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Are Large Windows Efficient? Evolution of Learning Rules in a Bargaining Model*

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Abstract

We endogenize the learning rules in a modified version of Young's (1993b). The Nash Demand Game is played by two different populations. Players choose their strategies in the light of some limited information about the strategies players from the other population have used in the past. The better informed population has higher bargaining power. The main drawback in Young's model is that the amount of information, and therefore the bargaining powers, are fixed exogenously. We endogenize players' learning rules and test for evolutionary stability. We study whether one population using a particular learning rule can be invaded by a mutant learning rules. We show that, when information is costless, the only evolutionarily stable learning rule maximizes players' information. If both populations follow the same learning rule, the equilibrium which is selected is the symmetric Nash bargaining solution. When information is costly there is a trade-off between costly learning and the rewards of being well informed. Finally we show that an economy populated by players who follow very simple imitative rules is socially more efficient than an economy of rational players.

1 Introduction

A standard feature of bargaining games is their multiplicity of equilibria. In the early fifties, Nash proposed two different approaches to solve the multiplicity problem. In a first paper, Nash [6] formulated a set of axioms (Invariance, Symmetry, Pareto Efficiency and Independence of Irrelevant Alternatives) which define properties that the outcome is required to satisfy, and which turn out to characterize a unique solution to the bargaining problem. Nash describes a 'bargaining problem' with all von Neumann and Morgenstern utility pairs representing the possible agreements available to the bargainers, and the utility pair that results in the case that no agreement is reached (the status quo). Nash shows that the unique solution satisfying the four axioms is given by the deal which maximazes the 'Nash product'. When the Symmetry axiom, which asserts that in a symmetric situation, neither player will accept an agreement giving him a lower utility than his opponent's, is abandoned, the other axioms together with the 'bargaining powers' associated to each player determine the 'asymmetric' Nash bargaining solution. In a second paper Nash [7] obtains precisely the same bargaining outcome by analyzing a static bargaining model, the Nash Demand Game, in which the players simultaneously announce demands, which they receive if and only if the demands announced are compatible. The Nash Demand Game has many Nash equilibria (for example, any Pareto-efficient outcome is a Nash equilibrium). In order to select only one equilibrium, Nash required that an equilibrium be robust to perturbations involving some uncertainty about the location of the Pareto frontier of the negotiation set S. When the perturbed Demand Game approaches the unperturbed game (for which the Pareto boundary is known with certainty), all the Nash equilibria of the perturbed game converge on the Nash solution (see Binmore [2] and [1])

The Nash solution is supported by various strategic models. Nash [7] himself, with the perturbed Demand Game, provides a noncooperative support to his axiomatic solution. Another noncooperative defense of the Nash solution is the model of Rubinstein [8]. In Rubinstein's model two players bargain over a pie of size 1. Each period, one of the players proposes a partition and the other player either rejects or accepts. In the latter case, the game finishes and the agreement is implemented. If the offer is rejected, the play goes to the next period, in which it is the rejecting player the one who makes the offer. The unique subgame-perfect equilibrium of the game converges to the Nash bargaining solution, when the time interval between subsequent offers approaches zero. Furthermore, the bargaining powers are determined by the players' time preferences. (See Rubinstein [8] for the assumptions under which his result holds). In the case when the players are equipped with different discount factors it turns out that the most patient player enjoys a larger bargaining power.

More recently, Young [10] has provided a new interpretation of the Nash bargaining solution that still uses the Nash Demand Game, but leads to an asymmetric outcome in which the players have different bargaining powers. However the interpretation of these bargaining powers differs markedly from Rubinstein's interpretation. The approach followed by Young [10] is to embed the Nash Demand Game in an evolutionary framework in order to explain the emergence and persistence of one particular outcome. An interesting feature of the model is that it provides an appealing interpretation of the bargaining powers that characterize the asymmetric Nash bargaining solution. In Young's model the Nash bargaining game, over how to share a pie, is played repeatedly by members from two different large populations. Two players, one from each population, are randomly selected to play the game; players announce a share of the pie which they get if the demands are compatible, otherwise they get nothing. A crucial assumption of Young's model is that players learn how to play the game from the past behaviour of members from the other population. Players have access to a random sample, whose size may differ among players, drawn from the most recent demands which have been announced by the opponents. They take their sample as a predictor of the behaviour of the player they will face, and usually play a best reply to the empirical distribution derived from the sample. However, this behaviour is perturbed by rare 'mutations' so that sometimes the players make mistakes and announce a demand that is not a best reply to any possible sample.

An important feature of Young's dynamic process is that, in the limiting case when the mutation rate goes to zero, the system converges to a fixed Pareto-efficient division that corresponds to the asymmetric Nash bargaining solution, with the bargaining powers determined by the distribution of sample sizes. The model implies that, when all members of the same population observe a sample of the same size, it is the better informed population which gets the larger share of the cake. A less appealing result is obtained when people with different sample sizes coexist in the same population. In this case poorly informed players exert a negative externality on the better informed members of their population. The population's bargaining power is determined by the members who draw the smallest sample, even though such individuals may be present only in very small numbers.

Young obtains the Nash bargaining solution under very weak informational assumptions: Players only know their own preferences and a small sample of what happened in some recent past. This is what typically happens in many real world situations. Students seeking houses to rent know how much landlords have asked in the past for similar apartments while landlords know by experience how much students are willing to pay for a flat in some particular areas. The model has several drawbacks. The crucial elements in determining the population's bargaining powers and therefore their shares of the cake are fixed exogenously. Young leaves the most important element unexplained. The model has the unsatisfactory prediction that the share received by a population with one million types who use large samples is determined by just one further type who uses a small sample. It does not explain why different types of players, probably receiving different payoffs, may co-exist in the same population.

In this paper we endogeneize the size of the samples drawn by the players. We will present a model similar to Young's, but with the added feature that people can change their learning rule by altering the sample size or 'window' used. We will assume that players observe the payoffs received by other members from the same population and from time to time decide to imitate more successful behaviours. It is therefore as though people care about their relative performances within the social class to which they belong. If a learning rule performs better than its rivals, it is natural to expect that it will be employed by a growing proportion of people over time.

We can identify two opposite forces that affect the amount of information gathered by the players. On the one hand, players with small samples are more likely to draw a 'wrong sample' when the system is close to but not at a convention and to play a non-optimal strategy. If information is free, big samples will give a higher expected payoff. On the other hand, when the probability of mistakes is small, the system is close to a convention most of the time and those players who sample few elements will play optimally almost as often as players with big samples. If sampling costs grow with the size of the sample taken, 'small windows' will do better, and evolution will tend to reduce the amount of information gathered by the players.

In the paper we show that:

(i) When there are no sampling costs and the level of noise is arbitrarily small, people with 'larger' window sizes perform better, on average, than people with smaller sample sizes within the same population.

(ii) When there is no noise, for any positive sampling cost 'smaller' sample sizes perform better than larger samples. In this extreme case the evolutionary process converges with probability one to a convention and remains there for ever. In this case there is no need for agents to collect more than one unit of information - it is enough to see one car to realize that Londoners drive on the left. If we assume some type of imitation or Darwinian selection, we will observe, in a noiseless world, a tendency for 'well informed' people to disappear.

(iii) When there are sampling costs, one can always find small enough levels of noise such that the 'smallest' sample size will always pay best. The intuition underlying this fact is that, as the noise vanishes, so also does the advantage of sampling.

Finally, we characterize the evolutionarily stable sample sizes. The idea is to allow the entry of new people who bring with them new behaviours and to test their fitness in the enviroment. If there are samples that perform better than others, they will invade the population because everybody will adopt them. In the limiting case in which the noise tends to zero, we can characterize not only the evolutionarily stable sample size, but also the long-run convention of the system. We compare our results with a situation in which each population can decide how much to sample. In particular we consider a thought experiment in which the different populations elect a representative to play the game who is committed to choosing a certain sample size. On comparing the Nash equilibrium of this game, where sample sizes are the strategies, with the outcome of an economy populated by uncoordinated myopic players who follow very simple imitative behaviour, we find that the latter is socially more efficient. We shall show that the economy of myopic players will always converge to the symmetric bargaining solution while this is not necessarily true for the economy populated by 'rational' players.

We offer an informal discussion of the case in which the asymptotic results do not hold. The assumption of very small mistake rates is made, in the works of Kandori et al. [4] and Young [9] and [10], for reasons of tractability rather than because the noisy case is thought uninteresting. The results of some simulations of the model show that, when the noise is large, the difference in the profitability of different sample sizes depends not only on the level of noise but also on the distribution of sample sizes in the populations. We provide a very simple example in which, when sampling is costly, large sample sizes are better than smaller ones for 'intermediate' levels of noise while they are worse for both small and large noise rates.

The paper is organized as follows: In the next section we introduced the model. In the third section we characterized players' behaviour in terms of their windows sizes which is useful to compare expected payoffs to players using different learning rules. In the last two sections we present the main results of the paper.

2 The model.

Suppose that the unperturbed Nash Demand Game is played once every period by two players respectively drawn at random from two large populations which we follow Young in calling population I (landlords) and population II (tenants). Each player announces a share of the crop, and receives his demand only when the pair of demands is compatible. Each player forms his beliefs about the enviroment he is facing knowing some part of the available information about what other people have done in the past. Landlords (tenants) have access to a 'library' that contains information about m past demands of members of the other population. Players have access only to their own population's library. A landlord (tenant) decides what strategy to use by taking a random sample of size k (w) from his population's library and then playing the best reply to it. Players from the same population may use samples of different sizes.

The information stored in the landlord's (tenant's) library evolves as follows. Every time the game is played, the strategy played by the tenant (landlord) is stored. However, since the library has a limited capacity of m units of information another record of a play must leave the library.

The main difference of our model as compared with Young's lies in the definition of the state space. A state of the system in the model of Young is the 'ordered sequence' of the last m plays of the game. In our model, the demands are not ordered according to the time they entered the libraries. Every time the game is played a new element enters the library and the element it replaces is chosen at random from those previously present. Such a change in the model reduces considerably the state space. Our model has the advantadge of being much more tractable than Young's. The changes in the model simplifies the analysis without doing any violence to the essentials of the process.

We are interested in characterizing the evolution of the information stored in the libraries, since it determines the probability distribution of the future behaviour in the two populations. The evolution of the information in the libraries can be represented by a Markov chain defined on the state space Z. Let Z, the set of all possible stocks in the library, be characterized as follows:

$$Z = \{\{(z_1^I, z_2^I, \ldots, z_n^I), (z_1^{II}, z_2^{II}, \ldots, z_n^{II})\} | z_i^j \in \{0, 1, 2, \ldots, m\}, \sum z_i^j = m\}$$

where z_i^j is the number of times the strategy i is recorded in the population j's library and n is the dimension of the strategy space (all possible announcements). In order to characterize the evolution of the state of the system we make the following assumptions:

> *Assumption 1.* Every sample is drawn with the same positive probability.
>
> *Assumption 2.* Every record of a past play leaves the library with the same positive probability.
>
> *Assumption 3.* With positive probability ϵ, players make mistakes[1] by playing a strategy chosen at random. When a mistake is made, all strategies are possible. We will assume that all strategies occur with the same probability ϵ/n.

[1] For sake of simplicity we assume that the probability of mistakes is the same in both populations. The results do not change if different rates are assumed.

Assumption 4. The probability densities for window sizes k and w are $f(k)$ and $g(w)$ in populations I and II respectively. The probability that a landlord uses sample size $\tilde{k}$ is equal to $f(\tilde{k})$. The probability that a tenant uses sample size $\tilde{w}$ is $g(\tilde{w})$.

We now define a convention. Consider the state in which $z^I_{1-x} = m$ and $z^{II}_x = m$. Whatever samples are drawn, the landlords will then demand x and the tenants $(1-x)$. The states consisting of such a Pareto efficient division of the crop are the *'conventions'* of the system, i.e. the states that reproduce themselves[2]. The main feature of such conventional behaviour is that any player prefers to conform to it if everybody else does so. For notational convention we will refer to the m-repetions of the same partition $(x, 1-x)$ as c_x.

Since our model differs from Young's, it is neccesary to confirm the following result. The proof is similar, although the current model allows a much less restrictive constraint on the minimum necessary sample size. Young requires that at least some players sample at most $m/2$ records in their libraries, whereas the following proposition works with $m/2$ replaced by $m-1$. ($(m-1)$ instead of $m/2$).

Proposition 1 *If at least one agent in each population samples at most m-1 elements the system converges almost surely to a convention.*

Proof. We need to prove that it is possible that the same sample will be drawn for some time with the result that the same best reply until is made, until we have built up homogeneous library records, one for each population, that correspond to Pareto efficient divisions of the crop.

To this end, we consider the extreme case in which some players in each population sample exactly $m-1$ records, while the remainder may sample all the records. Suppose that players from population I who sample $(m-1)$ records happen to be selected to play the game $(m-1)$ times and that they happen to sample the same elements and so all play the same best reply x. This possibility occurs with positive probability, because the last element which enters the library can leave it in the following period. We can obtain a state of population I that contains $(m-1)$ copies of the same demand x. These $(m-1)$ elements can remain for some time in the library of population II, and so be drawn by the players from this population, who will, therefore, demand $(1-x)$. We thereby build a state of the system that contains $(m-1)$ copies of the observation $(1-x)$ in the library of population I and $(m-1)$ copies of the observation x in the library of population II. There is a positive probability that these are the samples drawn next period and that the elements that are different leave the corresponding library. We have reached a convention (c_x) in $(2m-1)$ periods

[2]The conventions have the property that they are the only absorbing sets of the model we are considering.

with positive probability p. The probability that a convention is not reached in $s(2m-1)$ periods is $(1-p)^s$, which goes to zero as $s \to \infty$. □

A convention can be abandoned only when some people start deviating from the behaviour prescribed by it. This is why in our model we introduce the possibility that people may make mistakes and play a strategy that is not a best reply to the sample they have drawn.

To illustrate this point consider the 2×2 bargaining game of Figure 1:

	LOW	*HIGH*
LOW	b, b	b, a
HIGH	a, b	0, 0

$$a > b > 0$$

Figure 1: Game 1.

The state space is:

$$Z = \{(z_1, z_2) | z_1, z_2 \in \{0, 1, 2, \ldots, m\}\}$$

where z_1 (z_2) denotes the number of Low's in the library to which players from I (II) have access.

This simple game has only two conventions: (m,0) and (0,m). Let us assume that the established convention is (m,0) and that some tenents start making mistakes. Sometimes they demand Low although the best reply to any sample containing all Lows is High. The mistakes will, with positive probability, remain for some time in the landlords' library. It is possible that a landlord will draw the mistakes and, if there are sufficiently many, he will, then, play a strategy that is not the conventional one. If all players have the same utility function, the probability that agents deviate from a convention in response to mistakes made in the other population will depend on the size of the sample they draw.

The introduction of mistakes keeps the system continuously in motion. Under assumptions (1)-(4) we can obtain a Markov Chain defined on the state space Z with the transition matrix:

$$M(\epsilon) = M(\epsilon; m, f(k), g(w), G) = [p_{i,j}], \quad (i, j \in Z) ,$$

where the transition probability $p_{i,j}$ is the probability of moving from state i to state j in one period [3].

Introducing mistakes makes the Markov chain irreducible, i.e; all the states intercomunicate. As the Markov chain is also aperiodic, it is therefore ergodic and has a unique stationary distribution, i.e.,there exists a unique distribution $(1 \times |Z|)$ vector μ_ϵ such that:

$$\mu_\epsilon M(\epsilon) = \mu_\epsilon \tag{1}$$

thus, system settles down in the long run to a distribution which is independent of the initial conditions. The solution to (1) is a correspondence $\Gamma : \epsilon \Longrightarrow \Delta^{|Z|-1}$ which is upper hemicontinuous. The equilibrium selection is continuous with respect to perturbations (see Kandori and Rob [5]).

The interpretation of the probabilities attached to each state in the long-run distribution, is the time spent by the system in the corresponding state.

It turns out that this long-run distribution often put most of its mass at just one of the possible conventions when the mutation rate is small. As the mutation rate tends to zero, all other states are assigned zero mass. We then say that the remaining convention has been selected in the long run. This conclusion is no longer valid when the mutation rate is set to zero from the outset. The convention that is then observed in the long-run depends on the initial conditions. When players do not make mistakes, the conventions are absorbing states, i.e. once the system is at a convention it is impossible to escape.

The trick in selecting a particular stationary distribution out of all possible conventions is the introduction of a small amount of noise into the system. Out of all the possible conventions we select one, by introducing noise in the system and letting it tend to zero.

We follow Young in assigning a 'resistance' to each convention. The convention that is selected is that from which it is most difficult to escape or, seen from another perspective, the one which is easiest to reach from any other convention. The computation of the 'resistance' associated to one particular convention involves counting the minimum number of mutations needed to reach such a convention from any other. As the mutation rate tends to zero only those states which are easiest to reach will be observed in the long run.

The convention which has the smallest resistance is therefore the one from which it is most difficult to escape. When we consider the possibility of going from one convention to another we have only to consider the minimum number of mistakes one of the libraries has to contain for 'the most mistake-sensitive player' to be capable of drawing a sample that prescribes a non-conventional choice. When all players from the same population have the same utility function, the most sensitive player is the person who draws the smallest sample.

[3]In what follows we will write $M(\epsilon)$ instead of $M(\epsilon; m, f(k), g(k), G)$. The game, the memory size and the distributions of window sizes are fixed.

Proposition 2 *(Young [9]) When the rate ϵ of mistakes goes to zero, the stationary distribution will put weight only on the convention or conventions with minimal resistance .*

The main result of Young's study of the Nash Demand Game is the selection of the asymmetric Nash bargaining solution as the long-run convention of the system. There are two conditions that need to be satisfied. The level rate of mistakes has to be positive but vanishingly small. Also a very finely meshed stategy space has to be considered. That is to say

$$S = \{0, \delta, 2\delta, 3\delta, \ldots, 1-\delta, 1\}$$

where the mesh-size $\delta > 0$ must be taken to be sufficiently small.

Definition 1 (Asymmetric Nash Bargaining Solution) *The Asymmetric Nash Bargaining Solution is the division $(x, 1-x)$ that maximizes*

$$\{u(x)\}^a\{v(1-x)\}^b \textit{ subject to } 0 \leq x \leq 1$$

where u and v are the utility functions of players 1 and 2 respectively, and a and b are their bargaining powers.

The following proposition, drawn from Young [10], is true in our model:

Proposition 3 *(Young [10]) The evolutionary process described above converges to the asymmetric Nash solution as $\delta \to 0$, with each population's bargaining power equal to the smallest sample size used by an individual in that population.*

Proof. Young's proof also applies in our model. There is a correspondence between the conventions in our model and those in Young's. The proof of the theorem depends on computing the number of mistakes needed to abandon one convention in order to enter the basin of attraction of another convention and the considering the limit as $\delta \to 0$. □

The proposition above implies that determining the convention that will be observed most of the time, requires focusing only on the evolution of $\underline{k} = \min\{supp\ f(k)\}$ and $\underline{w} = \min\{supp\ g(w)\}$, where f and g are the densities that describe the distribution of sample sizes in populations I and II (see Assumption 4). The long run convention of the system is determined by the members of the populations who draw the smallest sample. In the conventions, the payoffs obtained by members from the same population are the same independently of the amount of information gathered.

The fact that people make mistakes implies that with positive probability the system is not at a convention at any particular time. In such cases, the expected payoff to different window sizes may differ. In order to compare the profitability to different sample sizes, we need to obtained the expected payoffs in each state as well as the long-run distribution of the system.

3 Long-run payoffs

We will assume that all players have the same utility function. This assumption is necessary to rule out the influence of different levels of risk aversions in the selection of the long run convention. The difference in the expected behaviour of two players from the same population then depends only on how much information they gather from their library (sample size) and not on their attitudes towards risk.

Risk-aversion plays an important role in the models of Young [10] and Rubinstein [8]. In both models it is the player who is more risk averse who gets the smallest share of the cake.

Players' strategies. Players use best replies against random samples drawn from their libraries. Before the sample is chosen, we can characterize the anticipated behaviour of each player as a "mixed strategy", with the probabilities attached to each strategy being determined by the current stock of information on record. It is important to notice that these are "mixed strategies" of a special type. They are observed not because players randomize over pure strategies but because they base their choice of strategy on the information provided by a random sample drawn from their library. Each landlord (tenant) drawn to play, faces a tenant (landlord) who behaves as if he were using a mixed strategy. Notice that each player has some limited knowledge about the past behaviour of the other population, but not about his own.

Consider Game 1 of Figure 1 and a landlord who draws a sample of size k. The state of the system is $z = (z_1, z_2)$ and the recorded history contains m past plays of the game.

Let us define $p_i^s(z_{-i}, x)$ as the probability with which player i plays strategy s when the state of the system is z and he draws a sample of size x.

When the library contains m records, there are $\binom{m}{k}$ possible samples of size k. High will be the best reply when the sample drawn contains at least $l_b(k)$ lows [4]

$$l_b(k) = \left[\frac{b}{a}k\right]^+$$

The probability with which a player from population 1, sampling k records, plays high in state z is therefore

$$p_1^H(z_2, k) = \binom{m}{k}^{-1} \sum_{l_b \leq l \leq k} \binom{z_2}{l} \binom{m - z_2}{k - l}$$

[4] $[x]^-$ ($[x]^+$) denotes the greatest (smallest) integer smaller or equal (greater or equal) than x.

This probability is non-decreasing in z_2 and is zero for $z_2 < l_b$ and one for $z_2 > m - (k - l_b)$.

The strategy played by any member of population I (II) depends on the state of the system in the other population, the payoffs (through l_b) and on the size of the sample.

Example Choose the payoffs a and b of Game 1 to obtain:

	LOW	*HIGH*
LOW	1.2, 1.2	1.2, 3
HIGH	3, 1.2	0, 0

Figure 2.

Assume that there are two types of landlord. When called upon to play, Type 1 samples only one past record; Type 2 samples three. All tenants sample 2 units of information. Tables 2, 3 and 4 report the mixed strategy (the probability of playing High) used by tenants, Type 1 and Type 2 landlords respectively. Each entry corresponds to one state of the system. The horizontal dimension is the state of the tenants, i.e. the number of times in landlords' library that a tenant played Low. The vertical dimension corresponds to the state of the landlords. Both range from 0 to 8. The entry (6,3) in Table 1 says, for instance, that a tenant will play High with probability 27/28 when the landlords' library records that tenants played Low 3 times and the tenants' library record that landlords played Low 6 times. The same entries in tables 2 and 3 represent the probabilities assigned to High by landlords who sample 1 and 2 records respectively.

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
2	$\frac{13}{28}$	$\frac{13}{28}$	$\frac{13}{28}$	$\frac{13}{28}$	$\frac{13}{28}$	$\frac{13}{28}$	$\frac{13}{28}$	$\frac{13}{28}$	$\frac{13}{28}$
3	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{9}{14}$
4	$\frac{11}{14}$	$\frac{11}{14}$	$\frac{11}{14}$	$\frac{11}{14}$	$\frac{11}{14}$	$\frac{11}{14}$	$\frac{11}{14}$	$\frac{11}{14}$	$\frac{11}{14}$
5	$\frac{25}{28}$	$\frac{25}{28}$	$\frac{25}{28}$	$\frac{25}{28}$	$\frac{25}{28}$	$\frac{25}{28}$	$\frac{25}{28}$	$\frac{25}{28}$	$\frac{25}{28}$
6	$\frac{27}{28}$	$\frac{27}{28}$	$\frac{27}{28}$	$\frac{27}{28}$	$\frac{27}{28}$	$\frac{27}{28}$	$\frac{27}{28}$	$\frac{27}{28}$	$\frac{27}{28}$
7	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1

Table 1

	0	1	2	3	4	5	6	7	8
0	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
1	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
2	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
3	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
4	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
5	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
6	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
7	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
8	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1

Table 2.

	0	1	2	3	4	5	6	7	8
0	0	0	$\frac{3}{28}$	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{5}{7}$	$\frac{25}{28}$	1	1
1	0	0	$\frac{3}{28}$	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{5}{7}$	$\frac{25}{28}$	1	1
2	0	0	$\frac{3}{28}$	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{5}{7}$	$\frac{25}{28}$	1	1
3	0	0	$\frac{3}{28}$	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{5}{7}$	$\frac{25}{28}$	1	1
4	0	0	$\frac{3}{28}$	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{5}{7}$	$\frac{25}{28}$	1	1
5	0	0	$\frac{3}{28}$	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{5}{7}$	$\frac{25}{28}$	1	1
6	0	0	$\frac{3}{28}$	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{5}{7}$	$\frac{25}{28}$	1	1
7	0	0	$\frac{3}{28}$	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{5}{7}$	$\frac{25}{28}$	1	1
8	0	0	$\frac{3}{28}$	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{5}{7}$	$\frac{25}{28}$	1	1

Table 3.

Notice that the strategy used by each population depends neither on its own state (horizontal dimension for tenants and vertical for landlords), nor on the distribution of types in the two populations. Notice also, comparing tables 3 and 4, that there are two subsets of states in which a pure strategy is played and that these subsets grow with the size of the sample.

In the proposition which follows we formalize the intuition provided by the previous example. We characterize the set of states in which players use a pure strategy, when not making a mistakes.

Let us consider Game 1 and define the following subsets of Z:

$$\underline{Z}_{x,y} = \{(z_1, z_2) | z_1 \leq x, z_2 \leq y\}$$

$$\overline{Z}_{x,y} = \{(z_1, z_2) | z_1 \geq x, z_2 \geq y\}$$

Consider a landlord who samples k elements, with l which are Low's. Low will be the best reply to that sample if $l \leq kb/a$. All possible samples drawn from:

$$\underline{Z}_{m,[\frac{kb}{a}]^-} = \{(z_1, z_2) | z_1 \leq m, z_2 \leq \left[\frac{kb}{a}\right]^- \}$$

will have at most $[kb/a]^-$ lows and every landlord who samples k elements will play Low with probability one.

High will be played with probability one by any landlord with a sample of size k in all the states in $\overline{Z}_{0,[m-k(a-b)/a]^+}$. In any other state it is possible to find samples of size k to which Low is the best reply as well as samples of equal size to which High is the best reply.

We are interested in identifying the sets of states in which only pure strategies are played.

Consider the following correspondence $s^i : R^2 \Rightarrow Z$,

$$s^i(m, x) = \{(z_1, z_2) | \ p_i^s(z_{-i}, x) = 1\}$$

Note that $L^1(m, k) = \underline{Z}_{m,[\frac{kb}{a}]^-}$ and $H^1(m, k) = \overline{Z}_{0,[m-k(a-b)/a]^+}$.

Proposition 4 *$L^1(m, \lambda k) \subseteq L^1(m, k)$ and $H^1(m, \lambda k) \subseteq H^1(m, k)$ for* $0 \leq \lambda \leq 1$.

Proof. It follows from the definitions of L^1 and H^1.

Proposition 4 states that the set of states in which pure strategies are played shrinks with the sample size.

This result can be easily extended to the case in which there are more than two strategies. Under the assumption of homogeneous utility functions the different sample sizes need the same 'proportion of mistakes' to start playing a nonconventional strategy.

Player's payoffs. In order to compare the profitabilities to different sample sizes we need to obtain the payoffs in each state as well as the long-run distribution μ_ϵ.

Different sample sizes will have the same expected payoffs in all those states in which the mixed strategies are the same. From the preceding proposition, we know that in some states different sample sizes prescribe the same pure strategy. Clearly, in all such states, players using different amounts of information will have the same expected payoff, independently of the opponent's strategy. In all other states different sample sizes prescribe different mixed strategies and, therefore, will have different expected payoffs.

The profitability of a learning rule (characterized by its sample size) depends not only on the rate of mistakes but also on the composition of the populations which determines the actual long-run distribution. Let $\pi^k(\epsilon; z, g(w))$ be the expected (gross) payoff in state z to a player who samples k units of information when the rate of mistakes is ϵ. We can decompose this expected payoff into two components:

$$\pi_k(\epsilon; z, g(w)) = (1-\epsilon)\pi_k^s(\epsilon; z, g(w)) + \epsilon\pi^t(\epsilon; z, g(w)) \tag{2}$$

The first part of the payoff, $\pi_k^s(\epsilon; z, g(w))$, is received when the player uses the information provided by the sample he has drawn. Players do not tremble, and therefore use the information available to them with probability $(1-\epsilon)$. The second component, $\pi^t(\epsilon; z, g(w))$, is the payoff obtained when the player trembles and plays an arbitrary strategy. This component which does not depend on the sample size is the same for all members from the same population.

Let $c(k) \geq 0$ be the cost of a sample of size k. We obtain the net payoffs by substracting $c(k) \geq 0$ from the right-hand side of equation (2).

The (gross) expected payoff to a player who uses a sample of size k when the long-run distribution of the system is μ_ϵ is given by:

$$\pi_k(\epsilon; f(k), g(w)) = \sum_{z \in Z} \pi_k(\epsilon; z, g(w))\mu_\epsilon(z)$$

where $\mu_\epsilon(z)$ is the weight of the state z in the long-run distribution μ_ϵ.

We are interested in pairwise comparisons of sample sizes. In accounting for differences in window sizes, we shall consider first the simplest case.

Consider Game 1. Assume that all tenants are characterized by the same sample size w. Consider two different samples k and $k' < k$ in the population of landlords.

Proposition 5 *Let us consider sample sizes k and $k' < k$. For m large enough there exist integers q_1, q_2 and $q_2' \geq q_2$ such that*

$$\forall\ z \in \underline{Z}_{m,q_2} \cup \overline{Z}_{q_1,0}\ \pi_k^s(\epsilon; z, w) \geq \pi_k'^s(\epsilon; z, w)$$

$$\forall\ z \in \underline{Z}_{q_1,m} \cup \overline{Z}_{0,q_2'}\ \pi_k^s(\epsilon; z, w) \geq \pi_k'^s(\epsilon; z, w)$$

Proof. Let q_1 as the smallest z_1 such that $p_2^H(z, w) \geq b/a$, and let q_2 and q_2' be the states such that,

$$\text{for all } z_2 < q_2 \qquad p_1^H(z_2, k) \leq p_1^H(z_2, k') \ \ \textit{and}$$

$$\text{for all } z_2 \geq q_2 \qquad p_1^H(z_2, k) \geq p_1^H(z_2, k')$$

The following figure is a graphical illustration of the previous proposition.

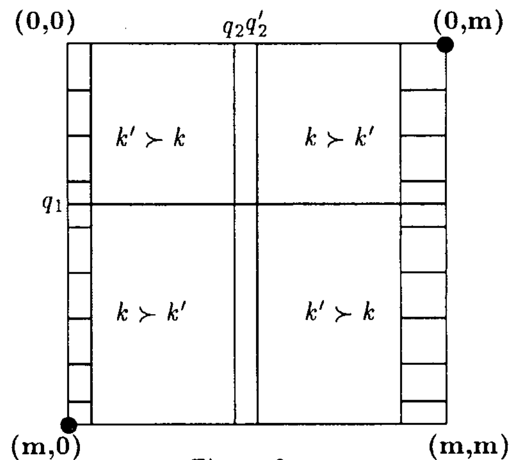

Figure 3.

The state of population II is represented in the horizontal dimension. It ranks from 0 to m and represents the number of Low's in the recorded memory. Similarly, the state of population I is represented in the vertical dimension, from up to down. The conventions are in lower-left and the upper-right corners (filled dots).

Let us consider the area labelled with $k \succ k'$ on the upper-right part of the figure, which are states in some neighbourhood of $(0, m)$. The history available to the tenants has many High's. Tenants will play Low with high probability. The best reply to $p^H < b/a$ is High. A player using a large sample will play High with higher probability than a player with a smaller sample. A similar argument applies to the states in the lower-left corner. The dashed areas correspond to the states in which landlors, either with k or k', play the same (pure) strategy and, therefore, have the same expected payoff. The smallest sample pays better in the sets labeled with $k' \succ k$.

4 The evolution of the learning rule.

The aim of this section is to endogenize the amount of information gathered by players. People can observe the payoffs of some members of their own population and imitate the most successful behaviour. Students may know how much other students are paying and how much they searched. We will assume that together with the average payoffs, players can observe the sample sizes drawn by other members from the same population.

Comparison of payoffs. We will assume that the comparison of payoffs takes place relative to the long-run distribution and so does the evolution of sample sizes. We can justify this assumption on the grounds that the adjustment periods are negligible compared with the time the system spends in the long-run distribution.

When comparing the payoffs to two different sample sizes from the same population, we have only to consider the first part in equation (2).

The difference in payoffs to window sizes k and k', evaluated relative to the long-run distribution is given by

$$D\pi(\epsilon; k, k', f(k), g(w)) = (1-\epsilon)\sum_{z\in Z}(\pi_k^s(\epsilon; z, g(w)) - \pi_{k'}^s(\epsilon; z, g(w)))\mu_\epsilon(z))$$

The function $D\pi$ is a polynomial in ϵ and it is therefore continuous.

Consider the simplest possible case. There are two types of landlords, in proportions θ and $(1-\theta)$. The first type samples k records the second type sample k'. All tenants are of the same type, and sample w past records. The rate of mistakes is the same in both populations.

Let $M(r_I, r_{II})$ be the transition matrix when population I (II) follows rule r_I (r_{II}) to play the game. The rule can be either to take a sample (k, k' or w) or to tremble (t). We can decompose the Markov matrix $M(\epsilon; \theta k + (1-\theta)k', w)$ as follows:

$$\begin{aligned} M(\epsilon; \theta k + (1-\theta)k', w) &= \theta((1-\epsilon)^2(M(k,w) + (1-\epsilon)\epsilon M(k,t)) \\ &\quad (1-\theta)((1-\epsilon)^2(M(k',w) + (1-\epsilon)\epsilon M(k',t)) \\ &\quad \epsilon^2 M(t,t) + (1-\epsilon)\epsilon M(t,w) \end{aligned}$$

Each time the game is played, a player sampling k is drawn with probability θ. With probabiliy $(1-\epsilon)^2$ neither he nor the opponent, who samples w with probability 1, tremble. The transition matrix is given in this case by $M(k,w)$. With probability ϵ^2 both players tremble; $M(t,t)$ describe the transition probabilities. With probability $\epsilon(1-\epsilon)$ only one player trembles. The markov matrices when only the first or only the second player tremble are $M(t,w)$ and $M(k,t)$ respectively. The terms multiplied by $(1-\theta)$ have an analogous interpretation, with k' being the sample size used by the player drawn from the first population.

When $\epsilon = 0$,

$$M(0; \theta k + (1-\theta)k', w) = \theta M(k,w) + (1-\theta)M(k',w)$$

and the system has as many absobing states as there are Pareto-efficient divisions of the cake. Once a convention has been reached, and it will happen with positive probability, the economy will remain there for ever. The set of absorbing states is independent of the composition of the populations. Independently of the value of θ, the conventions are the only states with 1 on the diagonal of $M(0,)$. Changes in θ only affect the transition probabilities but do not change the absorbing states. From any other state, it is possible to find a chain of transitions which ends up in one convention. The convention which will be selected depends on the initial conditions. In other words, history matters. In a convention, all players obtain the same payoff and, therefore $D\pi(0; k, k') = 0$. The information given by a single unit of information is as good as the whole history.

When $\epsilon = 1$,

$$M(1; \theta k + (1-\theta)k', w) = \epsilon^2 M(t,t) \tag{3}$$

The long-run distribution depends neither on the composition of the population nor on the sample sizes. Players play an arbitrary strategy. The expected payoff is the same for all players and $D\pi(1; k, k') = 0$.

When $0 < \epsilon < 1$, there are no absorbing states. The diagonal elements of $M(\epsilon; \theta k + (1-\theta)k', w)$ are all smaller than 1. The long-run distribution will depend on the specific way trembles are modeled and on the sampling process which is assumed. We shall assume that the probability of sampling an individual with sample size k is equal to its proportion in the population. The trembles have been modeled as the choice of any of the possible strategies with equal probability.

As the noise tends to zero, the long-run distribution concentrates around the convention whose basin of attraction is hardest to escape. If the noise is vanishingly small we can easily characterize the long-run distribution and the long-run payoffs. For very small noise rates, the system will be almost always in a convention, although all other states will be visited with positive probability. The closer the states are to the conventions, the higher will be their weights in the long-run distribution. For very small mistake rates we need to consider only states in neighbourhoods of the conventions to compare expected payoffs. In those states, as we have seen in the previous section, larger sample sizes have a higher expected payoff, due to the fact that, close to the conventions, it responds with smaller probability to the mistakes coming from the other population.

Proposition 6 *Let us consider two sample sizes k and $k' < k$. For $0 < \epsilon < \tilde{\epsilon}$ and large m, $D\pi(\epsilon; k, k') > 0$.*

Proof. See Appendix 1. We prove that for arbitrarily small positive ϵ, the payoffs in the states where the larger window size pays best compensate the disadvantadge in all other states.

In Appendix 2, we report the results of a simulation of Game 1 with $m = 4$ and different values of ϵ. Each entry is the probability attached to the corresponding state in the long-run distribution. We report the weight (p) of the conventions and the six neighbouring states (three for each convention) in the long-run distribution.

The larger the noise, the smaller the probability p. As the level of noise grows p decreases and the probability mass shifts towards states in which the smallest sample performs relatively better.

We can conclude that $D\pi$ is 0 at $\epsilon = 1$ and $\epsilon = 0$ and growing at this last point.

The expected payoff to the different sample sizes depends on the mass put by the long-run distribution on all the states of the system. As the rate of mistakes ϵ grows the probability weight moves from the conventions to the other states. The set of states in which the smallest sample size has higher expected payoff depends on the payoffs, the sample sizes and the memory length. It is possible to find examples for which the largest sample is the most profitable for all levels of noise. There exists, also, the possibility that at high levels of noise, the smallest sample has a higher expected payoff. The states in which miscoordination is common are more likely when the noise is large. The smallest sample may be better as we shall see in the following numerical example.

An example Consider the game considered in the previous section. All tenants draw a sample of size 2. There are, as before, two types of tenants [5]. Type 1 tenants draw a sample of size $k = 3$. Type 2 sample $k' = 1$ units of information. The proportion of members from the landlords' population who sample $k = 3$ is given by θ. The columns report, respectively, the level of noise of the system and the difference in payoffs ($D\pi(\epsilon; 3, 1)$ for the different levels of noise, in the long-run distribution.

ϵ	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
0	0	0	0
0.05	0.024999	0.025198	0.02521
0.1	0.039749	0.04082	0.041329
0.2	0.048659	0.049013	0.050820
0.3	0.036386	0.039811	0.042752
0.4	0.023438	0.026059	0.028503
0.5	0.012609	0.014197	0.015741
0.6	0.0053594	0.0061561	0.0069475
0.7	0.0014224	0.0017458	0.00207
0.8	-0.00008059	0.00001237	0.00010585
0.9	-0.00019779	−0.00018624	-0.00017463
0.95	-0.000071602	−0.000070145	-0.000068685
1	0	0	0

Table 4 .

The last column shows the difference in expected payoffs when all members of population I sample $w = 3$, evaluated relative to the long-run distribution. For all noise rates smaller than 0.8, the larger window size pays better than the smaller. Any mutant using a unit less of information will die out.

[5] We have selected k and k' in such a way that the structure represented in Figure 1 is preserved.

The function $D\pi$ is defined for fixed $f(k)$ and $g(w)$. In the following section, we introduce dynamics in the distributions of sample sizes. As the proportion of players using different sample sizes change so does $D\pi$.

5 Evolutionary stability.

We can only make qualitative statements about the relation between costs, noise and evolution of learning rules in the system. It is important to realize that all the results of Young [9], Kandori et al. [4] and Young [10] are valid when the level of noise is close to zero. Only in this special case in which can we characterize the long-run distribution of the system.

In this section we characterize the evolutionarily stable sample sizes. The idea is to perturb the distributions of sample sizes by introducing new people with different learning rules. Stable distribution are those that survive such a disturbance. The dynamics in the compositions of the populations will be driven by some type of imitation or Darwinian selection (the survival of the fittest). It is important to notice that we have two different levels of evolution. On the one hand we have the evolution of the system, as in Young [10], which is driven by the adaptive play and the mistakes. On the other hand we have the evolution of the learning rules which is driven by the imitation of more profitable learning rules and by mutations which affect the sizes of the sample. We do not take the distributions of sample sizes as given. We can consider two different relevant time horizons. In the long run we take the distributions of sample sizes, f and g, as given with the system being in the long run distribution. We can think of a situation in which people adjust very slowly their learning rules compare to adjustments in the environment. In the ultra long run players have had time to adjust their learning rules. Our aim is to find two sample sizes k^* and k^* which are evolutionarily stable, i.e, cannot be invaded. In the ultra long run, the distributions f and g will put weight only on k^* and w^* respectively. As we have seen in the previous section, there is always a sample size which dominates the others, i.e has a higher expected payoff. Under darwinian dynamics the populations will be invaded by such a sample. Selection implies, in this case, homogeneous populations.

We will compare the results with an hypothetical situation in which sample sizes are selected at the population level. For this purpose we will asume that players, in each population separately, elect a representative to play the game on their behalf. The representative is characterized by the size of the sample he draws. Both populations behave this way, knowing the long-run implications of their choices.

5.1 Asymptotic results.

The asymptotic results apply in the case when the noise is very small. We can focus on the behaviour of $\underline{k}$ and $\underline{w}$, which are the sample sizes which determine the bargaining powers and the long-run distribution (Proposition 2).

The distributions of sample sizes, $f(k)$ and $g(w)$, may change over time. When different sample sizes are present in the same population, there will be a process of selection that will wipe out inefficient learning rules. Only when these changes affect either $\underline{k}$ or $\underline{w}$ will the system move to a new convention.

5.1.1 Costless window sizes

From the previous section, we know that when information is free, and the noise tends to zero, big samples have higher expected payoff than small ones, although the advantage of sampling vanishes with the noise.

Proposition 7 . *When players can change their sample sizes without cost, the only evolutionarily stable sample sizes are* $\overline{k}$ *and* $\overline{w}$.

Proof. Let us assume that all members in population 1 (2) are sampling $k < \overline{k}$ ($w < \overline{w}$) and that sampling is costless. These sample sizes are not evolutionarily stable. By proposition 6 any mutant who enters the population and samples more will have a higher expected payoff. □

When sampling is costless we will observe an endless process of growth in the samples. If there is a limit in peoples' capacity to retain information, there will be full employment of this capacity, which will be the only uninvadable sample size, with the population gifted with higher capacity receiving a greater share.

Nash equilibrium in sample sizes. In the analysis developed in the previous section, players do not have any concious choice of strategies. They simply apply the simple rule of playing a best reply to some observation about the past and sometimes imitate more succesful learning rules. We have assumed very little about players' information. Players only know some limited information about previous demand and payoffs and window sizes of players from the same population. In what follows we compare the results obtained above with those obtained in the extreme case of perfect forsight. We shall assume that players are committed to play as before, but they are able to computing the long run distribution and know that with their choice of sample size can affect the convention which will be selected. The situation can be modeled as a one-shot game, with sample sizes as strategies and payoffs computed in the long-run conventions.

We consider the following thought experiment: Imagine that landlords and tenants have to elect a representative (a type) to play the Nash Demand Game on behalf of the population. The rules of the game are as before, with the difference that the player is not randomly selected but chosen by the population.

The representative decides how to play by sampling the number of records that characterize his type. Players are aware that their joint choices will determine the bargaining powers and their shares of the crop in the long-run. They only care about long-run payoffs. Which sample size will they choose? They will select a player with a sample size that maximizes their payoffs given the other population's choice of sample size.

The strategies spaces,

$$S^1 = \{1, 2, 3, .., \overline{k}\}$$
$$S^2 = \{1, 2, 3, .., \overline{w}\}$$

are all the possible sample sizes. The capacity limits, $\overline{k}$ and $\overline{w}$, are not necessarily the same.

Let us consider the simplest case in which the utility functions are linear. The asymmetric Nash bargaining solution, with bargaining powers are k and w, is the partition $(x^*, 1 - x^*)$ which solves,

$$\max_x x^k (1 - x)^w$$

The solution is

$$x^* = \frac{k}{w + k}$$

Landlords and tenants have to elect a representative to play Game 2. Each entry correspond to the (asymmetric) Nash bargaining solution $(x^*, 1 - x^*)$ for the different sample sizes.

	1	2	3	4	5
1	$\frac{1}{2}$, $\frac{1}{2}$	$\frac{1}{3}$, $\frac{2}{3}$	$\frac{1}{4}$, $\frac{3}{4}$	$\frac{1}{4}$, $\frac{4}{5}$	$\frac{1}{5}$, $\frac{5}{6}$
2	$\frac{2}{3}$, $\frac{1}{3}$	$\frac{2}{4}$, $\frac{1}{2}$	$\frac{2}{5}$, $\frac{3}{5}$	$\frac{2}{6}$, $\frac{4}{6}$	$\frac{2}{7}$, $\frac{5}{7}$
3	$\frac{3}{4}$, $\frac{1}{4}$	$\frac{3}{5}$, $\frac{2}{5}$	$\frac{1}{2}$, $\frac{1}{2}$	$\frac{3}{7}$, $\frac{4}{7}$	$\frac{3}{8}$, $\frac{5}{8}$
4	$\frac{4}{5}$, $\frac{1}{5}$	$\frac{4}{6}$, $\frac{2}{6}$	$\frac{4}{7}$, $\frac{3}{7}$	$\frac{1}{2}$, $\frac{1}{2}$	$\frac{4}{9}$, $\frac{5}{9}$

Figure 4. Game 2.

The entries on the diagonal correspond to the symmetric Nash bargaining solution, the fifty-fifty division, because both populations have the chosen the same sample size and therefore have the same bargaining powers. The optimal choice for each population, given the other's sample size, is to elect a player who samples the most he can. In the unique Nash equilibrium of this game, the two populations select a player with the largest possible sample size. If the strategy spaces, S^1 and S^2, are the same for both populations, we will observe the fifty-fifty division and people sampling at the limit of their capacity. Notice that the Nash equilibrium corresponds to the evolutionarily stable sample size. It is interesting the fact that the observed behaviour is the same with myopic imitative players and with fully rational players. Evolution leads to the same result that concious rational choice.

5.1.2 Costly sample sizes.

We now study the more realistic setting in which sampling is costly. We assume that the cost of a sample is proportional to its size; all members of the same population have the same marginal cost, c_1 for landlords and c_2 for tenants.

Proposition 8 . *When sampling is costly, the only evolutionarily stable sample sizes are $k = w = 1$. Furthermore, if players have the same utility fuction, the long-run convention will be the fifty-fifty division.*

Proof. When small samples are less costly a reduction in the sample size implies a saving in the cost while the worsening in the performance is negligible:

$$lim_{\epsilon \to 0} D\pi(\epsilon; k, 1) = 0 \quad \forall k > 1$$

$$\lim_{\epsilon \to 0} c(k) - c(1) > 0 \quad \forall k > 1$$

The only uninvadable sample size is 1. This result is independent of the relative costs and of the shape of the cost function. In the particular case of homogeneous utility functions the fifty-fifty division will be the rule and decision costs will be minimized.

The results differ from the situation in which representative player is chosen by the populations.

Nash equilibrium in sample sizes Landlords and tenants choose sample sizes k^* and w^* which maximize they long-run payoffs, taking the rival's sample sizes as given,

The Nash equilibrium of the following one shot game is selected,

	1	2	3	4	5	
1	$\frac{1}{2} - c_2$ $\frac{1}{2} - c_1$	$\frac{2}{3} - 2c_2$ $\frac{1}{3} - c_1$	$\frac{3}{4} - 3c_2$ $\frac{1}{4} - c_1$	$\frac{4}{5} - 4c_2$ $\frac{1}{4} - c_1$	$\frac{5}{6} - 5c_2$ $\frac{1}{5} - c_1$	
2	$\frac{1}{3} - c_2$ $\frac{2}{3} - 2c_1$	$\frac{1}{2} - 2c_2$ $\frac{2}{4} - 2c_1$	$\frac{3}{5} - 3c_2$ $\frac{2}{5} - 2c_1$	$\frac{4}{6} - 4c_2$ $\frac{2}{6} - 2c_1$	$\frac{5}{7} - 5c_2$ $\frac{2}{7} - 2c_1$	
3	$\frac{1}{4} - c_2$ $\frac{3}{4} - 3c_1$	$\frac{2}{5} - 2c_2$ $\frac{3}{5} - 3c_1$	$\frac{1}{2} - 3c_2$ $\frac{1}{2} - 3c_1$	$\frac{4}{7} - 4c_2$ $\frac{3}{7} - 3c_1$	$\frac{5}{8} - 5c_2$ $\frac{3}{8} - 3c_1$	
4	$\frac{1}{5} - c_2$ $\frac{4}{5} - 4c_1$	$\frac{2}{6} - 2c_2$ $\frac{4}{6} - 4c_1$	$\frac{3}{7} - 3c_2$ $\frac{4}{7} - 4c_1$	$\frac{1}{2} - 4c_2$ $\frac{1}{2} - 4c_1$	$\frac{5}{9} - 5c_2$ $\frac{4}{9} - 4c_1$	

Figure 5. Game 3.

Game 3 is obtained from Game 2 by simply substracting the sampling costs which are proportional to the window sizes.

The unique Nash equilibrium of Game 3 is given by,

$$k^* = \frac{c_2}{(c_1 + c_2)^2}$$

$$w^* = \frac{c_1}{(c_1 + c_2)^2}$$

The bargaining powers are inversely related to the relative costs:

$$\frac{k^*}{w^*} = \frac{c_2}{c_1}$$

When the marginal costs are the same, $c_1 = c_2$, we will observe the fifty-fifty division. In this case there is social inefficiency because players incurr in a costs of sampling which are saved in the case in which players follow the very simple imitative behaviour we have assumed. We have an evolutionarily stable sample size ($k = w = 1$) which is not a Nash equilibrium of the game in samples sizes. The reason is that players, when changing their sample sizes, do not take into account neither the effect of their action in the long-run nor any strategic consideration. Both populations could be better-off if they agreed on sampling only one unit of information. In this case they would save the sampling costs getting half of the cake. Both parts have incentives to deviate from such an agreement. It is prissoner's dilemma situation.

An economy populated by myopic players is more efficient that one in which strategic considerations are taken into account and intra-population coordination is possible.

When the marginal costs are different the two populations get different shares, the higher one being received by those which have the smallest marginal cost.

5.2 Non asymptotic results.

In section 3 we have obtained a relation between rates of noise and differences in expected payoffs to two different sample sizes. If sample sizes are costly, the same relation defines a locus of noise rates and differential costs which makes players indifferent between two different windows.

Let $c(k)$ ($c'(k) > 0$) be the cost of keeping a window of size $k > 0$. For each ϵ and two given sample sizes k and $k' < k$ we can find a function $d(\epsilon, k, k')$, such that

$$if \;\; c(k) - c(k') = d(\epsilon; k, k') \quad then \quad \pi(\epsilon; k) - c(k) = \pi(\epsilon, z; k') - c(k')$$

Clearly, $d(\epsilon; k, k') = D\pi(\epsilon; k, k')$.

The analysis of the evolution of learning rules for non-negligible rates of noise requieres the study of the evolution of the whole $D\pi$ function. The asymptotic results do not hold. For any difference in sampling costs we can find rates of noise for which small sample sizes are more profitable, as well as other noise rates for which the largest sample is prefered. The characterization of the evolutionarily stable sample sizes requires a better understanding of how changes in the proportions of sample sizes in the populations shift the $D\pi$ function.

Consider Figure 2. The shape of $D\pi$ corresponds to the example reported in Table 4. Let us assume thet the rate of noise is $\epsilon = \hat{\epsilon}$ and the difference in sampling costs $c(k) - c(k') = d$

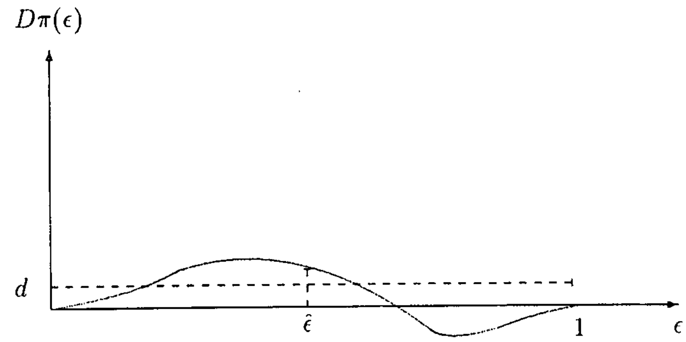

Figure 6.

the largest sample size k has, at $\hat{\epsilon}$, higher expected payoff

$$\pi(\hat{\epsilon}; k) - c(k) > \pi(\hat{\epsilon}, z; k') - c(k')$$

The proportion of player using k will grow (change in θ and f). If the change in the distribution of sample sizes f moves $D\pi$ upwards, the system will end up with an homogeneous population of k-players. Instead, if $D\pi$ moves downwards it may happen that the process of growth of k-users stops. This will occur if the new $D\pi$ falls below d at $\hat{\epsilon}$. In this last case we could have, in the ultra long-run, people in the same population using different window sizes.

The following table reports the difference in payoffs for the simulation described in section 3 but keepeng fixed the noise rate at $\epsilon = 0.8$. The proportion of players sampling $k = 3$ and $k' = 1$ are θ and $(1-\theta)$ respectively. All members from population II sample $w = 2$. The memory is $m = 4$. As θ increases (the proportion of k-strategist grows) the long run distribution put more weight on those states where the larger sample size has higher expected payoff.

θ	$D\pi(0.8;3,1)$
0	-0.00008059
0.05	-0.000071324
0.1	-0.000062045
0.2	-0.00004347
0.3	-0.000024876
0.4	$-6.261\,10^{-6}$
0.5	0.00001237
0.6	0.00003102
0.7	0.00004970
0.8	0.0000684
0.9	0.000087115
0.95	0.000096481
1	0.00010585

Table 5.

The population will evolve, depending on the initial value of θ, towards $\theta = 1$ (everybody sampling k) or towards a $\theta = 0$ (everybody sampling k'). For small initial proportions of k'-players the smallest sample size performs better and will invade the population. The opposite is true for high enough $\theta's$. No general statement can be done about evolutionary stability for non-negligible rates of noise. The ultra long run distributions will depend on the initial distribution of sample sizes, the sampling costs and the rate of noise.

6 Conclusions

In this chapter we have developed an evolutionary model of bargaining with endogenous bargaining powers. In the model there are two levels of evolution and noise. On the one hand there is the evolution of the state of the system which is driven by the adaptive play and the trembles affecting players' demands. On the other hand there is the evolution of the distribution of window sizes which is continuously perturbed by mutants who employ different learning rules. When the second level of evolution is absent our model is observationally equivalent to Young's. In this case the model predicts the negative externality exerted by poorly informed players on the whole population. This result, which is obtained under the assumption of fixed samples sizes, leaves unexplained the main determinant of the bargaining powers. The model does not explain either the co-existence of different behaviours in the populations. By allowing players to imitate more succesful behaviours we endogenize the bargaining powers. We show that there will be a tendency towards homogeneous populations. All members from the same populations will, in the ultra-long run, receive the same share. It will happen not because there is a marginal player who determines the share received by everybody but because all players behave the same way. When sampling is costless both populations tend to be informed as much as they can. If there are no differences in the informational capacities of the two populations the process converges to the symmetric Nash bargaining solution. The same is true when sampling is costly, though in this case both populations sample only one unit of information. Any asymmetry in the populations' sampling costs are not reflected in the shares received. When we compare the results with those obtained with populations of rational players we observe that the economy of myopic imitative players is more efficient. The main problem with the model is that all the results are obtained in the limiting case of very small rates of mistakes. More interesting situations are those in which the rate of mistakes are not necessarily small. In this case the symptotic results do not hold and we cannot characterize the long run distributions. Some simulations seem to suggest that a closer study of the relation between the rate of mistakes and the sampling costs is needed in order to characterize the evolutionarily stable learning rules.

7 Appendix 1

Let $p_{(i,j)}^{(k,l)}$ be the transition probability between state (i,j) and (k,l). Let us consider a memory size m and a state (m,j) and consider thet all players sample 1 unit of information.

$$p_{(m,j)}^{(m,j)} = \alpha(m,j)\delta(m,j)$$

$$p_{(m,j-1)}^{(m,j)} = \beta(m,j-1)\delta(m,j-1)$$

$$p_{(m,j+1)}^{(m,j)} = \gamma(m,j+1)\delta(m,j+1)$$

$$p_{(m-1,j)}^{(m,j)} = \alpha(m-1,j)\phi(m-1,j)$$

$$p_{(m-1,j-1)}^{(m,j)} = \beta(m-1,j-1)\phi(m-1,j-1)$$

$$p_{(m-1,j+1)}^{(m,j)} = \gamma(m-1,j+1)\phi(m-1,j+1)$$

where

$$\alpha(i,j) = (1-\epsilon)(\frac{m-i}{m}\frac{j}{m} + \frac{i}{m}\frac{m-j}{m}) + \frac{\epsilon}{2}$$

$$\beta(i,j) = (1-\epsilon)\frac{m-i}{m}\frac{m-j}{m} + \frac{m-j}{m}\frac{\epsilon}{2}$$

$$\gamma(i,j) = (1-\epsilon)\frac{i}{m}\frac{j}{m} + \frac{j}{m}\frac{\epsilon}{2}$$

$$\delta(i,j) = (1-\epsilon)(\frac{m-j}{m}\frac{i}{m} + \frac{j}{m}\frac{m-i}{m}) + \frac{\epsilon}{2}$$

$$\phi(i,j) = (1-\epsilon)\frac{m-j}{m}\frac{m-i}{m} + \frac{m-i}{m}\frac{\epsilon}{2}$$

Let $\mu = \mu(\epsilon, m)$ be the long run distribution.

$$\begin{aligned}\mu_{(m,j)} \quad = \quad & \mu_{(m,j)}p_{(m,j)}^{(m,j)} + \mu_{(m,j-1)}p_{(m,j-1)}^{(m,j)} + \\ & \mu_{(m,j+1)}p_{(m,j+1)}^{(m,j)} + \mu_{(m-1,j)}p_{(m-1,j)}^{(m,j)} + \\ & \mu_{(m-1,j-1)}p_{(m-1,j-1)}^{(m,j)} + \mu_{(m-1,j+1)}p_{(m-1,j+1)}^{(m,j)}\end{aligned}$$

Let $\mu_{(i,j)}^{m} = \lim_{m\to\infty} \mu_{(i,j)}$

$$\mu_{(m,m)}^{m} = \mu_{(m,m)}^{m}(\frac{\epsilon}{2})^2 + \mu_{(m,m-1)}^{m}(\frac{\epsilon}{2})^2$$

$$lim_{\epsilon \to 0} \frac{\mu^*_{(m,m)}}{\mu^*_{(m,m-1)}} = 0$$

Solving recursively, we obtain

$$lim_{\epsilon \to 0} \frac{\mu^*_{(m,j)}}{\mu^*_{(m,j-1)}} = 0$$

We can always find an $\tilde{\epsilon}$ such that for all $\epsilon \leq \tilde{\epsilon}$, the larger window always pays best.

8 Appendix 2

	0	1	2	3	4
0	$3.8\,10^{-6}$	0.0000446	0.000187	0.000821	0.00466
1	0.000186	0.000706	0.000908	0.000697	0.000331
2	0.00532	0.00535	0.00171	0.000285	0.0000213
3	0.1	0.0247	0.00198	0.00008	$1.24\,10^{-6}$
4	0.785	0.0656	0.00171	0.0000151	$3.17\,10^{-8}$

Table 6. $\epsilon = 0.05$, $p = 0.98149$

	0	1	2	3	4
0	0.0000186	0.000184	0.000749	0.00265	0.00745
1	0.000791	0.0026	0.00344	0.00258	0.00111
2	0.0156	0.016	0.00635	0.00122	0.000105
3	0.158	0.0565	0.0073	0.000421	$8.43\,10^{-6}$
4	0.607	0.104	0.00561	0.000102	$4.28\,10^{-7}$

Table 7. $\epsilon = 0.1$, $p = 0.939487$

	0	1	2	3	4
0	0.000109	0.000856	0.00317	0.00801	0.011
1	0.00327	0.00998	0.0136	0.00987	0.00362
2	0.0371	0.0464	0.0239	0.00576	0.000599
3	0.19	0.113	0.0247	0.00233	0.0000758
4	0.35	0.128	0.0148	0.000597	$6.15\,10^{-6}$

Table 8. $\epsilon = 0.2,\ p = 0.81265$

	0	1	2	3	4
0	0.000299	0.00207	0.00684	0.0134	0.012
1	0.00654	0.0204	0.0285	0.0201	0.00654
2	0.0503	0.0768	0.0479	0.0138	0.00165
3	0.166	0.142	0.0445	0.00596	0.000284
4	0.196	0.114	0.0215	0.00146	0.0000286

Table 9. $\epsilon = 0.3,\ p = 0.671295$

	0	1	2	3	4
0	0.000583	0.0037	0.0109	0.0172	0.0114
1	0.00961	0.0313	0.0445	0.0308	0.00916
2	0.0543	0.099	0.0724	0.0242	0.00326
3	0.129	0.148	0.0614	0.011	0.000716
4	0.109	0.0909	0.0248	0.00256	0.0000835

Table 10. $\epsilon = 0.4$, $p = 0.545698$

	0	1	2	3	4
0	0.000945	0.00558	0.0147	0.0194	0.0101
1	0.012	0.0411	0.0594	0.0405	0.0113
2	0.0522	0.112	0.094	0.0358	0.00539
3	0.0948	0.139	0.074	0.017	0.00146
4	0.061	0.0686	0.0258	0.00382	0.000193

Table 11. $\epsilon = 0.5$, $p = 0.444821$

	0	1	2	3	4
0	0.00137	0.0076	0.0179	0.0201	0.00861
1	0.0137	0.0491	0.0718	0.0485	0.0128
2	0.0471	0.117	0.111	0.048	0.00799
3	0.0677	0.124	0.0826	0.024	0.00263
4	0.0345	0.0508	0.0256	0.00526	0.00039

Table 12. $\epsilon = 0.6$, $p = 0.367024$

	0	1	2	3	4
0	0.00187	0.00966	0.0203	0.0198	0.00719
1	0.0148	0.0552	0.0815	0.0547	0.014
2	0.0409	0.116	0.125	0.0601	0.0111
3	0.0476	0.107	0.0881	0.0319	0.00437
4	0.0198	0.0374	0.0249	0.00698	0.00073

Table 13. $\epsilon = 0.7$, $p = 0.307522$

	0	1	2	3	4
0	0.00245	0.0117	0.0221	0.0188	0.00592
1	0.0154	0.0594	0.0884	0.059	0.0148
2	0.0346	0.111	0.134	0.0719	0.0146
3	0.0332	0.0907	0.0914	0.0408	0.00689
4	0.0115	0.0277	0.0241	0.00912	0.0013

Table 14. $\epsilon = 0.8$, $p = 0.261521$

	0	1	2	3	4
0	0.00312	0.0137	0.0231	0.0173	0.00483
1	0.0157	0.0618	0.0924	0.0616	0.0153
2	0.0287	0.103	0.139	0.0832	0.0187
3	0.0229	0.0757	0.0932	0.0509	0.0105
4	0.00668	0.0207	0.0236	0.0119	0.00227

Table 15. $\epsilon = 0.9$, $p = 0.225114$

	0	1	2	3	4
0	0.0035	0.0147	0.0234	0.0165	0.00435
1	0.0157	0.0623	0.0934	0.0622	0.0155
2	0.026	0.0986	0.14	0.0886	0.021
3	0.0189	0.0689	0.0936	0.0565	0.0128
4	0.00511	0.018	0.0235	0.0136	0.00298

Table 16. $\epsilon = 0.95$, $p = 0.209543$

	0	1	2	3	4
0	0.00391	0.0156	0.0234	0.0156	0.00391
1	0.0156	0.0625	0.0938	0.0625	0.0156
2	0.0234	0.0938	0.141	0.0938	0.0234
3	0.0156	0.0625	0.0938	0.0625	0.0156
4	0.00391	0.0156	0.0234	0.0156	0.00391

Table 17. $\epsilon = 1$, $p = 0.195312$

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