. In fact, aggregate annual consumption data are available from 1954 to 2003. The Spanish National Accounts do not distinguish between non-

durable and durable consumption; however, there exists a detailed classification of consumption expenditure by type of goods in two alternative sources: Uriel, Moltó and Cucarella (2000) for the 1954-1994 period, and those published in the Spanish National Accounts for the years between 1995 and 2003. We have linked these series including, as non-durable consumption goods and services, the following items: food products, beverages and tobacco; clothing and footwear; household rental, heating and lighting; household entertainment goods and services; medical and healthcare services; maintenance of means of personal transport; use of public transport; communications; entertainment and culture; other goods and services. The following concepts are classified as durable consumption: furnishing, accessories and household goods; purchase of vehicles; articles related to entertainment, sport and culture; books, newspapers and magazines; teaching. In both cases, we use data at constant 1986 prices and the series are adjusted for seasonality.

Unfortunately, disaggregated quarterly data by type of consumption are not available for the Spanish case. We have converted annual into quarterly figures applying the procedure described in Casals, Jerez and Sotoca (2005).¹³

The nondurable good is entirely consumed in the period of purchase, whereas the durable good provides service flows for more than one period. We compute the quarterly service flow for period t , which we denote by D_t , using the following motion

$$D_t = \sum_{\tau=0}^{48} (1 - \tau dep) G_{t-\tau}, \quad (19)$$

where G_t is durable consumption expenditure in quarter t , dep is the quarterly depreciation rate which takes the value of 1.875 percent, that is consistent with the depreciation rates of motor vehicles published by the Ministry of Economy and Public

¹³ The desegregation has been made following standard state-space techniques. The desegregation employs the information contained in a quarterly instrument and the specific technique is based on the principle of empirical consistency. This implies that, given the aggregation constraint, the models relating the variables in high and low sampling frequencies should be mutually compatible. We use quarterly car registrations published by ANFAC (Spanish Motor Vehicle Manufacturers' Association) as the instrument for computing quarterly durable consumption and the total production of manufactured goods index published by the INE (Spanish Statistics Agency) as an indicator of quarterly non-durable consumption goods. The estimation procedures are implemented in a Matlab toolbox for time series modeling called E4, which can be downloaded at www.ucm.es/info/icae/e4.

Finance. The quarterly service flow series is computed using 48 durable consumption expenditure lags.¹⁴

Finally, we have real returns on the equally-weighted market portfolio and ten equally-weighted size-sorted portfolios from 1963 to 2003. Moreover, quarterly real interbank rate is used as the risk-free rate between 1963 and 1987. Since then, the real three-month Treasury bill rate is employed as the proxy for the riskless rate. The consumer price index has been used to deflate all nominal figures.

The Hansen and Jagannathan (1991) bound shows that the volatility of the stochastic discount factor satisfies the following relation:

$$\sigma(M) \geq \left[(I_N - E(M)E(R))' \Sigma^{-1} (I_N - E(M)E(R)) \right]^{1/2} \quad (20)$$

where I_N and R are the N -vector of ones and returns respectively, and Σ is the variance-covariance matrix of returns. Any sensible SDF should satisfy this bound. We will therefore use this expression to select feasible consumption-based SDFs.

In particular, the Hansen-Jagannathan volatility bound is estimated with realized returns on the ten size-sorted portfolios and for a range of different values for $E(M)$. Figure 3 displays the feasible region for the SDF implied by the available equity data from 1963 to 2003. The minimum standard deviation of the SDF associated to the realized mean risk-free rate (1.5 percent quarterly) is about 0.35, corresponding to a mean SDF of about 0.985.

Figure 4 displays the volatility of the SDF using expression (20) but now calculated with overlapping sub-periods of 5 years of quarterly data from ten size-sorted portfolios. Each point shown in Figure 4 is the volatility bound for the given average level of the risk-free interest rate for each of the sub-periods. As long as this volatility is associated with the distance between the risk-neutral and objective probabilities, as suggested in Section 2, we may identify these changes in volatility with time-varying

¹⁴ It must be pointed out that purchase of vehicles constitutes the principal component of the durable good expenditure. A linear depreciation assumption consisting in a 1.875 percent rate gives the durable good a value of 10 percent after 48 quarters, which concurs with the official depreciation rates published by the Ministry. The first observations had to be calculated with a smaller number of lags, since the original data started in 1954.

economic fears.¹⁵ Figure 4 also contains Spanish macroeconomic recession bars in grey and stock market recessions in salmon as roughly identified by a continuous decreasing period in the level of the Spanish stock market index.¹⁶ The stock market recessions tend to be slightly ahead of macroeconomic recessions. It is clear that there are substantial differences over time in the volatility of the SDF which satisfies the Hansen-Jagannathan bound. The issue is of course whether these differences are associated with changing recession fears reflecting the distance between the probability measures. In any case, and from an option pricing point of view, it is interesting to point out that the volatility of the SDF has not experienced a permanent increase after the crash of 1987, even though it displays an increase just before October 1987.

5. The Estimation of Parameters for Alternative SDF Specifications

This section selects appropriate parametric SDFs using consumption data and stock returns from the Spanish economy. The idea is to study the behavior of the volatility of alternative specifications of consumption-based SDFs. Appropriateness is understood as those SDFs that generate pairs of $E(M)$ and $\sigma(M)$ which enter in the feasible region shown in Figure 3.

All specifications discussed in Section 3 are analyzed here. In particular, we explore the traditional power utility of equation (7), the recursive utility specification of expression (10), the long-run version of recursive utility (*recursive long*) given by (11), the durable SDF suggested by Yogo (2006) as in equation (12), its long-run version (*Yogo long*) given by (13), and the habit persistence model described in equations (14) and (18). We also consider a power utility model with both durable and non-durable consumption growth rates. This former specification of the SDF and the traditional power SDF are not able to generate pairs of means and volatilities of M which enter in the Hansen-Jagannathan region. For this reason, we do not discuss the behavior of the volatility of the SDF for any of these two models.

¹⁵ Since the risk-free rate is not constant over the whole period, we multiply the Hansen-Jagannathan volatility bounds by the average risk-free rate for each sub-period to generate comparable upper bounds across time. This is consistent with expression (6).

¹⁶ The macroeconomic recession periods are taken from Gómez-Biscarri (2005) who locates the expansionary and recessionary periods by dating the turning points of the industrial production index using the Bry-Boschan procedure.

Specifically, for each SDF, we try a large grid of feasible preference parameters. We then select sets of combinations of those parameters that generate a volatility of the SDF which lies above the Hansen-Jagannathan bound of 0.35 reported in Figure 3. Given the selected SDFs, we compute the pricing error of each of these SDF in valuating the ten size-sorted portfolios returns. Figure 5 displays the ten pricing kernels with the lowest mean-squared pricing errors for each of the five SDF specifications analyzed. Finally, we choose, for each SDF specification, the preference parameters that simultaneously make the SDF to enter inside the feasible mean-volatility space and have the lowest error in pricing the ten portfolios. The results are shown in Table 1.

Table 1 contains the parameter estimates, the volatility of M , the mean of M , and the mean-squared pricing error for each of the five SDFs chosen throughout the empirical exercise. The lowest mean-squared pricing error is obtained for the habit preference specification. However, it should be noted the large volatility and the low mean of M generated by this specification. It is interesting to note that *recursive long* and *Yogo long* also have a relatively low mean-squared pricing error, but their success is also accompanied by relatively large volatility and low mean of M . On the other hand, the contemporaneous recursive and Yogo models generate a pair of $E(M)$ and $\sigma(M)$ which is very close to the minimum historical figures for feasible SDFs. Unfortunately, the mean-squared pricing error is higher than in the other three cases. In general, habit persistence and long-run consumption growth models have very volatile SDFs and quite low pricing errors. Moreover, the two long-run versions are able to generate these characteristics with very reasonable levels of risk aversion. In fact, both *recursive long* and *Yogo long* SDFs have a very similar behavior both on average and over time. It seems that the combination of long-run consumption growth and the inclusion of the market portfolio return in the SDF through recursive preferences are key properties of potentially valid SDFs. Both models potentially capture the business cycle behavior of the economy which probably explains the success in pricing the ten size-sorted portfolio returns.

Figure 6 represents, over time and across recessions, the five SDFs reported in Table 1. In general, all SDFs tend to be high at the very beginning of (or even just before) recessions and low at the end of recessions (beginning of expansions). This is particularly the case for the habit, *recursive long* and *Yogo long* specifications.

However, as expected, given the estimates reported in Table 1, the habit persistence model displays a very volatile behavior.

Figure 7 displays the volatility of the five SDFs estimated with five years of overlapping data.¹⁷ The volatility of the SDF for the *recursive long* and *Yogo long* cases seem to experience an increasing behavior at the beginning of recessions. Our logic indicates that the overall absolute distance between risk-neutral and objective probability distributions becomes also larger just before or at the beginning of recessions. Therefore, it seems that $\sigma(M)$ for the cases of SDFs with ultimate consumption risk and recursive preferences capture time-varying economic fears of investors. However, more formal tests are necessary before reaching further and more precise conclusions.

6. Economic Cycles, Financial Market Uncertainty and the Volatility of Consumption-based Stochastic Discount Factors

As we have just argued, the volatility of the SDF may reflect the distance between the risk-neutral and objective probability distributions, which contains economic fears implicit in the investment behavior of investors. If so, the volatility of the SDF not only should contain information about the economic uncertainty, but it should also be able to predict future realized macroeconomic cycles. Both features are analyzed in this section.

The first analysis consists on determining whether the overlapping standard deviation of our five SDF specifications incorporates information about the future of two selected state variables: the growth rate of the industrial production index and the stock market returns.

We therefore perform the following OLS autocorrelation-robust-standard-error regressions for our five alternative SDF specifications:

$$\frac{IPI_{t+j} - IPI_t}{IPI_t} = \alpha + \beta\sigma(M_t) + \varepsilon_{t+\tau}, \quad \tau = 1, 2, 4, 8, 12 \quad (21)$$

¹⁷ These volatilities are multiplied by the corresponding average risk-free rate over the period.

where IPI_t is the quarterly Industrial Production Index for quarter t .

A similar regression is run for the market return index,

$$R_{mt+j} = \alpha + \beta\sigma(M_t) + \varepsilon_{t+\tau}, \tau = 1, 2, 4, 8, 12 \quad (22)$$

The estimation results of regressions (21) and (22) are reported in Panels A and B of Table 2, respectively. Both Panels show that for all specifications of the SDF, increases in the volatility of the SDFs are significantly associated with both a recession in the macroeconomic cycle of the Spanish economy and decreasing stock market behaviour. This is consistent with Figure 7. Moreover, for most cases, the predicting ability of the volatility of SDF increases with the horizon used in regressions. Hence, from an empirical point of view, the volatility of appropriated-selected consumption-based SDFs seems to be a powerful measure of future economic cycles. We may therefore be confident arguing that higher volatility of sensible SDFs reflects in fact a larger distance between the risk-neutral and objective probabilities. Hence, larger implicit fears in the stock market seem to be significantly negatively correlated with future changes in industrial production and stock market returns.

Because the volatility of the discount factor is very persistent, we also calculate the bias-corrected estimator and the corresponding bias-corrected t -statistic proposed by Amihud and Hurvich (2004). These authors suggest an augmented regression method for hypothesis testing in predictive regressions with multiple autoregressive predictor variables. Their simulations show that their adjustment outperforms other bias-correction methods such as those suggested by Stambaugh (1999) or Lewellen (2004). Although the new t -statistics tend to be lower than the ones reported, these adjustments do not change the qualitative conclusions drawn from Table 2.

Interestingly, the largest explanatory power, when forecasting macroeconomic cycles at any horizon, corresponds to the recursive preference specification with contemporaneous consumption. On the other hand, we need to incorporate ultimate consumption risk if we want to explain the future behaviour of stock market returns. This is especially true for longer horizons. It is probably reasonable to recall that SDF specifications with recursive preferences and long-run consumption growth provide a

low pricing error and a relatively high volatility of the SDF. In this sense, the results from Panel B of Table 2 are consistent with the evidence reported in Table 1.

The second question is whether the consumption-based SDFs and their volatilities are able to explain forward looking measures of stock market uncertainty and risk aversion.

As a measure of market uncertainty, we employ the implied volatility of ATM call and put options on the future of the Spanish market stock index. Unfortunately, we only have monthly implied volatilities from October 1996 to December 2004.¹⁸ On the other hand, the difference between the monthly probabilistic mass assigned to the 10 percent left tail of the risk-neutral distribution and the risk-adjusted distribution under a power utility function will be used as a measure of risk aversion. The extreme left-tailed events corresponds to bad states of nature which suggests that this difference should always be positive, since risk-neutral probabilities tend to pay more attention to unpleasant states relative to objective probabilities. This is the risk adjustment implicit in risk-neutral pricing and can be interpreted as a measure of ex-ante risk aversion. These differences in probabilistic mass are also available only from October 1996 to December 2004.¹⁹

Figure 8 displays our two measures of ex-ante uncertainty and risk aversion.²⁰ The similarities between both variables are striking. It suggests that implied volatility contains information about the distance between the risk-neutral and objective probability measures in the left tail of the distribution. Hence, implied volatility seems to incorporate implicit fears that investors have about potential crashes of the stock market. In other words, the difference between both probabilistic masses reflects the extra probability associated with unpleasant states of nature. Interestingly, it seems that implied volatility extracted from option prices also contains this extra probability.

¹⁸ These data are the same used by Alonso, Blanco and Rubio (2006) when estimating risk-neutral and risk-adjusted densities in the Spanish option market.

¹⁹ Alonso, Blanco and Rubio (2006) explore alternative utility specifications. Interestingly, independently of the stochastic discount factor employed, they cannot reject the hypothesis that risk-adjusted densities provide adequate predictions of the distributions of future realizations of the Spanish market index at four and eight-week horizons. Hence, all risk-adjusted densities generate similar forecasting statistics. In our case, we just take the simplest risk adjustment from the power utility specification. As discussed in footnotes 2 and 3 of this work, we impose a risk aversion coefficient of 1.67, which is the implied level of risk aversion that maximize the predicting ability of the resulting risk-adjusted density.

²⁰ They are transformed into quarterly figures.

Despite the fact that we have few observations on these measures, we perform a series of regressions to illustrate whether the SDF specifications and their volatilities are able to explain ex-ante uncertainty and risk aversion. In particular, we run the following OLS autocorrelation-robust-standard-error regressions for our five alternative SDF specifications:

$$\frac{IV_t - IV_{t-1}}{IV_{t-1}} = \alpha + \beta M_t + \varepsilon_{t+1} \quad (23)$$

$$IV_t = \alpha + \beta M_t + \varepsilon_{t+1} \quad (24)$$

$$IV_t = \alpha + \beta \sigma(M_t) + \varepsilon_{t+1} \quad (25)$$

where IV_t is the implied volatility of ATM options at the last month of quarter t , IV_{t-1} is the implied volatility at the last month of the previous quarter, and M_t is the stochastic discount factor for quarter t .

Moreover, we also run the following regressions,

$$\frac{LT_t - LT_{t-1}}{LT_{t-1}} = \alpha + \beta M_t + \varepsilon_{t+1} \quad (26)$$

$$LT_t = \alpha + \beta M_t + \varepsilon_{t+1} \quad (27)$$

$$LT_t = \alpha + \beta \sigma(M_t) + \varepsilon_{t+1} \quad (28)$$

where LT_t is the difference between the probabilistic mass assigned by the risk-neutral distribution and the risk-adjusted distribution for the 10 percent left-tail at the last month of quarter t .

The results from regressions (23), (24) and (25) are reported in Table 3, Panels A, B, and C respectively. Independently of the specification employed, the results show that the recursive SDF and Yogo's specification with contemporaneous consumption are able to explain market uncertainty as measured by implied volatility. Both, the SDF itself and its volatility are positively and significantly related with ex-ante uncertainty.

As expected, given the results from Figure 8, the results using our measure of risk aversion are the same. This evidence is contained in Table 4 with Panels A, B, and C for regressions (26), (27), and (28), respectively.

We may therefore conclude that, when trying to explain short-run uncertainty or risk aversion, the simplest recursive utility specification is powerful enough to significantly describe either levels or changes or uncertainty and risk aversion. The caveat is, of course, that we have a very limited time-series of uncertainty and risk aversion measures. In this sense, SDFs with ultimate consumption risk have even less observations. Moreover, these types of specifications clearly capture economic cycles. However, implied volatilities experience pronounced changes from one month to another. This represents a serious difficulty for either habit or ultimate consumption risk-based SDFs when trying to explain uncertainty or risk aversion from one period to another.²¹

7. A Robustness Analysis using U.S. Market Data

We use seasonally adjusted quarterly aggregate nominal expenditure on consumer nondurable and services for the period 1962-2003 from National Income and Product Accounts (NIPA). We also take population numbers and price deflator from NIPA to construct the time series of per capita real nondurable consumption numbers necessary for this section. On the other hand, durable consumption consists of items such as motor vehicles, furniture and appliances, and jewelry and watches. These are also taken from NIPA and we follow Yogo's procedure to construct the corresponding quarterly time series. The returns on the ten size-sorted portfolios, the risk-free return and the market returns are taken from Kenneth French's website.

The procedure is exactly the same followed for the Spanish case. First of all, Figure 9 displays the overlapping 5-years sub-periods of the volatility of the SDF estimated by equation (5).²² The volatility tends to increase in the quarters before macroeconomic recessions as defined by the NBER. Secondly, the consumption-based

²¹ See the related evidence of Beber and Brandt (2006). They find that when there is a lot of ex-ante uncertainty about macroeconomic fundamentals and new data is released, overall uncertainty and implied volatility significantly diminish in the U.S. market.

²² As in the Spanish case, we multiply the volatility by the average risk-free rate of each sub-period.

SDF of section 3 are estimated for the U.S. case. Once again, the smaller pricing errors are for the long-run consumption risk specifications. Interestingly, the habit persistence model displays a pricing error as high as the contemporaneous recursive and durable models. Thirdly, the selected SDFs are displayed in Figure 10, and their volatilities are contained in Figure 11. In both cases, it seems clear that the behavior of the SDFs closely follow the behavior of the Spanish counterparts. Fourthly, the predicting ability of the volatility of the SDFs is not as strong as in the Spanish case. However, once again, the recursive and Yogo specifications have more forecasting ability for macroeconomic recessions, while stock market returns are better captured by the long-run specifications. Finally, uncertainty is measured by the CBOE Volatility Index (VIX), which has become the benchmark for the U.S. stock market implied volatility. Although the magnitudes of the R -squares are lower than for the Spanish case, the SDFs based on recursive and Yogo specifications also show a better explanatory power than its long-run counterparts. These overall similarities tend to provide a reasonable level of confidence on the results using Spanish data.

8. Conclusions

In this paper we present convincing empirical evidence showing that the volatility of appropriated-selected consumption-based SDFs measure implicit recession fears of investors. The volatility of the SDF specifications with lowest pricing errors and high volatility are reported to have a reasonable predicting ability of future market recessions. In particular, a recursive utility SDF with long-run consumption risk (either with or without durable goods) are able to explain up to a 23.5 percent of future market returns at long horizons. The volatility of the habit-based SDF also has a good forecasting capacity at short horizons. This seems to be related with the extremely high volatility of this SDF.

On the other hand, the volatility of the SDF specification with recursive preferences and contemporaneous consumption growth, which is characterized by a relatively low volatility, seems to be the most powerful specification in capturing future macroeconomic cycles approximated by changes in the industrial production index. Thus, the volatility of this SDF explains between 9.3 and 34.4 percent of the future

macroeconomic growth at short and long horizons respectively. Interestingly, the SDF under the recursive-based utility specification also explains up to 40.5 percent of market uncertainty and risk aversion.

Given the results from the robustness analysis using U.S. data, we may conclude that the volatility of consumption-based SDFs seems to be a powerful indicator of both economic and stock market cycles. It suggests a strong connection between financial markets and the real economy which deserves future attention. Extending this work to additional economies and performing truly out-of-sample tests seem to be a promising future avenue of research. Additionally, the use of SDFs based on a rational disappointment aversion utility function that embeds downward risk, as discussed by Ang, Chen and Xing (2006), may be a potentially interesting way of directly modeling larger aversion to losses relative to the attraction to gains.

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Table 1
 Estimated Parameters and Moments for Alternative Stochastic Discount Factors
 with Lowest Pricing Error
 1965-2003

SDF	β	γ	η	α	ρ	δ	h	$E(M)$	$\sigma(M)$	<i>Pricing Error</i>
Recursive	0.880	29	-0.05	N.A.	N.A.	N.A.	N.A.	0.9777	0.3561	0.2891
Recursive Long-Run Growth	0.925	1.5	-0.05	N.A.	N.A.	N.A.	N.A.	0.9263	0.5458	0.0573
Yogo	0.820	25	-0.05	0.85	0.95	N.A.	N.A.	0.9650	0.3893	0.1472
Yogo Long-Run Growth	0.980	1.2	-0.02	0.95	0.85	N.A.	N.A.	0.9299	0.5536	0.0576
Habit	0.800	150	N.A.	N.A.	N.A.	0.50	0.99	0.9285	0.8668	0.0342

β is the subjective discount factor for future period utility; γ is the coefficient of relative risk aversion; η is the elasticity of intertemporal substitution; α is the expenditure share of the durable consumption good; ρ is the elasticity of substitution between durable and non-durable consumption; δ is the weight associated with past consumption; h is the global habit persistence parameter; and *pricing error* is the mean squared error over ten size-sorted portfolios.

Table 2

<i>Panel A: Future Industrial Production Index and the Volatility of SDFs</i>											
$\frac{IPI_{t+1} - IPI_t}{IPI_t} = \alpha + \beta\sigma(M_t) + \varepsilon_{t+1}$											
		<i>Recursive</i>		<i>Yogo</i>		<i>Habit</i>		<i>Recursive Long</i>		<i>Yogo Long</i>	
<i>1 Quarter</i>	<i>Constant</i>	0.024	(4.00) ^{1/}	0.019	(3.66)	0.016	(4.40)	0.019	(4.42)	0.019	(4.41)
	<i>Slope</i>	-0.049	(-2.78)	-0.039	(-2.38)	-0.012	(-2.53)	-0.030	(-2.92)	-0.030	(-2.90)
	<i>R² (%)</i>	9.27		6.73		7.83		7.90		7.95	
<i>2 Quarters</i>	<i>Constant</i>	0.047	(3.88)	0.038	(3.63)	0.033	(4.45)	0.038	(4.41)	0.038	(4.40)
	<i>Slope</i>	-0.095	(-2.71)	-0.077	(-2.38)	-0.024	(-2.66)	-0.060	(-2.90)	-0.058	(-2.87)
	<i>R² (%)</i>	16.08		12.07		14.59		13.91		13.99	
<i>4 Quarters</i>	<i>Constant</i>	0.095	(3.91)	0.078	(3.76)	0.063	(4.30)	0.074	(4.33)	0.074	(4.32)
	<i>Slope</i>	-0.191	(-2.79)	-0.155	(-2.53)	-0.045	(-2.66)	-0.115	(-2.82)	-0.112	(-2.79)
	<i>R² (%)</i>	20.74		15.83		16.32		16.42		16.52	
<i>8 Quarters</i>	<i>Constant</i>	0.189	(4.69)	0.156	(4.51)	0.108	(3.65)	0.147	(4.80)	0.146	(4.81)
	<i>Slope</i>	-0.382	(-3.55)	-0.315	(-3.35)	-0.064	(-2.13)	-0.234	(-3.36)	-0.227	(-3.34)
	<i>R² (%)</i>	31.05		24.74		12.25		25.69		25.66	
<i>12 Quarters</i>	<i>Constant</i>	0.271	(4.82)	0.225	(4.56)	0.154	(3.59)	0.220	(5.18)	0.218	(5.21)
	<i>Slope</i>	-0.531	(-3.56)	-0.442	(-3.33)	-0.085	(-2.02)	-0.352	(-3.92)	-0.342	(-3.92)
	<i>R² (%)</i>	34.41		28.04		12.47		33.23		33.32	
<i>Panel B: Future Market Portfolio Returns and the Volatility of SDFs</i>											
$R_{m,t+1} = \alpha + \beta\sigma(M_t) + \varepsilon_{t+1}$											
		<i>Recursive</i>		<i>Yogo</i>		<i>Habit</i>		<i>Recursive Long</i>		<i>Yogo Long</i>	
<i>1 Quarter</i>	<i>Constant</i>	0.074	(2.27)	0.040	(1.41)	0.080	(3.59)	0.093	(3.42)	0.092	(3.43)
	<i>Slope</i>	-0.155	(-1.73)	-0.055	(-0.65)	-0.086	(-3.31)	-0.199	(-2.92)	-0.194	(-2.91)
	<i>R² (%)</i>	1.93		0.28		8.39		7.16		7.17	
<i>2 Quarters</i>	<i>Constant</i>	0.150	(2.39)	0.083	(1.51)	0.159	(3.40)	0.192	(3.43)	0.190	(3.45)
	<i>Slope</i>	-0.311	(-1.85)	-0.113	(-0.70)	-0.166	(-3.12)	-0.414	(-3.02)	-0.403	(-3.02)
	<i>R² (%)</i>	3.14		0.48		12.55		12.39		12.45	
<i>4 Quarters</i>	<i>Constant</i>	0.329	(2.64)	0.187	(1.74)	0.289	(2.80)	0.416	(3.52)	0.412	(3.54)
	<i>Slope</i>	-0.684	(-2.24)	-0.273	(-0.91)	-0.277	(-2.40)	-0.888	(-3.31)	-0.866	(-3.31)
	<i>R² (%)</i>	5.52		1.03		12.79		20.54		20.70	
<i>8 Quarters</i>	<i>Constant</i>	0.637	(2.58)	0.354	(1.88)	0.517	(2.18)	0.894	(3.35)	0.884	(3.36)
	<i>Slope</i>	-1.178	(-2.27)	-0.337	(-0.68)	-0.402	(-1.54)	-1.860	(-3.46)	-1.811	(-3.47)
	<i>R² (%)</i>	4.28		0.41		7.10		23.39		23.49	
<i>12 Quarters</i>	<i>Constant</i>	0.787	(2.07)	0.463	(1.71)	0.733	(2.05)	1.295	(2.98)	1.282	(2.99)
	<i>Slope</i>	-1.065	(-1.43)	-0.070	(-0.10)	-0.454	(-1.24)	-2.501	(-2.90)	-2.433	(-2.91)
	<i>R² (%)</i>	1.68		0.01		4.35		20.35		20.40	

1/ Robust *t*-statistics in parentheses

IPI_t is the industrial production index in quarter t , R_{mt} is the market return in quarter t , and $\sigma(M_t)$ is volatility of the stochastic discount factor for quarter t . This volatility is estimated quarterly with five years of data.

Table 3

<i>Panel A: Changes in Implied Volatility and Stochastic Discount Factors</i>			
$\frac{IV_t - IV_{t-1}}{IV_{t-1}} = \alpha + \beta M_t + \varepsilon_{t+1}$			
SDF	Constant ($\hat{\alpha}$)	Slope ($\hat{\beta}$)	R^2 (%)
Habit	0.052 (0.58) ^{1/}	-0.002 (-0.05)	0.0
Recursive	-0.777 (-2.71)	0.889 (3.10)	36.4
Yogo	-0.822 (-2.66)	0.932 (2.96)	32.5
Recursive Long	0.191 (0.74)	-0.130 (-0.43)	0.9
Yogo Long	0.188 (0.71)	-0.125 (-0.41)	0.9
<i>Panel B: Implied Volatility and Stochastic Discount Factors</i>			
$IV_t = \alpha + \beta M_t + \varepsilon_{t+1}$			
SDF	Constant ($\hat{\alpha}$)	Slope ($\hat{\beta}$)	R^2 (%)
Habit	0.275 (8.10)	0.001 (0.08)	0.0
Recursive	0.073 (1.63)	0.221 (4.54)	40.7
Yogo	0.068 (1.39)	0.225 (4.13)	34.3
Recursive Long	0.170 (2.77)	0.114 (1.36)	11.3
Yogo Long	0.169 (2.76)	0.115 (1.38)	11.6
<i>Panel C: Implied Volatility and the Volatility of the Stochastic Discount Factors</i>			
$IV_t = \alpha + \beta \sigma(M_t) + \varepsilon_{t+1}$			
SDF	Constant ($\hat{\alpha}$)	Slope ($\hat{\beta}$)	R^2 (%)
Habit	0.297 (3.89)	-0.031 (-0.29)	0.3
Recursive	0.049 (0.32)	0.807 (1.49)	10.2
Yogo	-0.075 (-0.50)	1.480 (2.33)	17.4
Recursive Long	0.312 (3.43)	-0.147 (-0.42)	0.5
Yogo Long	0.291 (3.42)	-0.062 (-0.19)	0.1

1/ Robust t -statistics in parentheses

IV_t is the implied volatility of ATM options at the last month of quarter t , IV_{t-1} is the implied volatility at the last month of the previous quarter, M_t is the stochastic discount factor for quarter t , and $\sigma(M_t)$ is volatility of the stochastic discount factor for quarter t . This volatility is estimated quarterly with five years of data.

Table 4

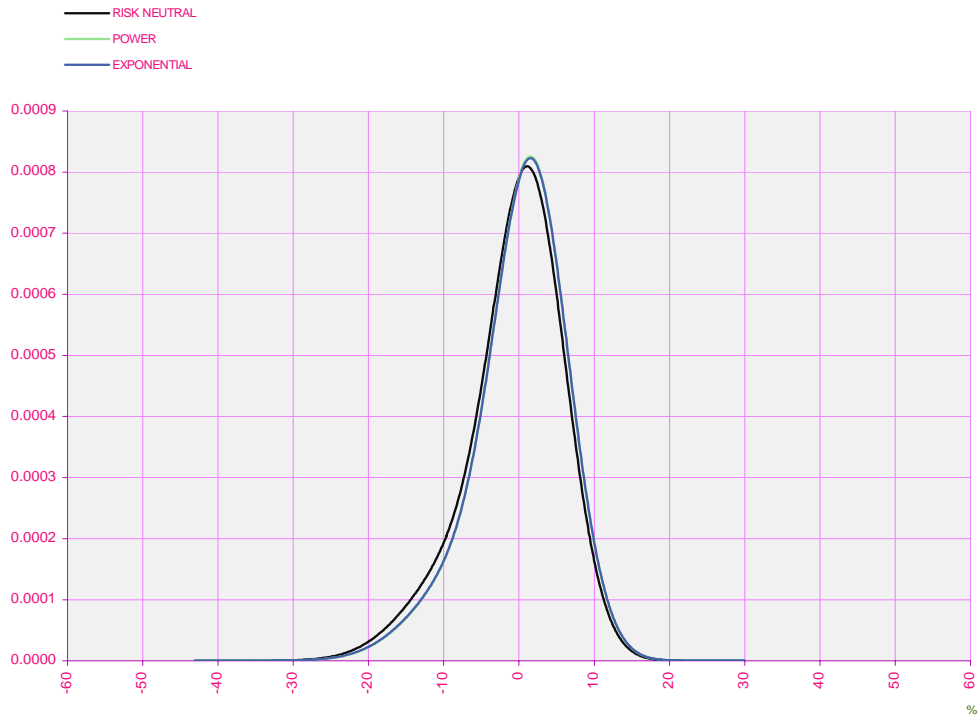
<i>Panel A: Changes in Left-Tailed Probabilistic Mass Differences and Stochastic Discount Factors</i>			
$\frac{LT_t - LT_{t-1}}{LT_{t-1}} = \alpha + \beta M_t + \varepsilon_{t+1}$			
SDF	Constant ($\hat{\alpha}$)	Slope ($\hat{\beta}$)	R^2 (%)
Habit	0.287 (1.24) ^{1/}	-0.002 (-0.01)	0.0
Recursive	-1.674 (-1.88)	2.108 (2.15)	25.8
Yogo	-1.716 (-1.86)	2.139 (2.10)	21.6
Recursive Long	1.130 (1.91)	-0.933 (-1.42)	9.9
Yogo Long	1.118 (1.87)	-0.914 (-1.38)	9.8
<i>Panel B: Left-Tailed Probabilistic Mass Differences and Stochastic Discount Factors</i>			
$LT_t = \alpha + \beta M_t + \varepsilon_{t+1}$			
SDF	Constant ($\hat{\alpha}$)	Slope ($\hat{\beta}$)	R^2 (%)
Habit	0.028 (4.22)	-0.001 (-0.52)	0.3
Recursive	-0.012 (-1.42)	0.042 (4.28)	40.6
Yogo	-0.014 (-1.41)	0.043 (3.93)	34.8
Recursive Long	0.004 (0.33)	0.024 (1.40)	12.4
Yogo Long	0.004 (0.31)	0.024 (1.43)	12.8
<i>Panel C: Left-Tailed Probabilistic Mass Differences and the Volatility of the Stochastic Discount Factors</i>			
$LT_t = \alpha + \beta \sigma(M_t) + \varepsilon_{t+1}$			
SDF	Constant ($\hat{\alpha}$)	Slope ($\hat{\beta}$)	R^2 (%)
Habit	0.034 (2.32)	-0.011 (-0.56)	1.2
Recursive	-0.018 (-0.65)	0.158 (1.55)	10.8
Yogo	-0.041 (-1.55)	0.282 (2.49)	17.4
Recursive Long	0.032 (1.79)	-0.026 (-0.38)	0.4
Yogo Long	0.028 (1.68)	-0.009 (-0.14)	0.1

1/ Robust t -statistics in parentheses

LT_t is the difference between the probabilistic mass assigned by the risk-neutral distribution and the risk-adjusted distribution for the 10% left-tail at the last month of quarter t , LT_{t-1} is the same difference at the last month of the previous quarter, M_t is the stochastic discount factor for quarter t , and $\sigma(M_t)$ is volatility of the stochastic discount factor for quarter t . This volatility is estimated quarterly with five years of data.

Figure 1
Estimated Risk-Neutral Probability Density Functions from a Cross-section of Option Prices and Risk-Adjusted Density Functions with Power and Exponential Utility Functions before and after September 11th, 2001

A. 24 AUGUST 2001



B. 21 SEPTEMBER 2001

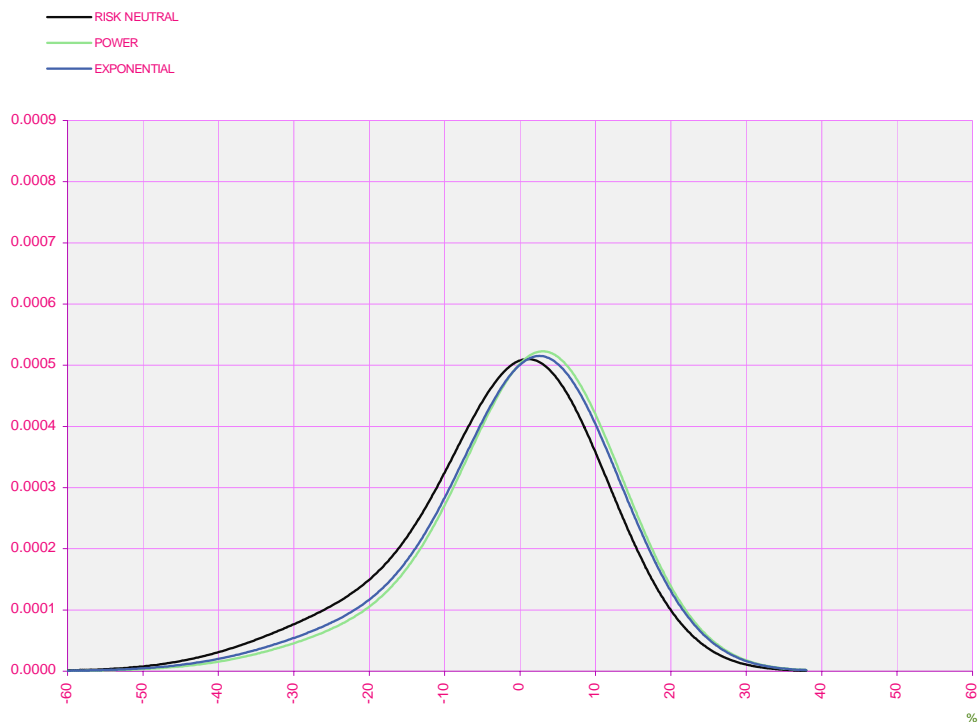


Figure 2
 Difference between the Monthly Probabilistic Mass Assigned to the 10% Left Tail of
 the Risk-Neutral and the Risk-Adjusted Density Functions from 1996 to 2004

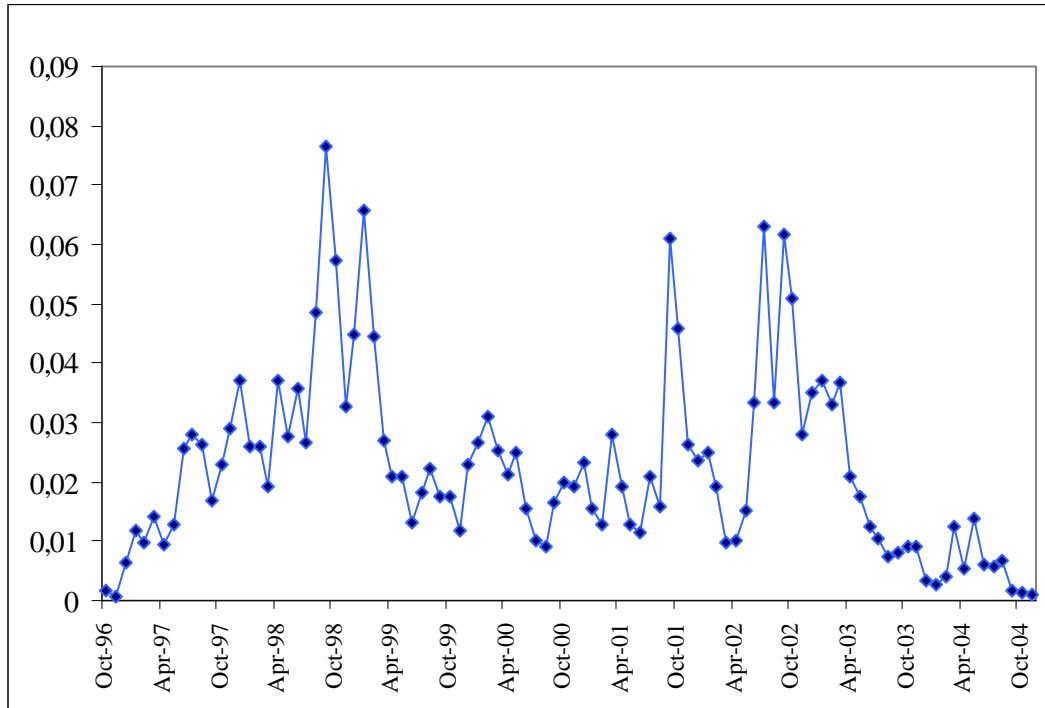


Figure 3
Hansen-Jagannathan Volatility Bound
1963-2003

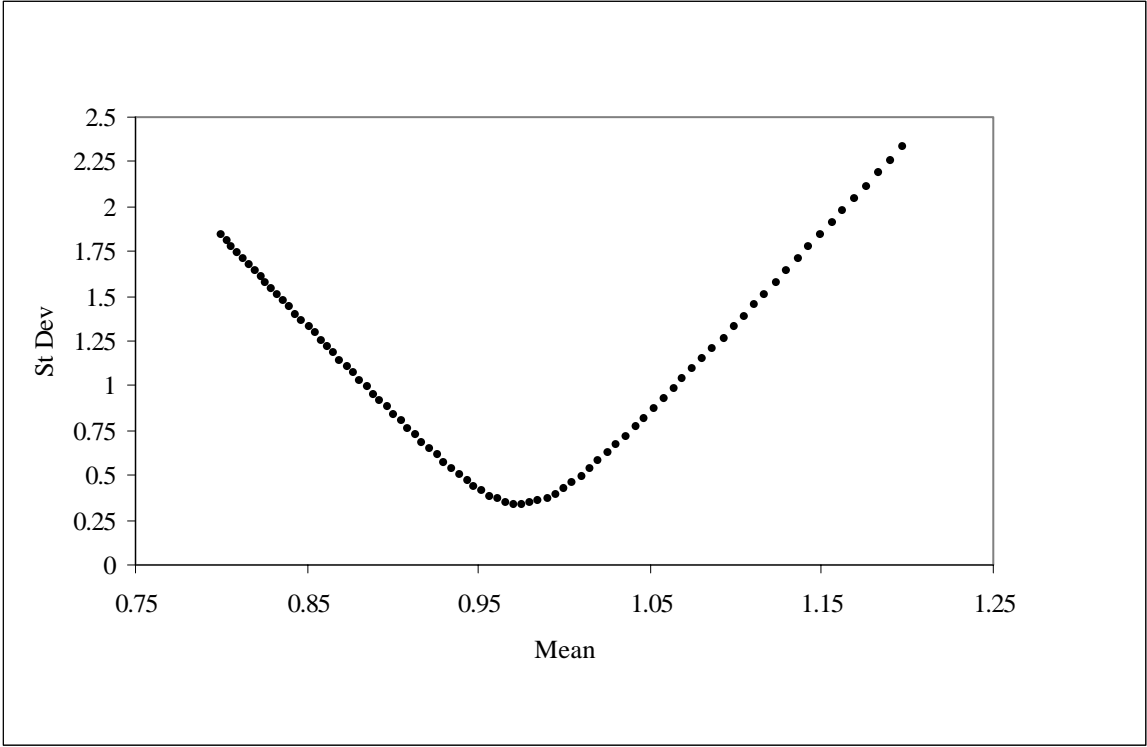
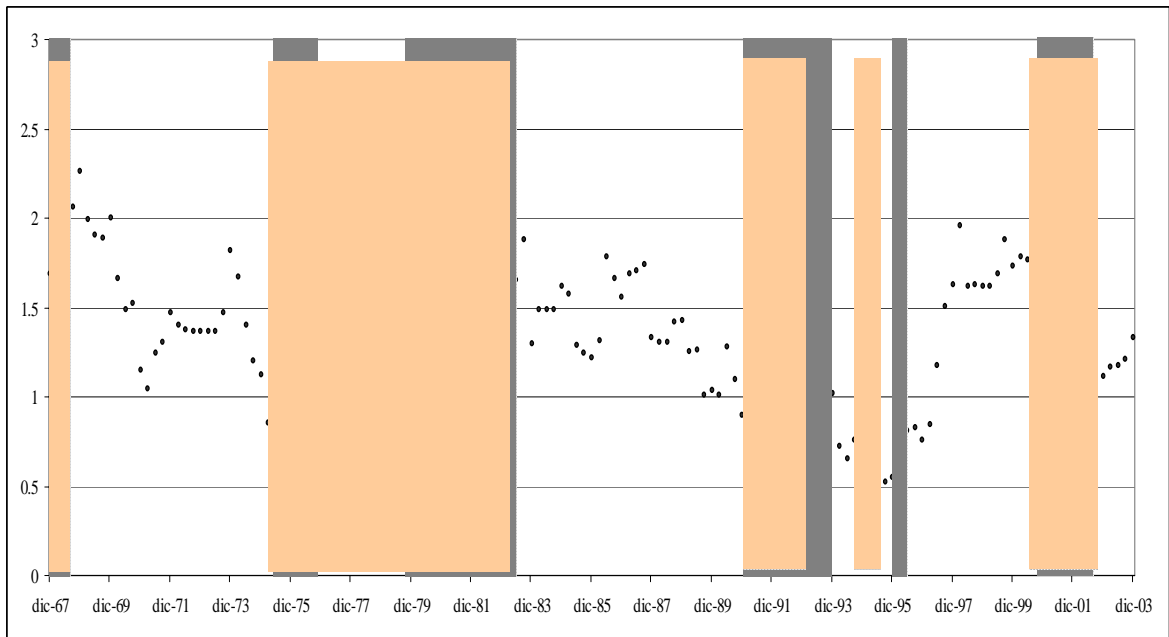


Figure 4
Hansen-Jagganathan Volatility Bound by Overlapping Five-Year Sub-periods
1963-2003



- HJ Bound
- Market Recessions
- Macroeconomic Recessions

Figure 5
Hansen-Jagannathan Volatility Bound and Stochastic Discount Factor Models
1963-2003

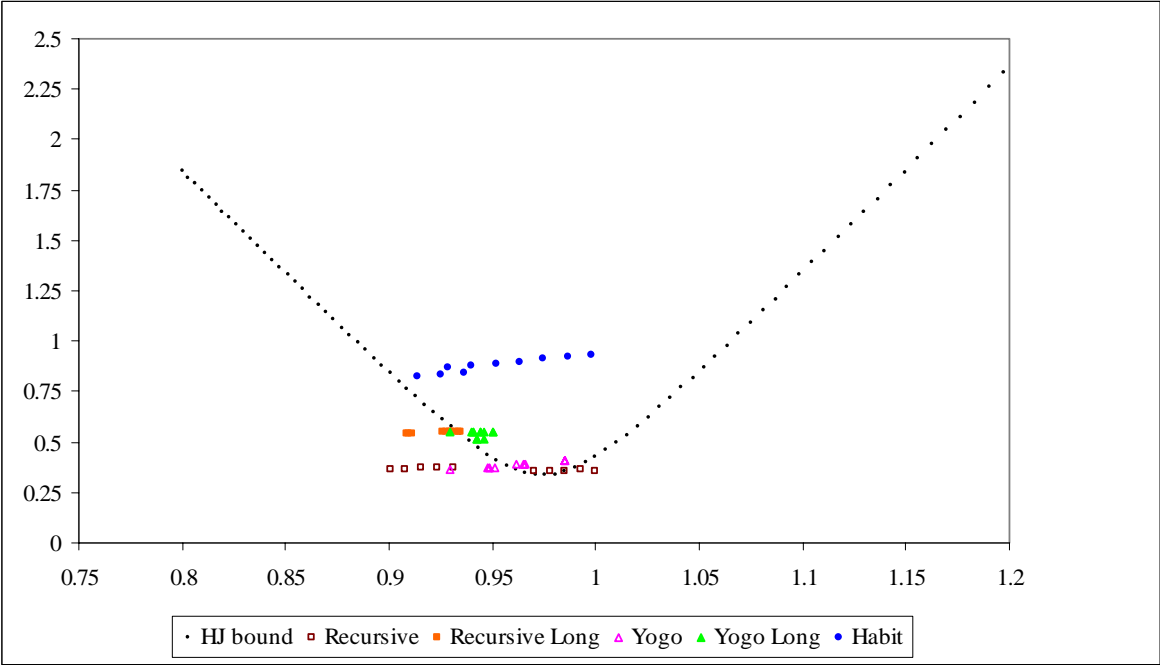


Figure 6
Stochastic Discount Factors during Market and Macroeconomic Recessions
1963-2003

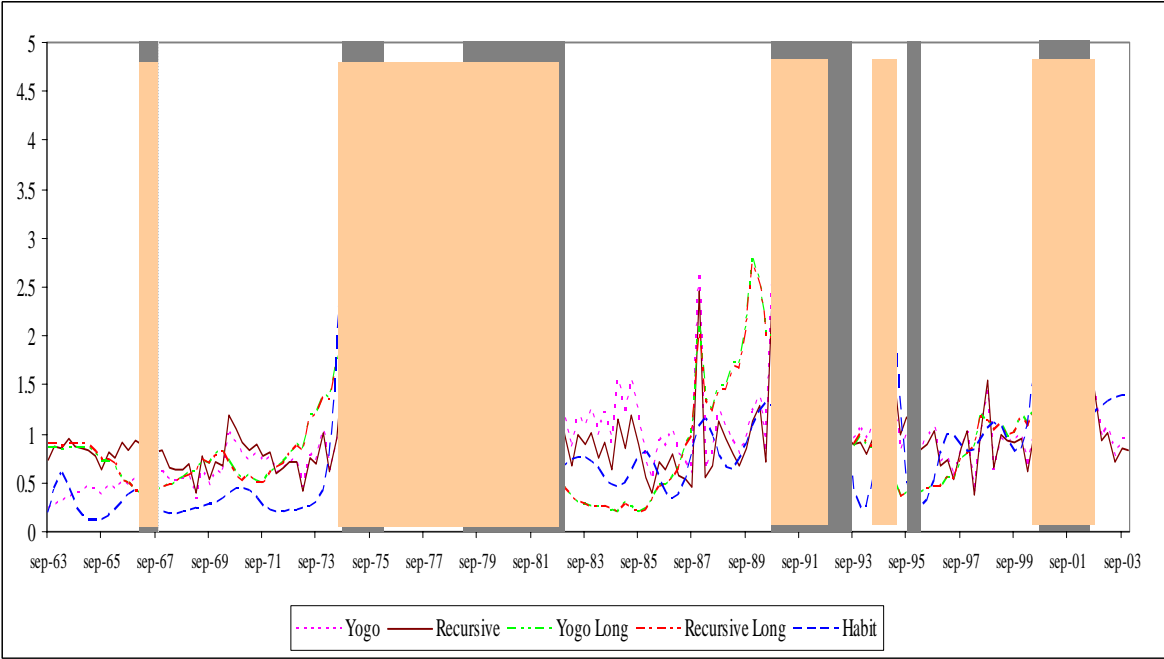


Figure 7
 Stochastic Discount Factor Volatilities during Market
 and Macroeconomic Recessions
 1963-2003

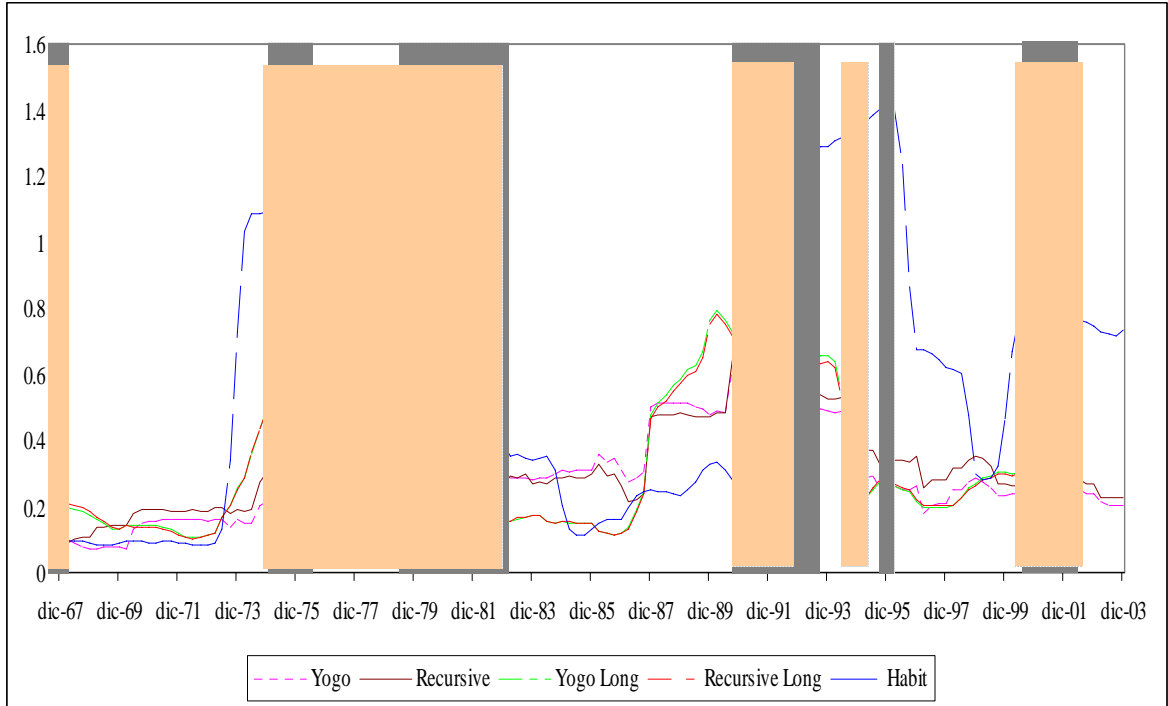


Figure 8
Capturing Uncertainty: Left-tail Probability Difference between Risk-neutral
and Risk-adjusted Density Functions and Option-Implied Volatility
1996-2004

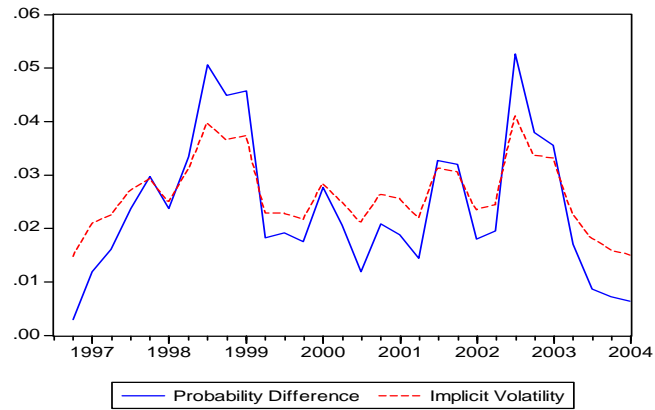


Figure 9
Hansen-Jagannathan Volatility Bound by Overlapping Five-Year
Sub-periods for the U.S. Market
1963-2003

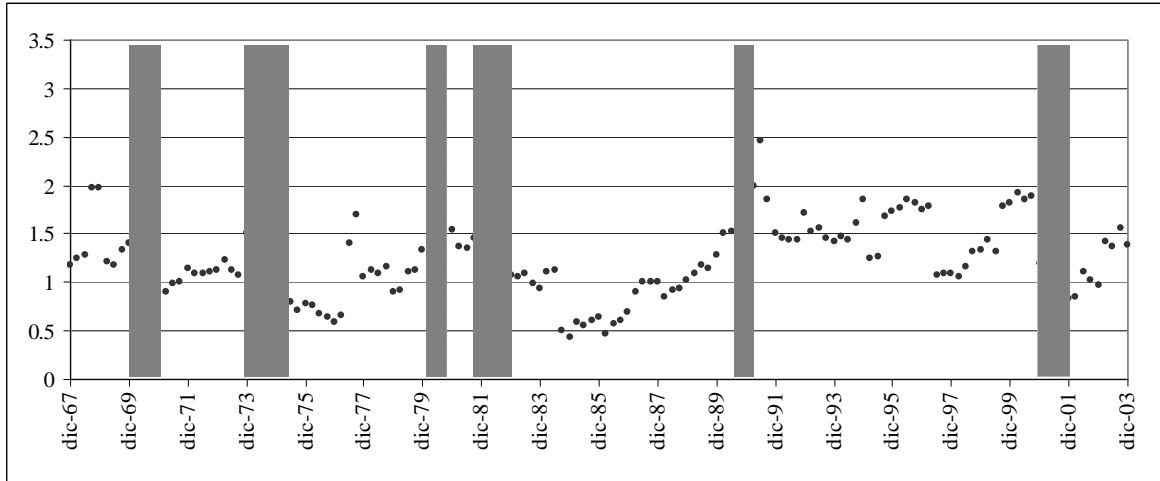


Figure 10
Stochastic Discount Factors during Macroeconomic Recessions for the U.S. Market
1963-2003

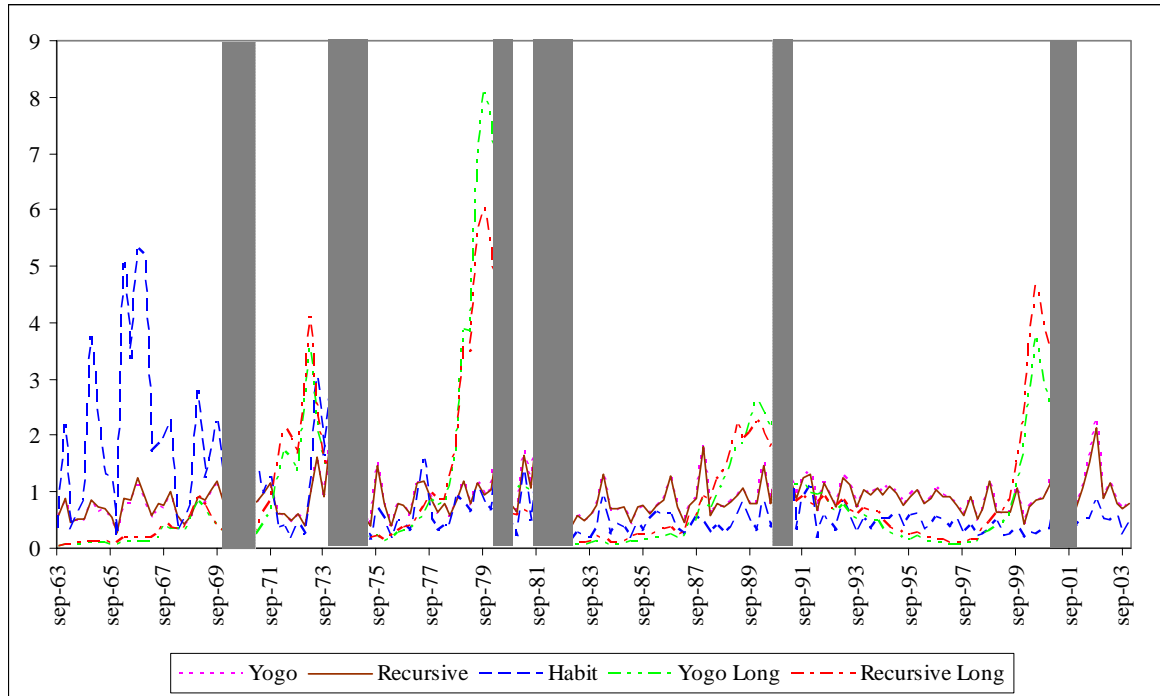


Figure 11
Stochastic Discount Factor Volatilities during Macroeconomic Recessions
for the U.S. Market
1963-2003

